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SEISMIC BEARING CAPACITY FACTORS FOR SHALLOW FOUNDATIONS THROUGH DIFFERENT METHODS OF ANALYSIS

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ABSTRACT

Past earthquakes provided evidence of the susceptibility of shallow foundations to undergo large deformations and failure also in situations where liquefaction was not triggered. Several examples of these failures occurred after the 1971 San Fernando earthquake, the 1978 Miyagiken-Oki earthquake and the Michoacan earthquake that shook Mexico City in 1985. Experimental evidence of bearing capacity failures similar to those observed in the field was also provided by shaking table and centrifuge model tests on shallow foundations under dynamic excitation.
In this study, the evaluation of earthquake-induced reduction of the bearing capacity is carried out for a shallow strip foundation using two different approaches, both relying on the application of the method of characteristics, extended to the seismic loading condition through the pseudo-static approach.
Empirical equations are available in the literature to compute corrective coefficients of static bearing capacity factors that allow considering separately the effects of soil inertia and superstructure inertia. The corrective factors provided by such equations, however, are less conservative than those obtained in the present paper.

Keywords: Foundations, seismic analysis, bearing capacity, kinematic effect, inertial effect

INTRODUCTION

Bearing capacity of shallow foundations is currently evaluated using the superposition formula introduced by Terzaghi (1943):

$$q_{\mathrm{ult}} = qN_{\mathrm{q}} + c'N_{\mathrm{c}} + \frac{1}{2}\gamma BN_{\gamma} \qquad (1)$$

where q_{ult} is the bearing capacity, i.e. the ultimate load that the soil can sustain under the assumption of rigid plastic behaviour. N_c, N_q and N_γ are the bearing capacity factors, depending on the angle of shear strength φ of the soil; γ and c are the unit weight and the cohesion of the soil, respectively; B is the width of the foundation and q is the vertical pressure acting on the ground surface aside the foundation. Equation (1) provides the bearing capacity of a strip footing resting on a homogeneous soil obeying the Mohr-Coulomb failure criterion and subjected to a vertical and uniformly distributed load, assuming that both the ground surface and the foundation base are horizontal.

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More general equations for the bearing capacity and modified expressions of the bearing capacity factors have been proposed over the years, taking into account, by means of corrective factors, the shape and the depth of the foundation, the inclination of the ground and of the foundation base, the inclination and the eccentricity of the applied load (Meyerhof, 1963; Brinch-Hansen, 1970; Vesic, 1973).
These solutions, available for both drained and undrained conditions, still form the basis of current design of shallow foundations and in seismic prone regions load inclination factors are commonly used to account for the effects of earthquake-induced inertia forces on bearing capacity.

Although earthquake-induced failures of shallow foundations are related to the occurrence of liquefaction of saturated sand deposits, past earthquakes provided evidence of the susceptibility of shallow foundations to undergo large deformations and failure also in situations where liquefaction was not triggered. Richards *et al.* (1993) reported several examples of these failures occurred after the 1971 San Fernando earthquake and the 1978 Miyagiken-Oki earthquake. Failures of shallow foundations occurred also during the Michoacan earthquake that shook Mexico City in 1985 (Mendoza & Auvinet, 1988).

Experimental evidence of bearing capacity failures of shallow foundations similar to those observed in the field was also provided by centrifuge model tests (e.g. Zeng & Steedman, 1998) and by shaking table tests (e.g. Knappett *et al.*, 2006).

During an earthquake, seismic waves travelling in the soil induce inertia forces in the soil mass underneath the foundation, thereby increasing the stress level. A further fraction of soil strength is required to balance shear stresses arising from inertia forces developing in the superstructure and transmitted to the soil.

Since earthquake shaking is a transient action, characterised by sudden changes in the amplitude of the motion, forces acting on the foundation or within the soil mass can attain the available resistance of the soil for short time intervals, without necessarily causing a general failure of the foundation but rather producing the accumulation of permanent vertical displacements.

