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PUSHOVER ANALYSIS OF AN ANCIENT MASONRY OIL-MILL IN THE SOUTHERN ITALY: A MESO-MACRO SCALE MODEL

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ABSTRACT

The paper presents a meso-macro scale numerical model –within the equivalent frame approach assumption– for the pushover analysis of in-plane loaded historical masonry walls constituted by an irregular assemblage of stones. The procedure is tested on an ancient masonry oil-mill in the Southern Italy. At the meso-scale, ultimate bending moment-shear force strength domains of piers and spandrels are derived by means of a heterogeneous upper bound FE limit analysis and the results are stored in a database. Assessing the capacity of both piers and spandrels is crucial when dealing with structural elements in irregular texture, since code of practice predictions are tailored for regular assemblages of bricks only. In this framework, heterogeneous limit analysis is particularly suitable for computing failure loads, since it permits a distinct modelling of stones and mortar joints and provides limit multipliers with a very limited computation effort. Appropriate static and kinematic boundary conditions are imposed to account for the complex interaction of internal forces and deformed shapes of single elements. At the macro-scale, a frame model of a masonry building is assembled, where piers and spandrels are modelled as elastic Timoshenko beams. At each analysis step, it is required that the internal forces in each structural element are smaller than the failure loads stored in the database created at the meso-scale. In order to test the reliability and efficiency of the approach proposed, an existing real scale old masonry oil-mill located in the Southern Italy is considered and one wall belonging to the building is analyzed when subjected to in-plane increasing horizontal loads up to collapse.

Keywords: Irregular masonry, Limit analysis, Equivalent frame approach, FE method

INTRODUCTION

Historical masonry buildings, especially in the Southern Italy, are usually realized with irregular stones joined through mortar with poor mechanical properties. For this reason, piers and spandrels ultimate resistance cannot be predicted by simplified formulas suggested by codes of practice, as for instance D.M. 2008 (2008), which are typically tailored to regular patterns. For this reason, to propose a comprehensive

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numerical model able to give reliable information on the non linear behavior of such typology of buildings –but taking at the same time into account properly the actual texture of the walls- seems at present a prohibitive challenge.

In this paper, a novel two step approach for the pushover analysis of historical masonry buildings is presented and applied to a case-study relying in an ancient oil-mill located in the southern Italy and constituted by walls in irregular texture. The model still uses for the global analysis of the masonry wall a simple equivalent frame model (Milani et al. 2009, Magenes & Della Fontana 1998, Belmouden and Lestuzzi 2009) and is able, at the same time, to estimate masonry macroscopic mechanical properties taking into account the disordered disposition of stones. The final aim is to put at disposal to practitioners a very simple instrument to be used in ordinary design, but at the same time capable of giving realistic estimations of the load carrying capacity of structural elements realized by irregular assemblages of stones with variable shape.

The tool is based on a two step procedure. In the first step (Step I), ultimate bending moment-shear force resistance of piers and spandrels are derived by means of a heterogeneous upper bound FE limit analysis (Sloan and Kleeman 1995) and the results are stored in a database. Piers and spandrels of an existing oil-mill located in Calabria (Southern Italy) are meshed through triangular elements following the actual disposition of the stones, on the base of a precise survey of the texture of each structural element available. A large database is collected, considering the behavior of each spandrel and pier belonging to the structure. A very refined discretization is used in order to take into account as close as possible the effect of the irregular texture and the presence of preferential planes of weakness. A heterogeneous limit analysis is adopted because is suitable for computing failure loads of complex structures with a moderate computational effort. The limit analysis is carried out for the range of expected axial loads in the structural elements and the expected relative rotations and displacements of piers and spandrels – two parameters that were found to affect the strength of unreinforced masonry significantly. Appropriate static and kinematic boundary conditions are imposed to account for the complex interaction of internal forces and deformed shapes of single elements.

In Step II (macro-level), specifically focused on the first step of the procedure proposed, a frame model of the masonry wall is implemented. In the model, for spandrels and piers, elastic Timoshenko beam elements are used. The strength of the piers and spandrels is defined by the strength domains stored in the database. At each analysis step, the bending moment and shear demands are compared to the respective capacities. If the capacity is exceeded, a rigid-plastic flexural hinge is introduced.

In order to test the reliability of the approach proposed, a small two-bay two storey wall is extracted from the structure and analyzed as isolated panel when subjected to increasing horizontal loads up to collapse. An expensive 2D plane stress elasto-plastic heterogeneous approach is also performed on the panel (Strand 7 2004), to have a reference solution to compare equivalent frame results. Pushover curves obtained with a standard commercial code, Aedes (2010), where code of practice formulas are implemented are also reported. It is found that the simple approach proposed seems generally more reliable with respect to an equivalent frame where code of practice formulas are reported, despite the fact that a reasonable estimation of the failure load may be obtained also in this latter case.

