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SEISMIC GROUND RESPONSE ANALYSIS: COMPARISON BETWEEN NUMERICAL SIMULATIONS AND OBSERVED ARRAY DATA

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ABSTRACT

In this paper the free-field response at the Large-Scale Seismic Test (LSST) site in Lotung, Taiwan, during the earthquake of May 1986 is studied, comparing the downhole motions recorded in-situ with the results of simple equivalent-linear visco-elastic analyses and advanced finite element numerical simulations. The comparison between recorded array data and numerical predictions obtained with approaches of different level of complexity allows to highlight the limits and benefits of the adopted numerical schemes and to assess the performance of the advanced non-linear approach with respect to the traditional equivalent-linear model.

Keywords: seismic ground response analysis, Lotung experiment site, constitutive models, numerical modelling

INTRODUCTION

Ground response analysis has traditionally been performed using one-dimensional frequency-domain numerical schemes based on the equivalent visco-elastic approach (Schnabel *et al.*, 1972; Idriss *et al.*, 1973). This approach has successfully been adopted in the last thirty years and it is widely accepted in the engineering practice, although its limitations are well-known.

Time-domain finite element (FE) schemes are nowadays available to solve the wave propagation problem in a more realistic way, accounting for the solid-fluid interaction by means of a coupled effective stress formulation (Zienkiewicz *et al.*, 1999). In those schemes, the behaviour of the soil can be described using either simple or sophisticated non-linear constitutive models. Such approaches are seldom adopted in engineering practice by non-expert users because both the model calibration procedures and the code usage protocols are often unclear or poorly documented.

In this paper the free-field response at the Large-Scale Seismic Test (LSST) site in Lotung, Taiwan, during the LSST7 event of May 1986 is studied, comparing the downhole motions recorded in-situ (Tang, 1987) with the results of simple and advanced numerical simulations. In particular, the simple approach includes 1D numerical analyses performed adopting the equivalent-linear scheme and 2D finite element simulations using a visco-elastic soil model (Amorosi *et al.*, 2010). In the advanced approach, the soil mechanical behaviour is described by a mixed isotropic-kinematic hardening model (Kavvasdas and Amorosi, 2000) implemented in a fully-coupled effective stress FE code.

The comparison between in-situ measurements and numerical predictions obtained with approaches of different level of complexity allows to highlight the limits and benefits of the adopted numerical schemes

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and to assess the performance of the advanced non-linear approach with respect to the traditional equivalent-linear model. The influence of plasticity on the numerical results is investigated with particular reference to the relation between the hysteretic and viscous damping effects and to the prediction of excess pore water pressures within the soil deposit during the seismic action.

THE CASE STUDY: LOTUNG LSST SITE

The Lotung Large-Scale Seismic Test (LSST) site is located in one of the most active region in the North-East of Taiwan. It was established in 1985 to study the dynamic behaviour and soil-structure effects of two scaled-down nuclear plant containment structures (1/4-scale and 1/12-scale models) constructed by the Electric Power Research Institute (EPRI) and the Taiwan Power Company (Tang *et al.*, 1990). The site response has been monitored by a number of surface and down-hole triaxial accelerometer arrays (Figure 1), down to a depth of 47 m, and pore pressure transducers. The down-hole accelerometers have been installed at depths of 0, 6, 11, 17 and 47 m, oriented in N-S, E-W and vertical (V) directions.

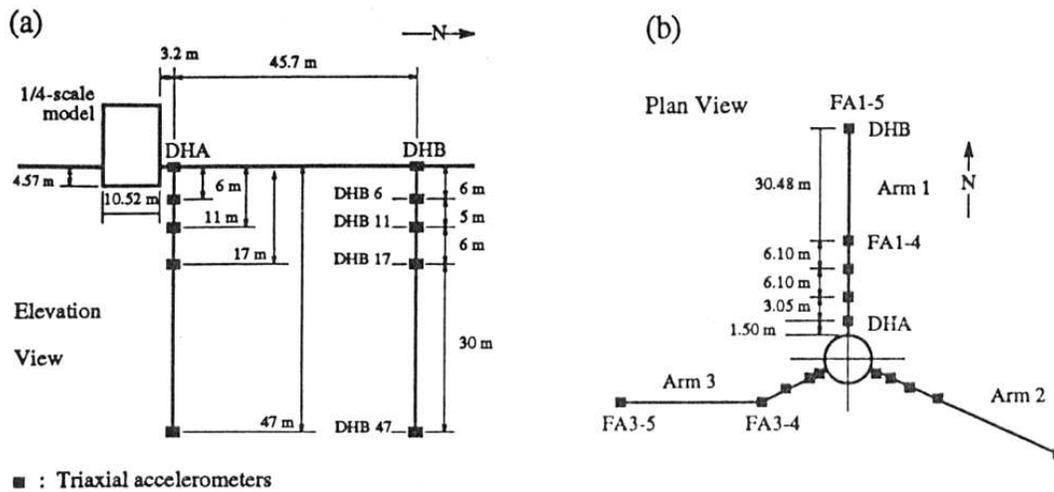


Figure 1. Instrumentation of Lotung experiment site: (a) down-hole arrays; (b) surface arrays (from Elgamal *et al.*, 1995).