Modern studies on seismic bearing capacity were initiated by Richards *et al.* (1990) and by Sarma & Iossifelis (1990). Since then, several studies were carried out approaching the problem by means of the three classical methods of stability analysis, namely, limit equilibrium (e.g Richards *et al.*, 1993; Choudhury & Subba Rao, 2005), limit analysis (e.g. Dormieux & Pecker, 1995; Conte, 1996; Paolucci & Pecker, 1997 a,b; Soubra, 1999) and the method of characteristics (e.g. Kumar & Mohan Rao, 2002; Maugeri & Novità, 2004; Cascone *et al.*, 2006). As a general result of these studies, under seismic conditions the bearing capacity (and more specifically the bearing capacity factors) undergoes a dramatic reduction as a consequence of inertia forces arising in both the soil and the structure. It is also generally recognised that the most significant reduction on the bearing capacity factors is due to inertia forces acting on the superstructure, the kinematic effect being negligible. In most of the available solutions, however, the two effects, kinematic and inertial, cannot be uncoupled and horizontal accelerations acting in the soil and on the superstructure are unrealistically assumed to be in phase and have the same magnitude, despite the soil and the superstructure are likely to exhibit different responses to a given seismic excitation.
Shi & Richards (1995) obtained solutions for the seismic bearing capacity factors following two different approaches: the method of characteristics and the method of limit equilibrium on a Coulomb failure mechanism with sliding active and passive wedge. In their analyses, kinematic and inertial effects were separated by introducing a shear transfer coefficient f, defined as the fraction of the horizontal inertia force acting on the superstructure that is actually transferred to the foundation. For $f = 0$ only the kinematic effect is taken into account while for $f = 1$ both kinematic and inertia effect are considered.

Shi & Richards (1995) showed that for isolated footings, and typically for bridge pier foundations, the coefficient f can assume values other than unity. In fact, the shear force at the base can be magnified by structural indeterminacy or reduced by base isolation. The results obtained following the characteristics and the limit equilibrium method showed a very good agreement and design charts determined through the limit equilibrium method were provided.

In this study, the evaluation of earthquake-induced reduction of the bearing capacity factors is carried out for a shallow strip foundation on a Mohr-Coulomb material under drained conditions using two different numerical approaches for the integration of the equations of plastic equilibrium, both relying on the application of the method of characteristics, extended to the seismic loading condition through the pseudo-static approach. The main difference in the integration of governing equations lies in the use of auxiliary variables in the former, more classical approach, while the latter tackles directly the integration without resorting to additional variables.

The kinematic effect, due to inertia forces in the soil, and the inertial effect, due to inertia forces in the superstructure, are separated in the analysis, thus allowing to apply different seismic acceleration coefficients in the evaluation of the inertia forces in the soil and in the superstructure.
Empirical equations, which satisfactorily approximate the numerical results, are provided to compute corrective coefficients of static bearing capacity factors allowing considering separately the effects of soil inertia and superstructure inertia. These equations can be introduced in current expressions of the bearing capacity to extend their applicability to seismic design of foundations.

METHODS OF ANALYSIS AND GOVERNING EQUATIONS

The governing equations are developed under the assumptions previously summarized:
(i) the soil exhibits rigid plastic behaviour and obeys the Mohr Coulomb criterion;
(ii) both the ground surface and the foundation base are horizontal;
(iii) the foundation is a strip footing subjected to a vertical and uniformly distributed load.
As it will be subsequently shown, the analyses have accounted for a load inclination other than zero, and can easily be extended by removing assumption (ii).