DESCRIPTION OF THE CASE STUDY UNDER CONSIDERATION

The building under study (see Figure 1) is an ancient oil-mill built in the early years of the twentieth century and located in the town of Bova Marina (Italy), in the center of a flat area between a small river (San Pasquale) to the west, the Ionian coast to the south and a hill (Agrillei) to the east. The building is

composed by a main body with rectangular plan connected perpendicularly to two smaller bodies, giving rise to a overall plant with "C" shape. The main body has two stories, whereas the other two have a single storey. Recently, a canopy realized using steel has been added in adherence to the entire eastern side. A traditional building technology was used to erect the oil-mill, which is typical of many constructions still present in the southern Italy. In particular, the building presents a 1.40 meters height basement of thickness 80 cm, realized using large stones and mortar. This portion of the perimeter walls actually coincides with the continuation of the foundation structure above the ground level. The remaining part belonging to the first story and the whole second story are realized with smaller irregular stones, pieces of bricks and mortar. With the aim of regularize the external walls realized with a disordered pattern, small piers constituted by a single row of solid clay bricks have been built. The first floor is made by small clay bricks vaults supported by double "T" steel beams, whereas the roof is sustained by a truss-like timber structure, currently very degraded.
For each pier and spandrel belonging to the building under consideration, strength domains are numerically obtained in terms of ultimate bending moment (Mu), ultimate shear (Vu) and imposed vertical pre-compression (N).

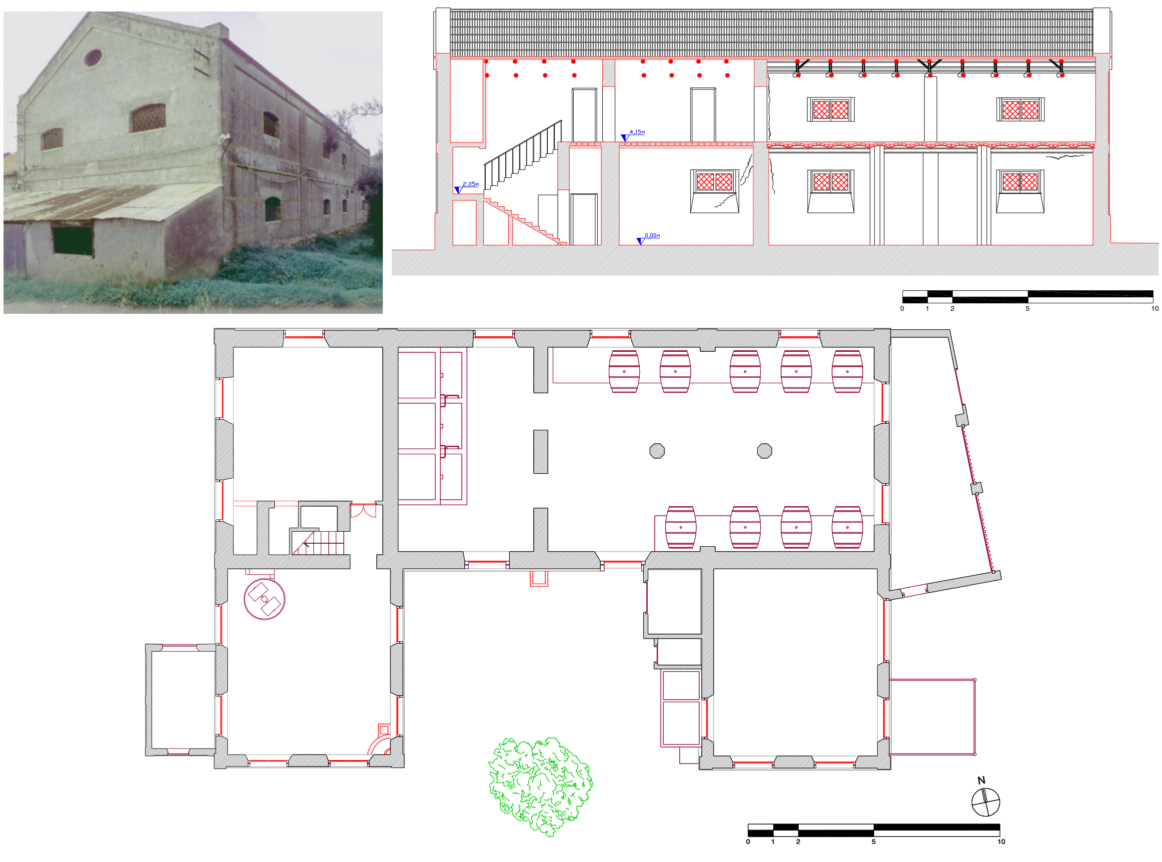


Figure 1: The ancient masonry oil-mill studied.