In this paper the ground motion recorded during the LSST7 event occurred on May 1986 (Table 1) is investigated by analysing the response of the “free-field” down-hole array DHB (i.e. the more distant array from the surface structures). This earthquake was selected due to the strong-motion characteristics of the two horizontal components.

The site geology consists of recent alluvium and Pleistocene materials over a Miocene basement. Field explorations show an upper alluvial layer (30-40 m thick), constituted by inter-layered silty-sand and sandy-silt with gravel (Anderson, 1993). The soil beneath consists mainly of clayey-silt and silty-clay. The Miocene basement rock is placed approximately 400 m below the ground surface (EPRI, 1993). The water table is located approximately at a depth of 1 m (Elgamal *et al.*, 1995). The local geological profile near the site of the 1/4-scale model shows a first layer of gray silty-sand and sandy-silt about 20 m thick,

Table 1. Characteristics of the LSST7 event (from Elgamal *et al.*, 1995)

Date	Magnitude (M_l)	Epicentral distance (km)	Peak accelerations for the DHB array		
			E-W (g)	N-S (g)	V (g)
20 May 1986	6.5	66.2	0.16	0.21	0.04

underlain by about 10 m of more gravelly layer resting on a thick deposit of silty clay, as indicated by the SPT log profile reported in Figure 2(a).

A series of geophysical seismic tests were performed to measure shear and compression wave velocities at the LSST site. Figure 2(b) shows the shear wave velocity profile obtained from the results of seismic cross-hole and up-hole tests conducted at the 1/4-scale structure location (Anderson and Tang, 1989). The shear wave velocity V_s assumes a value of about 100 m/s at the surface reaching a value of about 300 m/s at 60 m depth. The corresponding elastic shear modulus profile with depth, shown in Figure 2(c), was developed by Borja *et al.* (1999) assuming a total unit weight of 19.0 kN/m³ for the silty-clayey layer and a unit weight of 19.5 kN/m³ for the more gravelly layer.

Shear modulus and damping ratio curves were obtained from laboratory testing of Lotung soil samples by Anderson and Tang (1989) and subsequently by Stokoe (EPRI, 1993) through accurate resonant column and cyclic torsion tests on undisturbed specimens. As an alternative approach, Zeghal *et al.* (1995) proposed to back-figure the in-situ moduli ratio curves for Lotung soil based directly on its seismic response recorded along the down-hole arrays during 18 earthquakes occurred between 1985 and 1986. The technique, consisting in the double integration of the recorded acceleration time histories to obtain the absolute displacements and finite-differencing the depth to obtain the corresponding shear strain histories, allows to reduce the effects of sample disturbance and the uncertainties in laboratory testing, giving accurate results in the low-strain range (cyclic shear strain < 0.05%). Borja *et al.* (2000) adopted the G/G_0 and $D-\gamma$ curves developed by Zeghal *et al.* (1995) for the finite element simulation of the ground response at Lotung LSST site, assuming different sets of curves for the depths of 0-6, 6-11 and 11-17 m and a bedrock formation at the depth of 47 m. In particular, they considered the shear moduli and damping ratios developed from LSST7, LSST12 and LSST16 events, producing for each depth a least-square best fit as well as an upper and a lower bound curve indicative of the possible variations in the material dynamic properties. As the available data refer to the first 17 m only, the soil properties from 17-47 m depth were assumed to be the same as those from 11-17 m (Figure 2(d)).