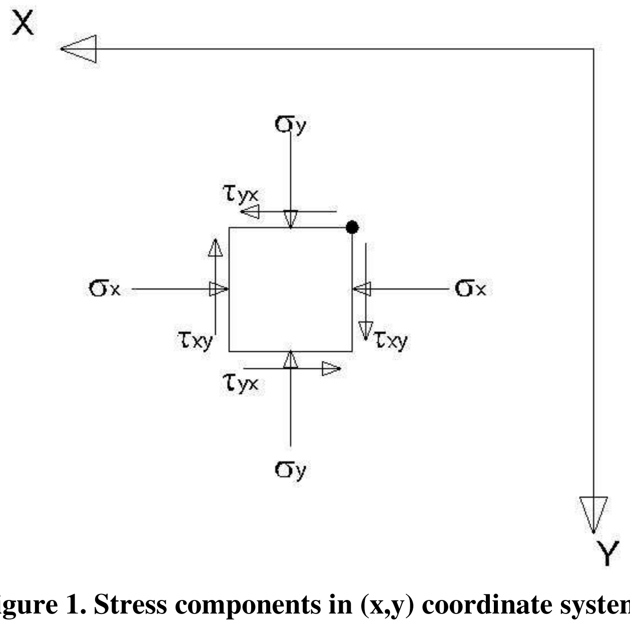

Figure 1. Stress components in (x,y) coordinate system

In a soil mass at incipient failure, both equilibrium equations and the failure condition must be satisfied. The resulting set of equations can be solved to obtain the ultimate load on a foundation. In two dimensions, the differential equations of equilibrium expressed in (x,y) coordinate system (fig. 1) are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = X \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = Y \tag{2a,b}$$

where the terms X and Y are body forces per unit volume. Stress components can be expressed by means of p and R, respectively the centre and the radius of Mohr's circle, as:

$$\begin{aligned} \sigma_x &= p + R\cos 2\psi \\ \sigma_x &= p - R\cos 2\psi \\ \tau_{xy} &= -R\sin 2\psi \end{aligned} \tag{3a,b,c}$$

as shown in fig. 2, while the Mohr-Coulomb criterion can be expressed in terms of p and R as:

$$R = p\sin\varphi + c\cos\varphi \tag{4}$$

Introducing eq. (4) in eqs. (3) yields:

$$\begin{aligned} \sigma_x &= p(1+\sin\varphi\cos 2\psi) + c\cos\varphi\cos 2\psi \\ \sigma_x &= p(1-\sin\varphi\cos 2\psi) - c\cos\varphi\cos 2\psi \\ \tau_{xy} &= -p\sin\varphi\cos 2\psi - c\cos\varphi\sin 2\psi \end{aligned} \tag{5a,b,c}$$

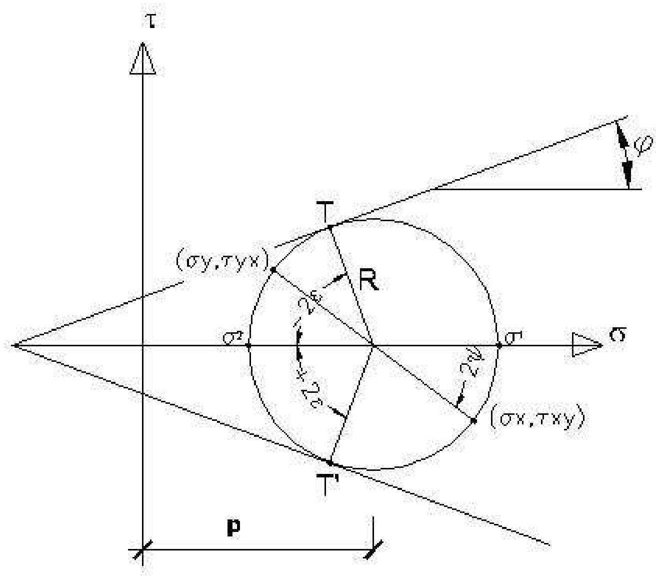

Figure 2. Mohr's circle at failure

Equilibrium equations (2) thus become:

$$\begin{aligned}&\frac{\partial p}{\partial x}(1+\sin\varphi\cos 2\psi)-2\sin 2\psi(p\sin\varphi+c\cos\varphi)\frac{\partial\psi}{\partial x}+\\&\qquad-\frac{\partial p}{\partial y}\sin\varphi\sin 2\psi-2\cos 2\psi(p\sin\varphi+c\cos\varphi)\frac{\partial\psi}{\partial y}=X\\&-\frac{\partial p}{\partial x}\sin\varphi\sin 2\psi-2\cos 2\psi(p\sin\varphi+c\cos\varphi)\frac{\partial\psi}{\partial x}+\\&\qquad+\frac{\partial p}{\partial y}(1-\sin\varphi\cos 2\psi)+2\sin 2\psi(p\sin\varphi+c\cos\varphi)\frac{\partial\psi}{\partial y}=Y\end{aligned} \tag{6a,b}$$