STEP I: LIMIT ANALYSIS NUMERICAL MODEL

All the numerical models used for limit analysis are based on an upper bound approach based on the kinematic discontinuous formulation originally presented by Sloan and Kleeman (1995), which has already been applied successfully to various masonry problems in Milani et al. (2006). The formulation is based on a triangular discretisation of the 2D domain and on the introduction of discontinuities of the velocity field along the edges of adjacent triangles. At each node j, a horizontal velocity u_x^j and a vertical velocity u_y^j are introduced. The resulting velocity field within a triangular element is linear whereas the strain rate field is constant. Across the interfaces a linear velocity jump is assumed and therefore for each interface four unknowns are introduced ($\Delta\mathbf{u} = \left[\Delta v^1 \ \Delta u^1 \ \Delta v^2 \ \Delta u^2\right]^T$) representing the normal (Δv^j) and tangential (Δu^j) velocity jumps with respect to the discontinuity direction evaluated at the nodes $j=1$ and $j=2$ of the interface.

A full description of the heterogeneous FE upper bound limit analysis model used in this paper is given in Cundari et al. (2010) and Milani et al. (2009) and the reader is referred there for further details. Here, it is worth remembering that, from a numerical point of view, the evaluation of the ultimate load bearing capacity of both piers and spandrels can be evaluated solving a suitable linear programming problem, where the objective function consists in the minimization of the total internal power dissipated:

$$\begin{cases} \min\left\{\left(\mathbf{b}_{ass}^{in}\right)^T \dot{\boldsymbol{\lambda}}_E^{ass} + \left(\mathbf{b}_{I,ass}^{in}\right)^T \dot{\boldsymbol{\lambda}}_I^{ass}\right\} \\ \text{such that} \begin{cases} \mathbf{A}^{eq}\mathbf{U} = \mathbf{b}^{eq} \\ \dot{\boldsymbol{\lambda}}_E^{ass} \geq \mathbf{0} \\ \dot{\boldsymbol{\lambda}}_I^{ass} \geq \mathbf{0} \end{cases} \end{cases} \tag{1}$$

where $\mathbf{b}_{ass}^{in}$ and $\mathbf{b}_{I,ass}^{in}$ are the assembled right-hand sides of the inequalities, which determine the linearised failure surface of the material of the continuum and of the interfaces, respectively, $\mathbf{U} = \left[\mathbf{u} \quad \dot{\boldsymbol{\lambda}}_E^{ass} \quad \Delta\mathbf{u}^{ass} \quad \dot{\boldsymbol{\lambda}}_I^{ass}\right]$ is the vector of global variables, which collects the vector of assembled nodal velocities ($\mathbf{u}$), the vector of assembled element plastic multiplier rates ($\dot{\boldsymbol{\lambda}}_E^{ass}$), the vector of assembled velocity jumps on interfaces ($\Delta\mathbf{u}^{ass}$), and the vector of assembled interface plastic multiplier rates ($\dot{\boldsymbol{\lambda}}_I^{ass}$); $\mathbf{A}^{eq}$ is the overall constraints matrix and collects velocity boundary conditions, relations between velocity jumps on interfaces and elements velocities, constraints for plastic flow in velocity discontinuities, constraints for plastic flow in continuum and spandrels equality.

PIERS STRENGTH DOMAINS

Within an equivalent frame approach context, a generic pier can be schematically represented by a shear deformable beam with 4 dof represented by nodal rotation (ϑ_1 and ϑ_2) and displacements perpendicular to the beam axis (u_1 and u_2), see Figure 2. Due to the very low axial deformability of the piers, we suppose that displacements along beam axis are negligible. We consider that an interaction between axial force N and ultimate bending moment M_u, and axial force N and ultimate shear V_u occurs.

A rigorous approach for piers would require the determination of the ultimate shear (V_u) and the ultimate bending moment (M_u) failure surfaces taking into account all the possible combinations of the following kinematic/static input variables: (i) the ratio $\rho = \vartheta_2 / \vartheta_1$ between head and foot rotations, (ii) the ratio $\pi = \left(w_2 - w_1\right)/\left(H\vartheta_2\right)$, where w_2 and w_1 are top and bottom horizontal displacements and H is the pier height and (iii) the applied pre-compression N.

Nevertheless, this approach would require an almost prohibitive computational effort, also in light of the very refined discretizations adopted in this paper to reproduce the actual microstructure of the piers. For this reason, we reasonably assume that ultimate shear and ultimate bending moments are "uncoupled", thus evaluating ultimate shear V_u simply as a function of vertical pre-compression N and keeping $\vartheta_2 = \vartheta_1$=0. Considering the importance of relative head and foot rotations on the evaluation of the ultimate bending moment, M_u is computed as a function of ρ ratio and vertical pre-compression. On the other hand, within a limit analysis framework, velocity field has to be considered instead of displacement and rotation fields. It is therefore necessary to evaluate piers resistance as a function of rotations rates ratio $\rho = \dot{\vartheta}_2 / \dot{\vartheta}_1$.

In this way, the limit analysis procedure proposed furnishes, at fixed pre-compression, two values of the ultimate shear (depending if the pier is loaded from west to east or vice versa) and a series of bending moments associated each one to a single value of the non-dimensional parameter ρ.