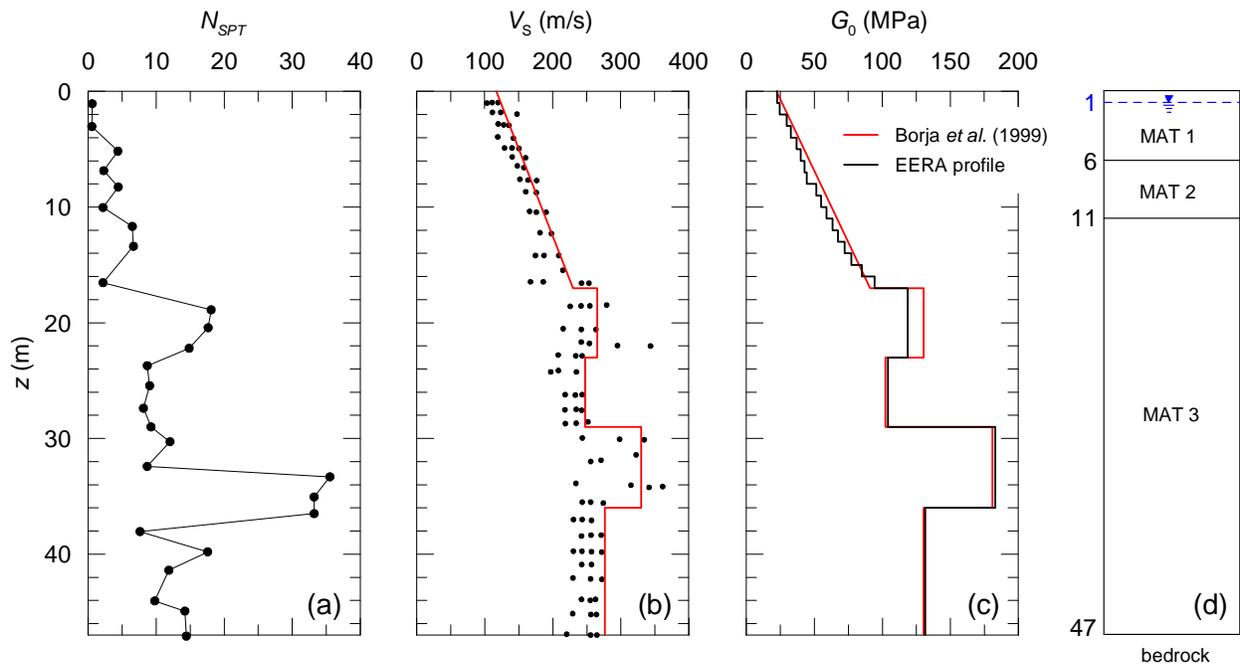


Figure 2. Local soil profile at LSST site: (a) SPT log; (b) shear wave velocity; (c) elastic shear modulus; (d) assumed geotechnical model.

NUMERICAL MODELS

The ground response analyses of the Lotung experiment site during the LSST7 event were performed using the equivalent-linear visco-elastic code EERA (Bardet *et al.*, 2000) and the finite element (FE) code SWANDYNE II (Chan, 1995), applying the two horizontal (E-W and N-S) components of the earthquake separately.

1D Equivalent-Linear Visco-Elastic Model

The code EERA is based on the assumption of equivalent-linear visco-elastic soil behaviour. The approach assumes that the shear modulus G and damping ratio D are function of the cyclic shear strain amplitude γ . The equivalent-linear analysis is repeated with updated values of G and D until the values of G and D are compatible with the so-called effective shear strain induced in all the layers of the numerical model.

In the presented EERA analyses the profile of small-strain stiffness shown in Figure 2(c) was discretised by constant stiffness sub-strata of 1 m thickness.