Following Sokolovskii (1965), equations (6) can be simplified introducing χ, defined as:

$$\chi=\frac{\cot\varphi}{2}\ln\frac{\sigma}{\sigma_r} \tag{7}$$

where σ_r is an arbitrary reference value and σ is linked to p by eq. (8):

$$\sigma=p+c\cot\varphi \tag{8}$$

The definition of χ implies that:

$$\frac{\partial\sigma}{\partial x}=\frac{2\sigma}{\cot\varphi}\frac{\partial\chi}{\partial x}\qquad\qquad\frac{\partial\sigma}{\partial y}=\frac{2\sigma}{\cot\varphi}\frac{\partial\chi}{\partial y} \tag{9a,b}$$

Multiplying by (cotφ)/(2σ), eqs. (6) are transformed into eqs. (10):

$$\begin{aligned}&(1+\sin\varphi\cos 2\psi)\frac{\partial\chi}{\partial x}-\sin\varphi\sin 2\psi\frac{\partial\chi}{\partial y}-\cos\varphi\left(\sin 2\psi\frac{\partial\psi}{\partial x}+\cos 2\psi\frac{\partial\psi}{\partial y}\right)=\frac{\cot\varphi}{2\sigma}X\\&-\sin\varphi\sin 2\psi\frac{\partial\chi}{\partial x}+(1-\sin\varphi\cos 2\psi)\frac{\partial\chi}{\partial y}-\cos\varphi\left(\cos 2\psi\frac{\partial\psi}{\partial x}-\sin 2\psi\frac{\partial\psi}{\partial y}\right)=\frac{\cot\varphi}{2\sigma}Y\end{aligned} \tag{10a,b}$$

Under static conditions, the body forces X and Y are respectively equal to 0 and γ, where γ is the soil unit weight, while in pseudostatic conditions they can be expressed as:

$$X=\gamma^*\sin\vartheta\qquad\qquad Y=\gamma^*\cos\vartheta \tag{11a,b}$$

where:

$$\gamma^*=\gamma\sqrt{(1-k_v)^2+k_h^2}\qquad\qquad\vartheta=\tan^{-1}\left(\frac{k_h}{1-k_v}\right) \tag{12a,b}$$

In eqs. (11) and (12) ϑ is the inclination of γ^* to the vertical direction (y axis), k_h e k_v being the horizontal and vertical seismic acceleration coefficients.

Governing equations (6) and their equivalent form (10) have been solved numerically, both in static and pseudo-static conditions, following two different approaches (hereafter referred to as approach *A* and approach *B*), both originally developed in static conditions, by Sokolovskii (1965) and De Simone (1989), respectively.

Approach *A* operates on eqs. (10), introducing a pair of auxiliary variables, namely (ξ, η), while approach *B* tackles directly eqs. (6), without introducing any auxiliary variable. The authors have extended both approaches to pseudostatic conditions, using eqs. (11) and (12); analytical details are respectively provided in Cascone *et al*. (2006) and Cillo (2009).
Before illustrating some results of the numerical analyses, attention should be paid to the definition of the arbitrary reference stress σ_r, which has been defined as:

$$\sigma_r = q + c + \frac{\gamma B}{2} \tag{13}$$

Since in both analyses the ultimate load q_{ult} is eventually determined in dimensionless form as a fraction of σ_r, namely:

$$N_{qc\gamma} = q_{ult} / \sigma_r \tag{14}$$

it follows that the dimensionless factor $N_{qc\gamma}$ respectively corresponds to N_q when $c = \gamma = 0$, to N_c when $q = \gamma = 0$, to N_γ when $q = c = 0$. If static conditions are assumed ($k_h = k_v = 0$), then the analyses are expected to provide the classical values of N_q, N_c and N_γ.