In the framework of a pushover analysis, at the end of the iteration i, the axial force $N_{(i)}$ acting on the beam as well as the rotations $\vartheta_{1(i)}$ and $\vartheta_{2(i)}$ at the beam extremes are known. The coefficient $\rho_{(i)} = \vartheta_{2(i)} / \vartheta_{1(i)}$ is defined as the ratio of the rotations at beam extremes at the iteration i and is kept also equal to rotation rates. Given $\rho_{(i)}$ and $N_{(i)}$, corresponding M_u and V_u values are computed from the collected database and compared to the actual bending moment and shear, to determine if the piers strength has been exceeded.

Differently from a standard limit analysis problem, additional kinematic boundary constraints must be imposed on piers to reproduce, in the collapse mechanism, a fixed ratio of rotations (rates) $\rho = \vartheta_2 / \vartheta_1$ of the upper ($\partial\Omega_1$) and lower ($\partial\Omega_2$) boundary, see Figure 2. For spandrels, the same considerations may be repeated, provided that such constraints are applied to the left and right boundaries. In a heterogeneous FE limit analysis, the kinematic constraint on ρ is taken into account by imposing a constraint on the rotational velocities of top and bottom edges ($\partial\Omega_1$ and $\partial\Omega_2$). A full description of the additional constraints to impose to the FE limit analysis model can be found in Milani et al. (2009) and they are omitted here for the sake of conciseness.

Here it is worth noting that ρ may range between –Inf and +Inf, since it is possible that $|\vartheta_2| > |\vartheta_1|$. In order to cover all the possibilities that can be encountered (at least theoretically) at structural level and considering that in limit analysis ρ represents a ratio between rotations rates, it may be useful to limit the range of variability of ρ between -1 and 1 and to introduce a further non-dimensional variable ρ' defined as $\rho' = \dot{\vartheta}_1 / \dot{\vartheta}_2$ ranging between -1 and 1. By means of the alternative imposition of boundary conditions related to ρ and ρ', all the possibilities that can be encountered in practice may be covered.

In Figure 3 and Figure 4, a few results of a massive numerical analysis campaign conducted using a heterogeneous FE limit analysis software and reported in Cundari et al. (2010) are summarized.

In particular, in Figure 3, two deformed shapes at collapse of one ground floor pier subjected to shear at increasing pre-compression and the corresponding ultimate shear-vertical pre-compression curve are represented. The role played by vertical pre-compression in the change of the failure mechanism is particularly evident. The presence of irregular cracks which zigzag between rigid stones suggests once again that simplified formulas provided by codes of practice may be not reliable and well suited for this typology of masonry.

Figure 4 shows some deformed shapes of the same pier subjected to bending moment. In the simulations, ρ is maintained equal to zero, whereas the vertical pre-compression is increased starting from zero and ending to the 80% of mortar compressive strength. The corresponding M_u-σ_v curves are also reported in Figure 4 (here σ_v indicates vertical stress acting at the top of the pier).