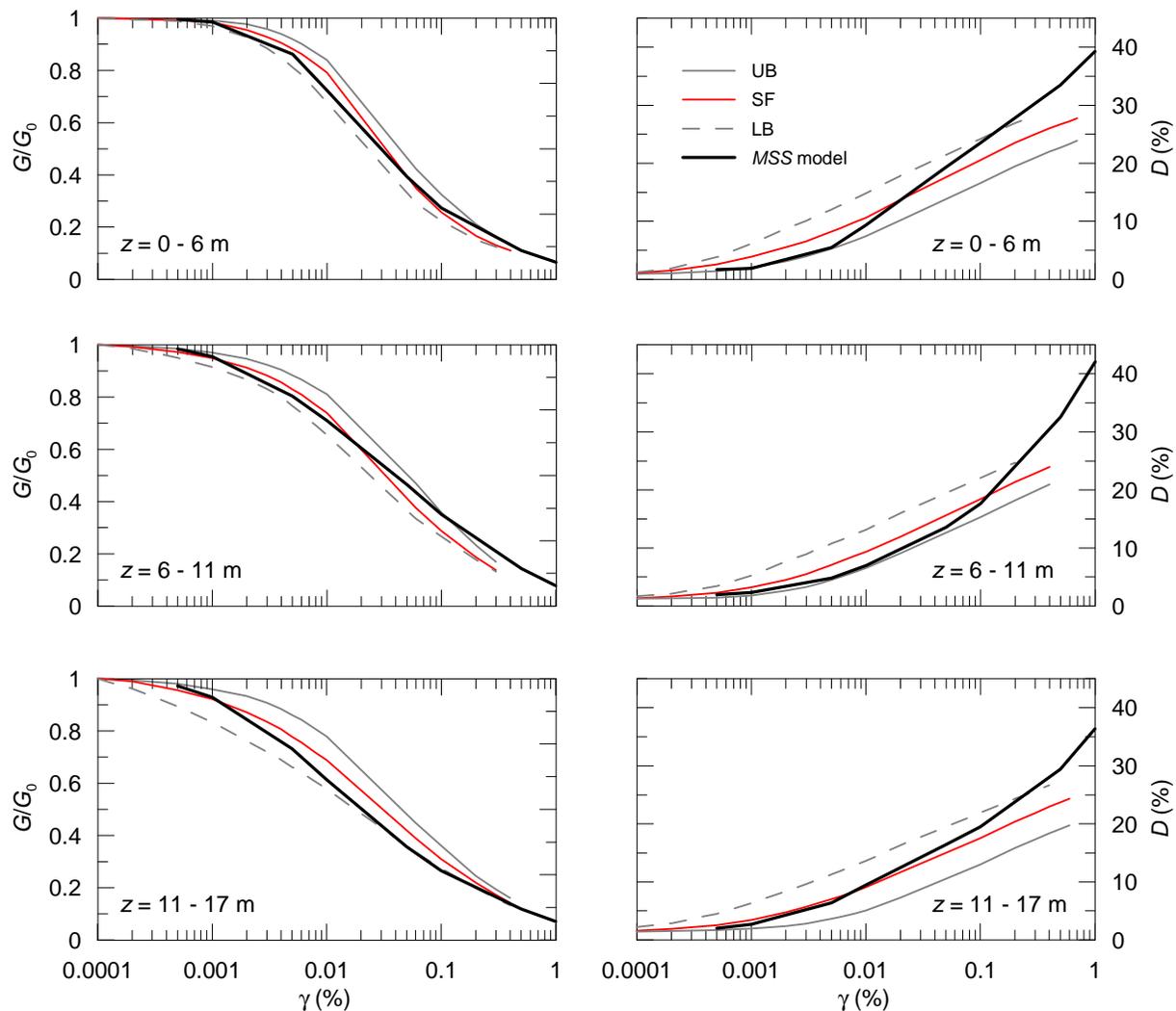


Figure 3. G/G_0 and D - γ curves adopted by Borja *et al.* (2000) and in this paper.

Figure 3 shows the modulus reduction curves G/G_0 and the variation of damping ratio D with shear strain level γ adopted by Borja *et al.* (2000) and already discussed in the previous section. As a good prediction of the peak accelerations was obtained by Borja *et al.* (2000) in both the E-W and N-S directions only using the statistical fit (SF) curves to describe the moduli ratio degradation, those curves (reported in red in Figure 3) were adopted in the equivalent-linear visco-elastic simulations presented in this work. From 17 to 47 m the same curves as those relevant to the depth 11-17 m are considered. EERA simulations were performed also using the upper bound (UB) and lower bound (LB) curves, but the best results were obtained adopting the SF curves, as discussed later.

2D Finite Element Visco-Elastic model

SWANDYNE II is a FE code that allows to perform linear and non-linear dynamic analyses: the Newton-Raphson integration scheme to solve the field equations is employed at the global level, while the Generalised Newmark method (Katona and Zienkiewicz, 1985) is adopted for time integration. In particular, the following values of the Newmark parameters were selected in all the FE analyses illustrated in this note: $\beta_1 = 0.600$ and $\beta_2 = 0.605$ for the solid phase and $\beta_1^* = 0.600$ for the fluid phase. Those values ensure that the algorithm is unconditionally stable, while being dissipative mainly for the high frequency modes (Zienkiewicz *et al.*, 1999).

In all the FE analyses presented in the paper, the mesh was characterised by a width equal to 5 m. The domain was discretised with 235 isoparametric quadrilateral finite elements with 8 solid nodes and 4 fluid nodes. In the dynamic analyses the bottom of the mesh was assumed to be rigid, while the nodes along the vertical sides were characterized by the same displacements (“tied nodes” boundary conditions).

The code SWANDYNE II performs fully-coupled dynamic analysis, solving a unique set of equations for the solid and the fluid phase at each time step. This solution scheme requires the hydraulic conductivity and void ratio to be defined as input parameters: they were assumed equal to $1.0E-08$ m/s and 0.7, respectively. Base and lateral hydraulic boundaries were assumed as impervious while drained condition was imposed at the top of the mesh. Although water flow was allowed within the mesh, such movement was not large enough to be detected due to the substantially undrained condition that characterises the dynamic analyses in relation to the earthquake duration and the assumed low hydraulic conductivity.

The characteristic dimension of the elements in the FE analyses satisfies the condition that the spacing of the finite element nodes, Δl_{node} , must be smaller than approximately one-tenth to one-eighth of the wavelength associated with the maximum frequency component f_{max} of the input wave (Bathe, 1982):

$$\Delta l_{node} \leq \lambda_{min} / (8 - 10) = V_{s,min} / (8 - 10) f_{max} \quad (1)$$

As the time discretisation can play a significant role on the accuracy of dynamic finite element analyses (e.g. Haigh *et al.*, 2005), a time step equal to 0.01 s was assumed in all the FE simulations discussed in this work. This value was selected based on a preliminary parametric study aimed at detecting the optimal time discretisation to achieve a satisfactory level of accuracy of the analyses and, at the same time, a reasonable calculation time to perform them.

In order to perform a comparative analysis with the EERA results, a linear visco-elastic constitutive model was first considered in the FE analyses. Plasticity was then introduced by adopting an advanced elasto-plastic constitutive soil model (see next paragraph).