RESULTS OF NUMERICAL ANALYSES

Comparison with benchmark solutions in static conditions

The accuracy of both numerical procedures has been preliminarily evaluated in static conditions against some classical benchmark solutions available for bearing capacity factors.
In static conditions, N_q and N_c factors and their corresponding inclination factors i_q and i_c can be obtained in closed form respectively as:

$$N_q = \frac{1+\sin\varphi}{1-\sin\varphi}\exp(\pi\tan\varphi) \qquad N_c = (N_q - 1)\cot\varphi \tag{15a,b}$$

$$i_q = \frac{1+\sin\varphi\cos(\Delta+\delta)}{1+\sin\varphi}\exp[-(\Delta+\delta)\tan\varphi] \qquad i_c = \frac{N_q i_q - 1}{N_q - 1} \tag{16a,b}$$

In eqs. (16) Δ is the Caquot angle, linked to load inclination δ by the following equation:

$$\Delta = \arcsin\frac{\sin\delta}{\sin\varphi} \tag{17}$$

The values of N_q and N_c obtained by means of both (*A* and *B*) numerical procedures, subdividing the Cauchy zone by means of 120 points, coincide perfectly with the analytical values provided by equations (15a) and (15b).
Perfect agreement between analytical and numerical values for both inclination factors has also been obtained.

Since the bearing capacity factor N_γ is not amenable to analytical solutions, comparisons have been carried out with numerical values provided by Absi and Kérisel (1984), which account also for the inclination of both ground and foundation base, as shown in fig. 3. When angle ω equals 90°, the bearing capacity problem of the vertical foundation shown in fig. 4 can be regarded also as a passive thrust problem on a vertical wall, which is actually the problem tackled by Absi & Kérisel (1984).

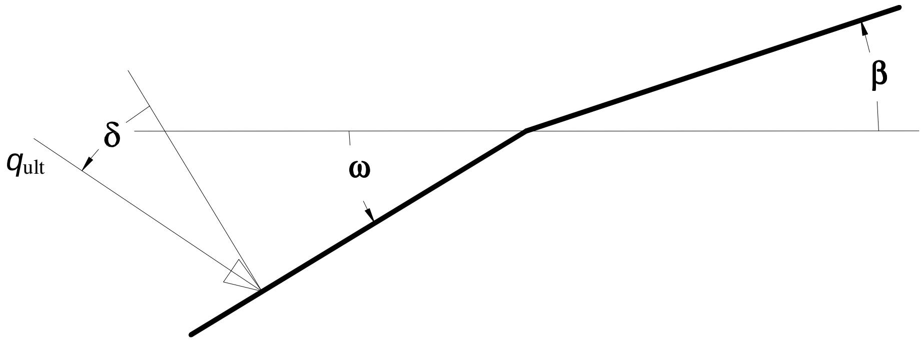

Figure 3. Ground and foundation base inclinations

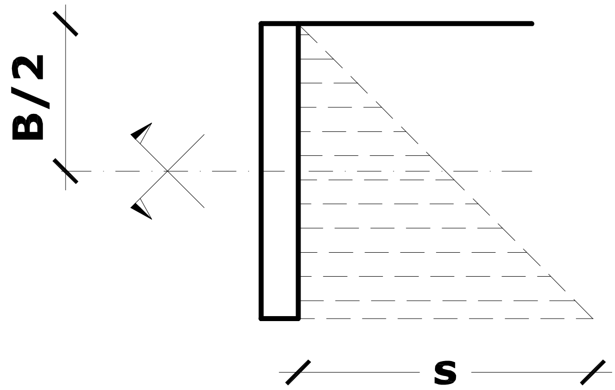

Figure 4. Bearing capacity problem as a passive thrust problem

Some results obtained by means of both approaches (*A* and *B*) are compared with the benchmark solutions provided by Absi & Kérisel (1984) in table 1, for δ = 0 and for various values of φ and ω, showing a satisfactory agreement, since mean error is less than 2%. The comparison between the results obtained in the present study and the benchmark solution is also shown in graphical form in fig. 5, where the results provided by approach *B* for the case ω = 30° and δ/φ = 1/3 are shown.