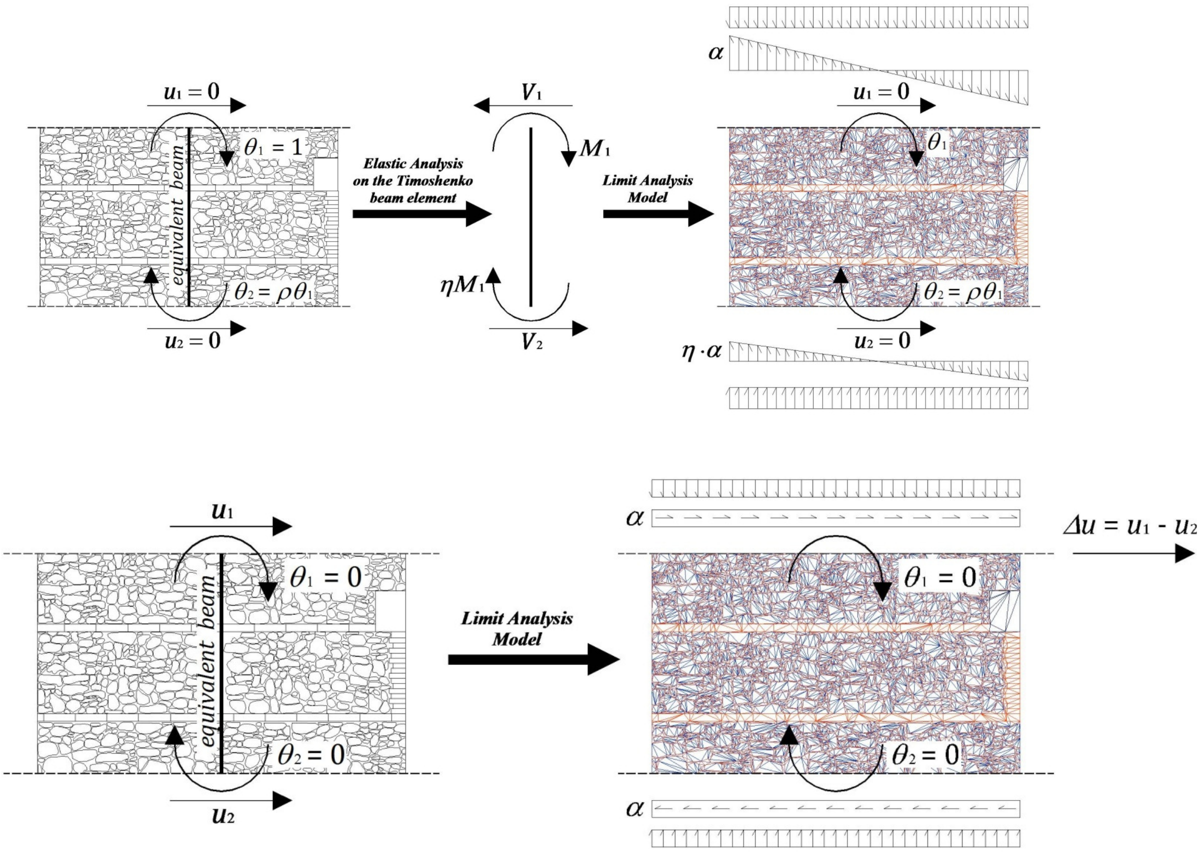


Figure 2: General procedure adopted for the determination of masonry piers strength domains.

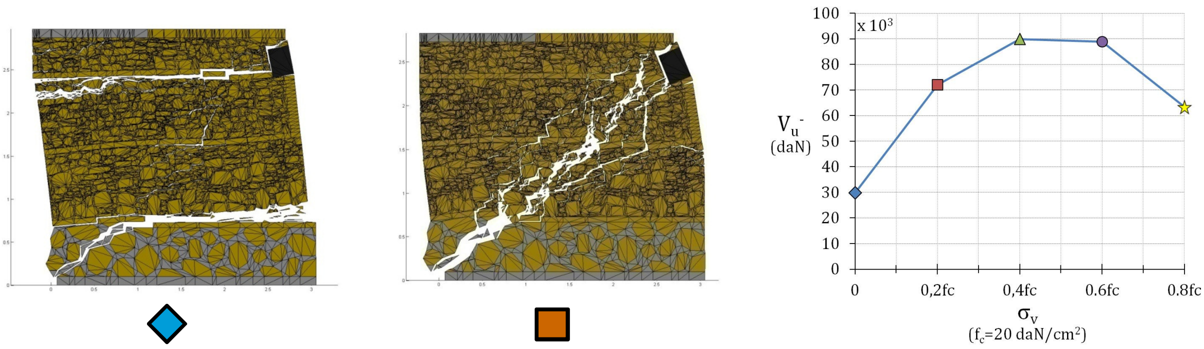


Figure 3: Pier 1, deformed shape at collapse corresponding to Vu at two increasing levels of pre-compression and corresponding shear strength as a function of vertical pre-compression.

As expected, a moderate vertical pre-compression increases considerably the ultimate bending resistance of the pier, whereas for σ_v exceeding 50% of the compressive strength of the joint the load bearing capacity of the structural element slightly decreases, which seems again in agreement with experimental evidences.