Viscous damping was introduced by means of the Rayleigh formulation, whose damping matrix is defined as follows:

$$[C] = \alpha_R [M] + \beta_R [K] \quad (2)$$

where $[\mathbf{M}]$ and $[\mathbf{K}]$ are the mass and the stiffness matrix of the system, respectively. The coefficients α_R and β_R are obtained considering the following relationship with the damping ratio D (e.g. Clough and Penzien, 2003):

$$\begin{Bmatrix} \alpha_R \\ \beta_R \end{Bmatrix} = \frac{2D}{\omega_m + \omega_n} \begin{Bmatrix} \omega_m \omega_n \\ 1 \end{Bmatrix} \quad (3)$$

where ω_m e ω_n are the angular frequencies related to the frequency interval $f_m - f_n$ over which the viscous damping is equal to or lower than D .

It is well-known that the solution of the wave propagation problem through FE analyses employing a linear visco-elastic model strongly depends on the assumed profile of the stiffness and damping coefficients with depth. In the visco-elastic FE analyses, G and D profiles were defined in order to match the ones resulting from the corresponding EERA analyses. To this aim, the numerical model in SWANDYNE was subdivided into the same number of layers employed in EERA and for each layer a value of G and D was selected with reference to the shear deformation level resulting from the EERA analyses at the corresponding depth. Figure 4(a) and 4(b) show the G and D profiles adopted in the FE analyses for both the E-W and N-S component of the LSST7 earthquake.

Different possible calibration procedures have been proposed in the literature to identify the interval $f_m - f_n$. In particular, a well established one (e.g. Hudson *et al.*, 1994) suggests to select f_m as the first natural frequency of the deposit f_1 , while f_n is assumed equal to n times f_m , where n is the closest odd integer larger than the ratio f_p/f_1 between the predominant frequency of the input earthquake motion (f_p) and the fundamental frequency of the soil deposit (f_1). This latter assumption is based on the evidence that the higher modes of a shear beam are odd multiples of the fundamental mode of the beam.

In order to obtain a better match between the linear time-domain and frequency-domain solutions, the new procedure for the selection of the two Rayleigh frequencies proposed by Amorosi *et al.* (2010) was here employed. The first natural frequency of the system which results as significantly excited by the earthquake, f_m , should be identified by comparing the EERA amplification function and the Fourier spectrum of the input motion. As regards the second frequency f_n , it should be identified considering the range over which the input motion is amplified during the propagation process: in particular, f_n should be selected equal to the frequency where the amplification function gets lower than one.

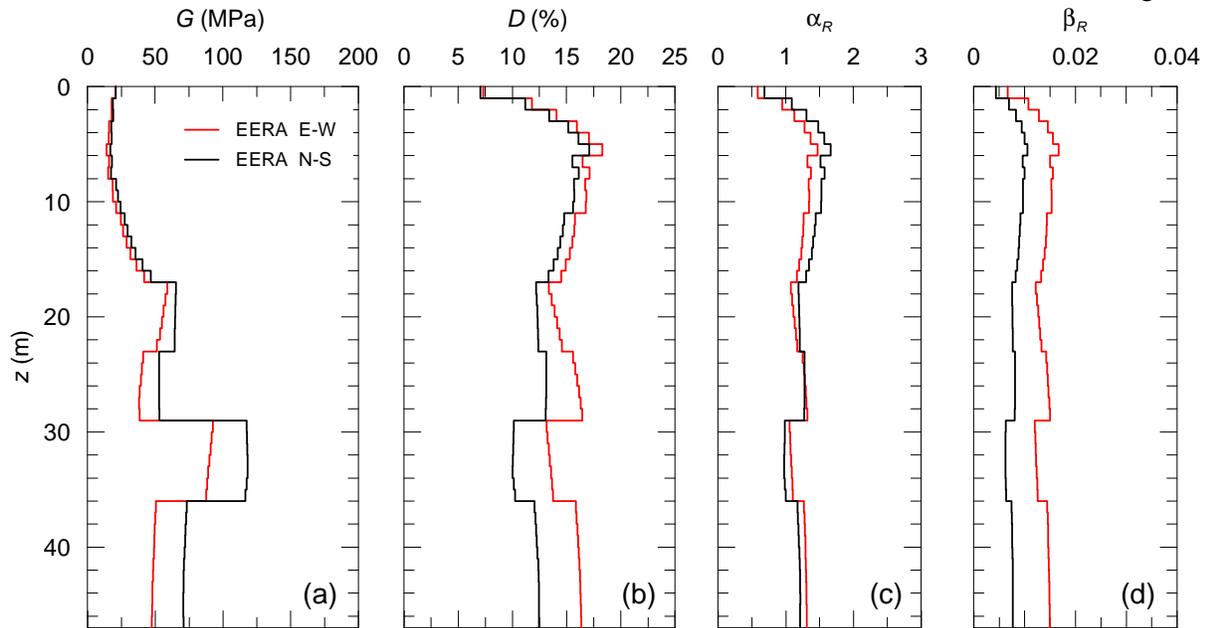


Figure 4. Profiles assumed in the FE visco-elastic analyses on the basis of EERA results.