Table 1. Factor N_γ: comparison between Absi & Kérisel (A&K) and present solutions (*A* and *B*)

	ω = 30°			ω = 25°			ω = 20°			ω = 15°		
	N_γ			N_γ			N_γ			N_γ		
φ	**A&K**	**A**	**B**	**A&K**	**A**	**B**	**A&K**	**A**	**B**	**A&K**	**A**	**B**
10	1.25	1.24	1.26	1.18	1.15	1.18	1.09	1.06	1.08	–	–	–
15	2.00	1.97	2.00	1.94	1.92	1.94	1.87	1.85	1.87	1.78	1.77	1.78
20	3.20	3.17	3.21	3.20	3.21	3.24	3.30	3.24	3.26	3.20	3.24	3.26
25	5.30	5.22	5.26	5.50	5.49	5.52	5.80	5.76	5.79	6.10	6.04	6.07
30	8.90	8.87	8.92	9.70	9.69	9.73	10.60	10.60	10.63	11.60	11.60	11.63
35	15.80	15.75	15.81	18.00	17.92	17.98	20.50	20.43	20.48	23.50	23.34	23.37
40	30.00	29.76	29.83	35.00	35.36	35.42	42.00	42.11	42.16	50.00	50.26	50.31
45	60.00	61.11	61.21	76.00	76.15	76.24	95.00	95.11	95.19	118.00	119.03	119.10

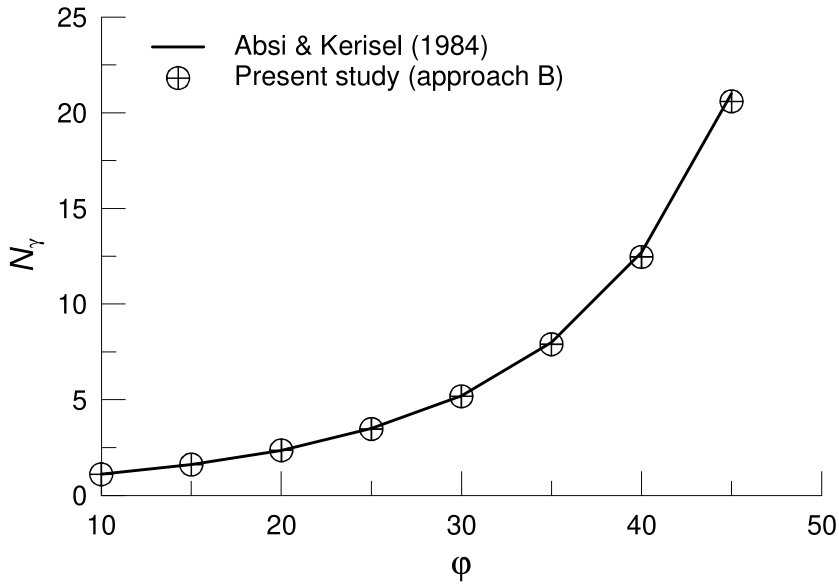

Figure 5. Factor N_γ: comparison between Absi & Kérisel and present solutions (approach *B*)

Results for pseudostatic conditions

Results for pseudostatic conditions are presented only for N_γ factor, which will be referred to as $N_{\gamma E}$ to distinguish it from its static value.

Both approaches allow treating separately kinematic effects, due to soil motion, and inertial effects, depending on inertia forces transmitted by the superstructure.

Kinematic effects are analyzed assuming no load inclination and coefficient $k_h = k_{hk}$ (see fig. 6a), while inertial effects are analyzed assuming $\tan\delta = k_{hi}$ and $k_{hk} = 0$, as shown in fig. 6b. Both effects are expressed as a ratio of the corresponding value of $N_{\gamma E}$ (which can thus be defined respectively as $N_{\gamma Ek}$ and $N_{\gamma Ei}$) to the static value N_γ, i.e., as reduction factors:

$$e_{\gamma k} = \frac{N_{\gamma Ek}}{N_\gamma} \qquad e_{\gamma i} = \frac{N_{\gamma Ei}}{N_\gamma} \tag{18a,b}$$

Cascone *et al.* (2006) have also shown that reduction factors can be superimposed, i.e.:

$$N_{\gamma E} = N_{\gamma} e_{\gamma k} e_{\gamma i} \tag{19}$$

The results obtained by the two approaches agree fairly well. As an example, values of reduction factors under kinematic and inertial effects provided by both approaches *A* and *B* are reported in table 2.