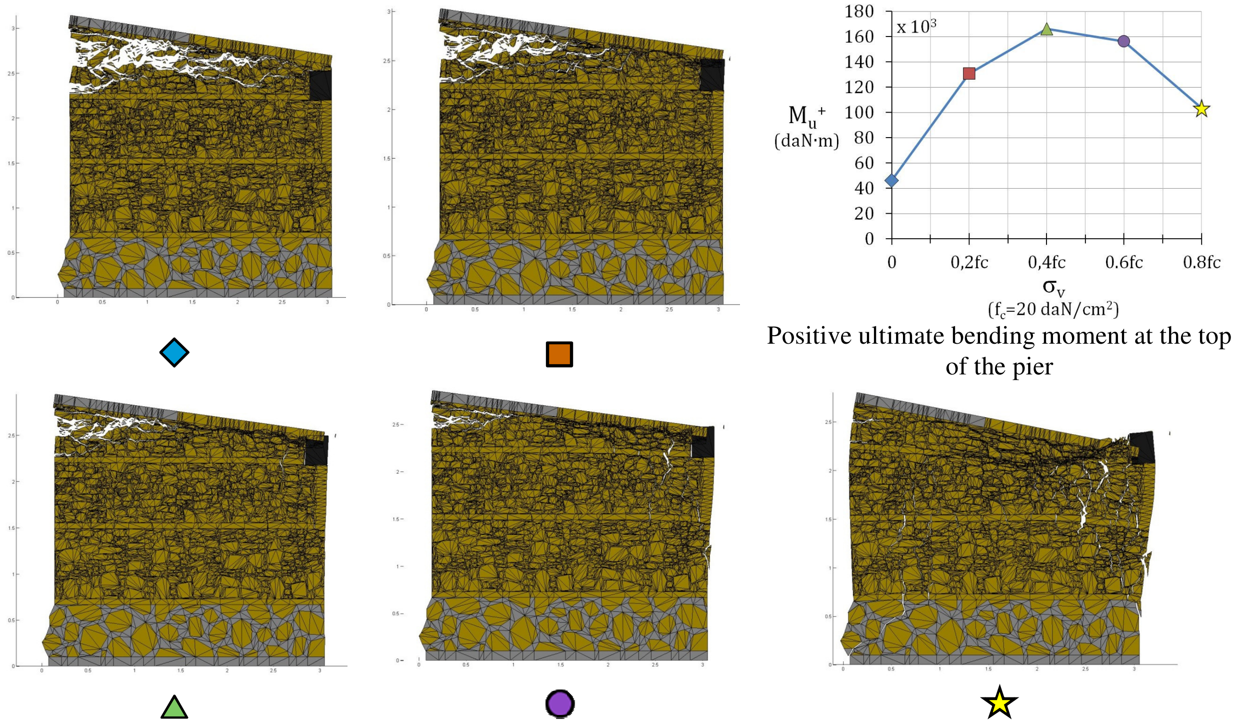
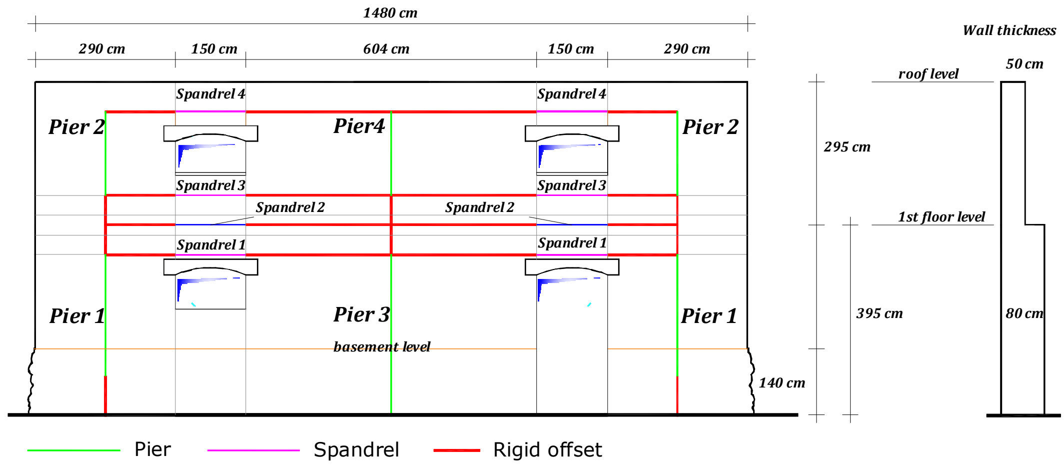


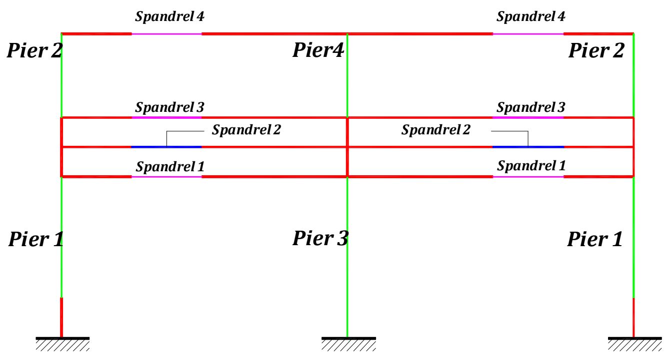
Figure 4: Pier 1, M_u (from left to right) strength domain at different pre-compression levels and corresponding deformed shapes at collapse.

STRUCTURAL IMPLEMENTATION ON A SINGLE WALL IN-PLANE LOADED

In order to test the reliability of the simple two-step frame model proposed at a structural level, a benchmark medium size masonry wall built in irregular texture, as depicted in Figure 5, is analyzed when subjected to a standard pushover analysis. A computationally very expensive plate heterogeneous mesh is built using a commercial elastic-plastic software, namely Strand 7 (2004), to have at disposal a reference global pushover curve, to compare to that obtained with the simple inexpensive frame model presented. Over 200000 elements, see Figure 5, have been used to discretize the wall, assuming for the constituent materials the hypothesis of perfect plasticity. For joints, a classic Mohr-Coulomb failure criterion has been adopted, obeying an associated flow rule, with friction angle and cohesion equal to 30° and 1.4f_t (f_t=0.06 MPa). Mortar Young modulus is assumed equal to 1300 MPa, shear modulus equal to 220 MPa and Poisson ratio equal to 0.25, in agreement with experimental data collected in the oil-mill, see Cundari et al. (2010) for details. Irregular blocks and bricks are modelled with linear elastic elements (Young Modulus: 15000 MPa, Poisson ratio: 0.20).
It is authors' opinion that only an expensive 2D heterogeneous approach may represent a reliable reference model to test the performance of a simplified frame approach. However, the non linear analysis required more than 8 days to run, using a PC equipped with 5 Gb RAM and a Pentium Dual Core 2.10 GHz, within a Windows 7 64 bit OS. Obviously, the long time needed to complete such a non-linear simulation precludes totally the usage at professional level by any practitioner and an alternative suitable

approach is needed.