The frequency content of both the E-W and N-S input motions suggests that the first natural frequency of the system should be selected as f_m , equal to 0.84 Hz and 0.95 Hz respectively, with no difference with respect to the standard procedure. At the same time, the standard procedure suggests to select a value of f_n equal to f_m in both cases ($n = 1$) being the dominant frequency of the two earthquakes lower than the first natural frequency of the deposit f_i , thus leading to an over-damped response of the system for the high frequencies. On the contrary, the new procedure leads to a value of f_n equal to 2.65 Hz for the E-W case and 4.20 Hz for the N-S case.

The resulting α_R and β_R profiles adopted in the visco-elastic FE analysis are shown in Figure 4(c) and 4(d), respectively, for both the E-W and N-S input motions.

2D Finite Element Plasticity Model

In order to investigate the effects of non-linearity on the wave propagation process, plasticity was added in the FE simulations through the advanced elasto-plastic constitutive model *MSS* (Model for Structured Soils) developed by Kavvadas and Amorosi (2000) for structured clayey soils. The model, based on multi-surface plasticity concepts, is characterised by two nested Cam-Clay like elliptical surfaces in the stress space: the external one, called Bond Strength Envelope (BSE), which represents the material states associated with the onset of degradation of structure at appreciable rate and the internal yield surface (PYE) geometrically similar to the BSE, but scaled by a factor $\xi \ll 1$.

For states inside the PYE, the behaviour is reversible and described by stress dependent bulk and shear moduli. For states on the PYE, early irreversibility is accounted for by the onset of plastic strain. In this case, when the PYE and BSE surfaces are not in contact the model predicts a realistic degradation of the soil stiffness, controlled by the smooth decay of the hardening modulus with the distance between the surfaces. Once the two surfaces are in contact the formulation coincides with that of a single surface Cam-Clay like model.

MSS includes both isotropic and kinematic hardening. The isotropic hardening rule controls the size of the BSE, i.e. it describes the evolution of material bonding by means of a damage-type mechanism to model volumetric and deviatoric structure degradation, whereas the kinematic hardening rules describe the motion of the two characteristic surfaces in the stress space and thus account for the evolution of

material anisotropy. The formulation of *MSS* allows the model to reproduce some of the key features of the cyclic behaviour of clays as the decay of the shear stiffness with strain amplitude, the corresponding increase of hysteretic damping and the related accumulation of excess pore water pressure under undrained conditions (e.g. Elia *et al.*, 2004).

The *MSS* model has been implemented in SWANDYNE II using an explicit integration scheme, with a constant maximum strain subdivision set by the user. Its predictive capabilities have been extensively tested under static and dynamic conditions for various boundary value problems (e.g. Amorosi *et al.*, 2008; Elia *et al.*, 2010). In this communication the discussion is limited to one-dimensional seismic wave propagation problems.

Numerical simulations of undrained cyclic simple shear tests were carried out with *MSS* in order to produce the curves of normalized shear modulus G/G_0 and damping D with cyclic shear strain γ . The secant shear modulus and the damping ratio for each shear strain amplitude were assessed after 500 load cycles, a number sufficient to reach steady-state condition. The results of the single element simulations performed with *MSS* are reported in Figure 3 with a solid black line for each considered depth. Model parameters similar to those adopted for the depth 11-17 m were used for the deeper soil layers (from 17 to 47 m).

Being the hysteretic damping provided by the advanced constitutive model sufficient to dissipate the energy introduced in the deposit by the seismic action, a small amount of viscous damping equal to 1% was added in the elasto-plastic FE simulations to reduce the high frequency spurious spikes.

RESULTS AND DISCUSSION

The results of the EERA simulations and the visco-elastic FE analyses performed applying the E-W and N-S components of the LSST7 earthquake are reported in Figure 5 in terms of acceleration time histories recorded at depths 0 and 11 m. The bedrock was assumed to be at a depth of 47 m, where the recorded motions were taken as input excitation (DHB-47).