In graphical form, fig. 7 shows the characteristics net: (a) in static conditions; (b) under kinematic effects; (c) under inertial effects, for a given set of values. Both kinematic and inertial effects alter significantly the shape of the failure domain obtained under static conditions; in addition, inertial effects cause a sharp reduction of the extension of failure domain, resulting in extremely low values of the reduction factor $e_{\gamma i}$.

In routine design, as an alternative to numerical analyses, simple approximating equations have been proposed. As an example, Cascone *et al.* (2006) have suggested eqs. 20:

$$e_{\gamma k} = \left(1 - \frac{k_{hk}}{\tan\varphi}\right)^{0.45} \qquad e_{\gamma i} = \left(1 - 0.7 k_{hi}\right)^{5} \tag{20a,b}$$

where eq. (20b) is the well known formula proposed by Brinch Hansen (1970). It has to be pointed out, however, that the results obtained by approaches *A* and *B* are more conservative than those provided by eqs. 20.

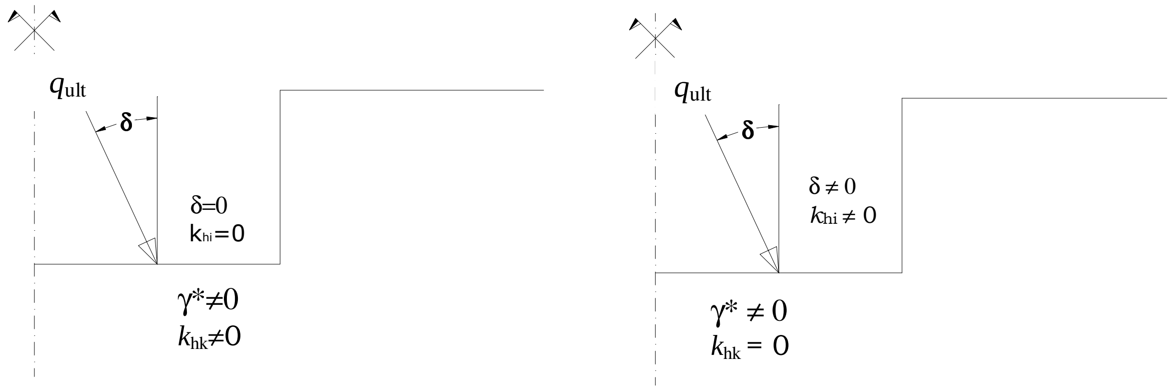

Figure 6. (a) Modelling kinematic effects; (b) Modelling inertial effects

Table 2. Numerical values of reduction factors

φ = 30°		N_γ = 7.65			φ = 30°		N_γ = 7.65		
k_{hk}	δ	$N_{\gamma Ek}$ (B)	$e_{\gamma k}$ (A)	$e_{\gamma k}$ (B)	k_{hi}	δ(°)	$N_{\gamma Ei}$ (B)	$e_{\gamma i}$ (A)	$e_{\gamma i}$ (B)
0	0	7.65	1.00	1.00	0	0	7.65	1.00	1.00
0.1	0	6.84	0.89	0.89	0.1	5.71	5.28	0.70	0.69
0.2	0	5.94	0.76	0.78	0.2	11.31	3.43	0.46	0.45
0.3	0	4.96	0.62	0.65	0.3	16.70	2.08	0.30	0.27
0.4	0	3.90	0.48	0.51	0.4	21.80	1.17	0.18	0.15