Section geometry [cm x cm]	
Pier 1	290x80
Pier 2	290x50
Pier 3	604x80
Pier 4	604x50
Spandrel 1	83x80
Spandrel 2	40x50
Spandrel 3	82x50
Spandrel 3	123x50

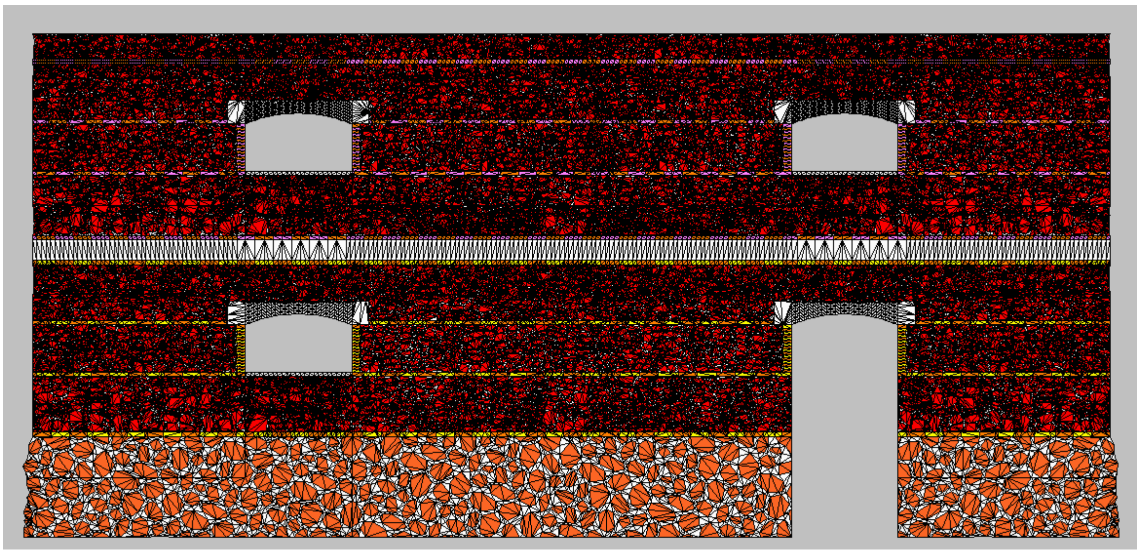


Figure 5: Geometry of the wall realized with irregular texture and its discretization into a frame model and a heterogeneous FE approach

With the aim of comparing results also with standard approaches available in common design practice, the commercial code Aedes (2010) has been also utilized. Within this software, the wall is discretized using the equivalent frame approach, where the Italian code of practice formulas D.M. 2008 (2008) are utilized to evaluate piers and spandrels ultimate resistance. When dealing with the Italian norm, reference is made to new or existing buildings made with regularly assembled masonry, raising doubts on its reliability for an ancient building with irregular stones. Due to the impossibility to set ad hoc masonry mechanical properties in the commercial code, two different models (hereafter labelled as I and II are tested). The first is a frame where cohesion for masonry is kept equal to $1.4f_t$, whereas for the second compression strength is assumed equal to 2 MPa, which gives an indirect evaluation of cohesion as c=0.06 MPa. Due to the impossibility to set properly all input data, it is expected that a standard software is unable to provide good predictions of the non linear behavior under horizontal static loads of walls assembled in irregular texture.

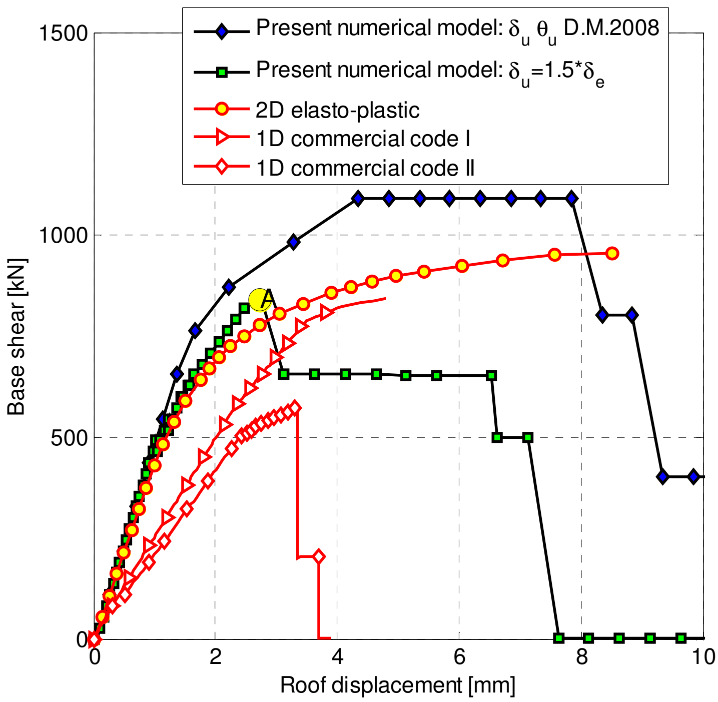


Figure 6: Comparison between pushover curve obtained with the present frame model, the expensive 2D heterogeneous approach and a frame model commercial software available.