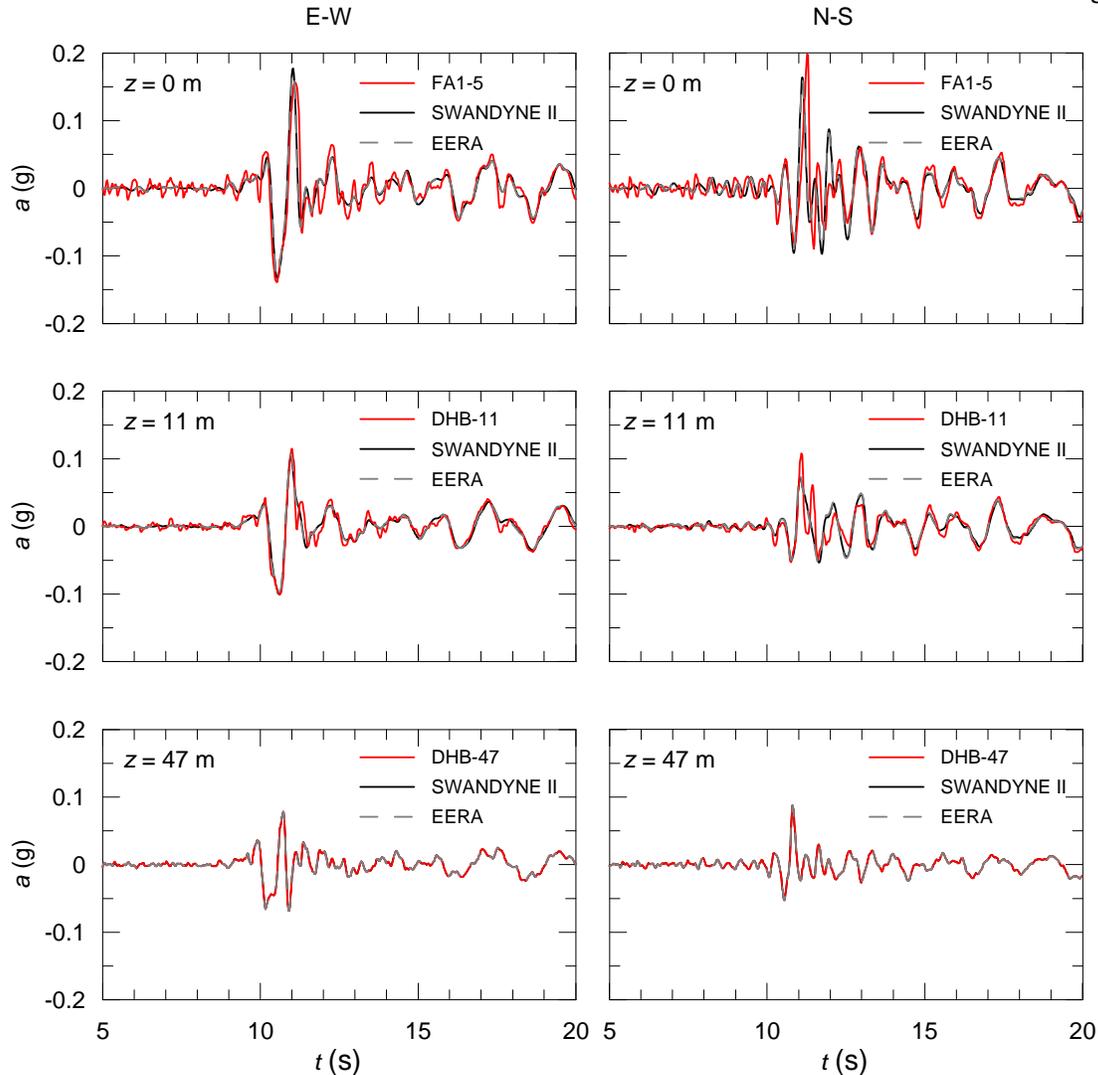


Figure 5. Acceleration time histories obtained with EERA and FE visco-elastic analyses.

The time-domain simulations carried out with SWANDYNE II employing the new procedure for the calibration of the viscous parameters are in good agreement with the results obtained at the same depths by the frequency-domain based EERA analysis. The acceleration time histories recorded at Lotung site along the downhole array at depths 0 and 11 m are also reported in the same figure and named FA1-5 and DHB-11, respectively. The figure shows that EERA and SWANDYNE II predict the E-W motion very well, particularly at the depth of 11 m both in terms of peak acceleration and zero crossing. The peak value of the E-W acceleration recorded at ground surface is slightly over-predicted. A poorer prediction was obtained applying the N-S component at bedrock level: the equivalent-linear and the FE visco-elastic analyses under-predict the peak acceleration quite significantly both at ground surface and at the depth of 11 m, while accelerations at 11.9 and 12.4 s are over-predicted. Moreover, a time shift in the acceleration peak between the recorded and predicted motions can be observed at surface.

Figure 6 shows the acceleration time histories obtained during the FE elasto-plastic analyses applying the E-W and N-S input signals at a depth of 47 m (as recorded at DHB-47). In this case, the peak acceleration of the E-W motion is under-predicted, especially at ground surface. Also for the N-S motion, a significant

under-estimation of the peak ground acceleration can be observed but an overall better prediction has been obtained with respect to the equivalent-linear and FE visco-elastic analyses.

In both cases, the wave signal arrives already significantly damped from the deeper soil layers for which a proper geotechnical characterization in terms of modulus reduction curves G/G_0 and variation of damping ratio D with cyclic shear strain γ was missing. Better predictions should have been obtained with the advanced non-linear time-domain approach if more detailed laboratory data relevant to the deeper material were available. Nevertheless, the advantage of using a sophisticated elasto-plastic constitutive models consists in the possibility to predict realistic plastic strains accumulation and development of excess pore water pressures inside the deposit throughout the shaking, providing sufficient hysteretic energy dissipation. Obviously, this is not possible if the soil is simulated as a single-phase visco-elastic material as in the simple equivalent-linear approach.