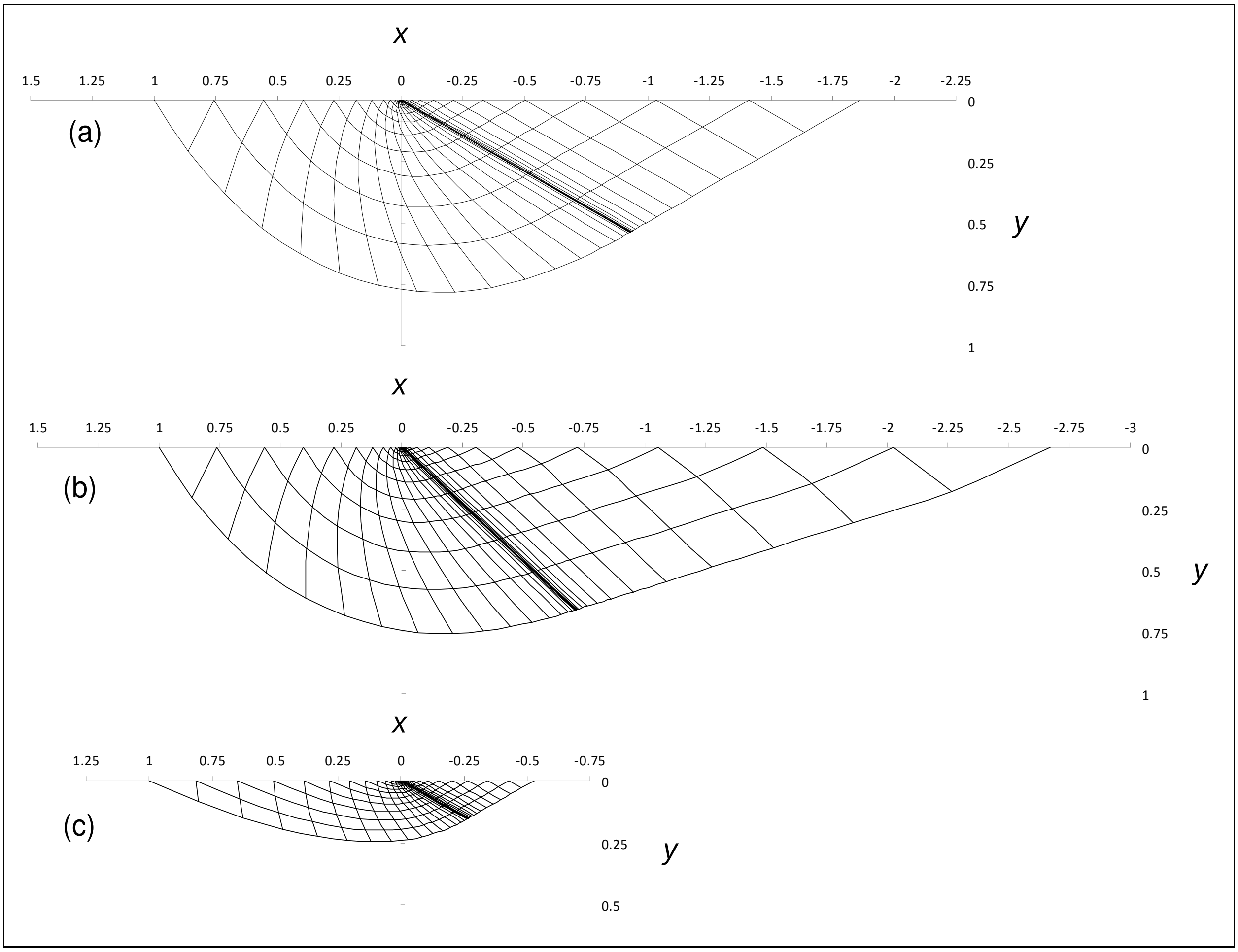

Figure 7. Characteristics net: (a) static conditions; (b) kinematic effects; (c) inertial effects

CONCLUDING REMARKS

The classical problem of bearing capacity of a shallow foundations resting on a Mohr–Coulomb medium has been extended to account for earthquake actions, modelled through the pseudostatic method, and treating separately inertia effects induced in the soil and in the superstructure.

Two different approaches have been developed to solve plastic equilibrium equations, both based on the method of characteristics.

Preliminarily, the results obtained using both approaches have been checked against benchmark solutions in static conditions, and excellent agreement has been found.

In pseudostatic conditions, corrective coefficients of the N_γ factor have been obtained accounting for kinematic and inertial effects separately and resulted to be more conservative than those provided by simple approximate expressions commonly used in routine analyses.

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