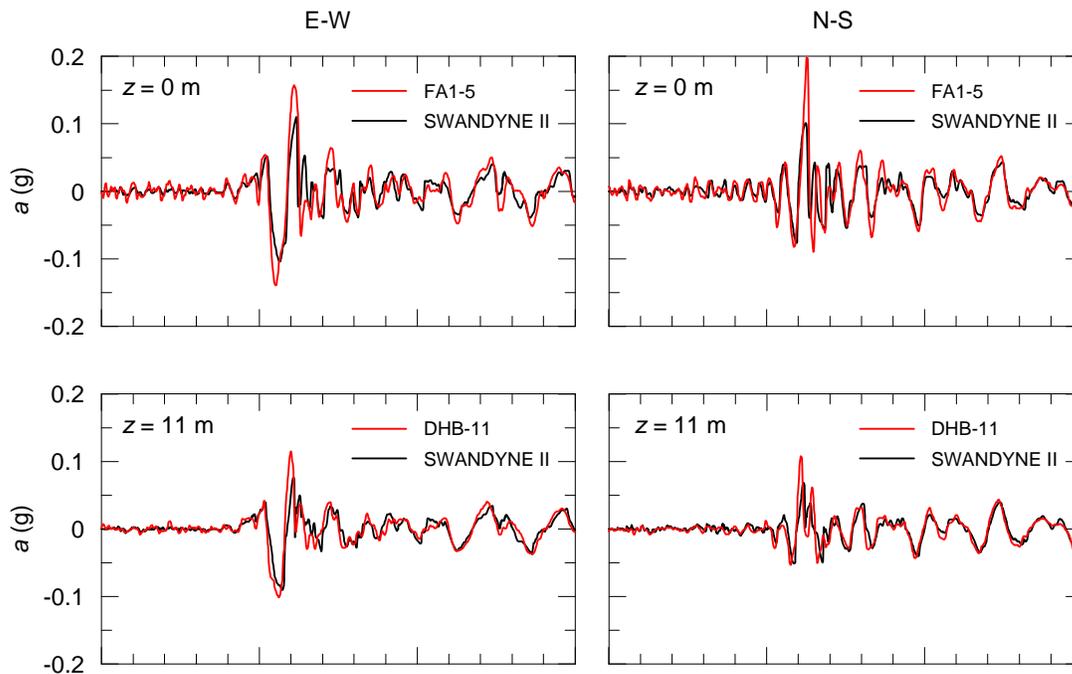


Figure 6. Acceleration time histories obtained during FE elasto-plastic analyses applying the input signal at a depth of 47 m.

With this respect, the excess pore water pressure distributions predicted at the end of the FE elasto-plastic dynamic analyses are shown in Figure 7 for both the LSST7 components. The numerical results are compared with a set of in-situ data recorded during the LSST16 event (Tang *et al.*, 1992), which was the only earthquake during which excess pore water pressures were measured. This event was characterised by similar values of peak ground acceleration, epicentral distance and magnitude of those relative to the LSST7 earthquake. The comparison shows a good agreement between recorded and predicted excess pore pressures in the first 20 m of the deposit. In-situ data for the deeper soil layers are not available.

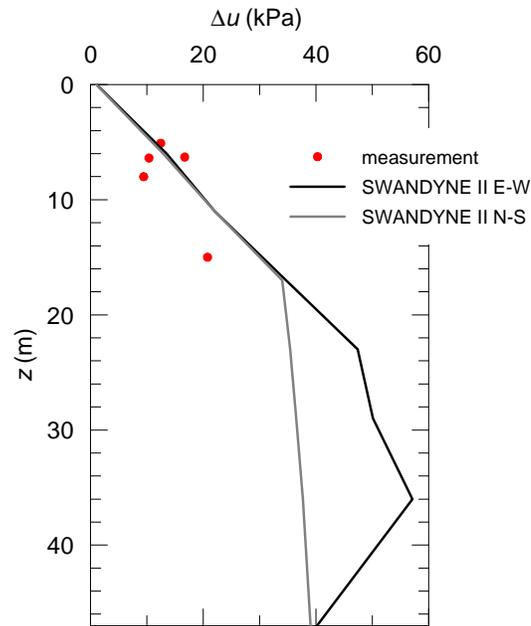


Figure 7. Excess pore water pressure distributions predicted by FE elasto-plastic analyses.

CONCLUSIONS

The ground response analysis of the Lotung experiment site during the LSST7 event has been studied in this work adopting numerical approaches of different level of complexity, ranging from simple equivalent-linear visco-elastic analyses to fully-coupled non-linear FE simulations. The direct comparison with recorded array data has allowed to highlight the limits and benefits of the different numerical schemes and to assess the performance of the advanced non-linear approach which is still seldom adopted in the engineering practice.

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