The structural example under consideration is a two storey two bay wall, with approximate dimensions equal to 14.50x6.35 m (length x height). Small openings regularly distributed are present both at the ground and first floor, making piers particularly squat. A basement 140 cm high and realized with large stiff and resistant irregular stones joined through thick mortar (modelled with plate elements) with poor mechanical properties is also present. This is a typical situation for existing buildings, where the first floor level was placed in elevation with respect to the ground. The remaining part of the wall is built with smaller and less stiff irregular stones, again bonded with thick mortar joints with irregular geometry, except for the presence of a few horizontal thin layers, equally stepped in vertical, obtained utilizing quite regular small clay bricks. A reinforced concrete ring beam with rectangular section (dimensions 40x50 cm), lightly reinforced (4 bars of diameter 10 mm at the corners) and with concrete with relatively poor mechanical properties (assumed here C20/25) is also present between the first and second floor. The

presence of the ring beam justifies the choice of an equivalent frame constituted by three separate spandrels at the first floor level. Indeed, it is reasonable to assume that the r.c. beam and masonry disposed below and on top of the ring beam undergo independent deformations. Ground floor thickness is assumed equal to 80 cm, whereas the last floor walls are assumed 50 cm thick.
Vertical loads are mainly constituted by masonry self weight, which is assumed equal to 2000 Kg/m^3. Horizontal load increased until failure is applied following a first mode distribution. While in the equivalent frame approach, horizontal static seismic action is simulated with concentrated forces on nodes belong to the first and second level, in the heterogeneous 2D model, loads are applied at each element centroid, which seems reasonable considering that the vertical loads are mainly due to masonry self weight.

In Figure 6, the pushover curve (load direction: west-east) obtained numerically with the model proposed is depicted and compared with that obtained with the commercial 2D heterogeneous software Strand 7 (2004) and with Aedes (2008) predictions. Additionally, the proposed model is slightly modified in order to reduce ductility of the elements proposed by the norm D.M. 2008 (2008). In this case, indeed, where piers are particularly squat and with very low deformability (thanks also to their thickness), it seems not reasonable to link the deformation capacity with the height of the elements. It is authors' opinion that this assumption probably would overestimate the ductility of the structure. Here, an alternative approach is proposed, limiting the ultimate deformation capacity of the elements to be equal to 1.5 times the elastic limit deformation, both in shear and flexion, in agreement with the classic POR approach, see D.M. 1981 (1981). The elastic limit deformation is defined as $\theta_e = M_u/(E_m I)$ and $\delta_e = T_u/(G_m A_s)$ for bending and shear respectively, where I is the sectional inertia modulus and A_s is the reduced shear areas.
From an overall analysis of pushover curves results, it can be stated that pushover curves provided by the present approach are very close to that obtained with the heterogeneous expensive 2D Strand mesh, meaning that the numerical procedure presented seems quite reliable. The extra resistance of the proposed model with deformation capacity from the D.M. 2008 (2008) slightly overestimate the maximum base shear reached by the structure, whereas the approach with the modified deformation capacity seems more conservative and therefore preferable in design phase.

From an overall analysis of the results obtained, the following key issues may be highlighted:

- The peak total shear at the base provided by the commercial frame software where Italian code of practice formulas are implemented is lower with respect to the present approach and the reference 2D solution, despite the fact that model I is capable of give acceptable predictions of the peak base shear. As expected, formulas provided by the Italian code difficultly can be adapted to a situation where texture is strongly irregular and where cracked zones, which zigzag inside thick mortar interspersed between stones, cannot be predicted easily with simplified approaches.
- The failure mechanism provided by the frame model approach involves piers of the first story in shear and all the spandrels, which typically fail in bending for rotation ratios at the extremes near 1. As expected, the central ground floor pier, which is very squat, fails for the formation of a shear hinge. Its contribution to the global non linear response of the wall is predominant with respect to the other deformable elements. The deformed shape of the 2D heterogeneous model seems similar to that provided by the frame approach.
- The role played by the spandrels in the numerical model is fundamentals at low levels of the external load and low deformations. Due to their small dimensions and small strength if compared to that of the piers, they fail very early during the deformation process, resulting in a premature decrease in the global stiffness of the pushover curve and, when their ultimate drift is exceeded, in moderate reductions of the load bearing capacity of the wall.

CONCLUSIONS

A simple model for the pushover analysis of ancient masonry walls in-plane loaded and constituted by the assemblage of irregular stones and mortar with poor mechanical properties has been presented. The approach is based on the preliminary determination of piers and spandrels strength domains through a heterogeneous FE limit analysis package and on the successive implementation of the strength domains collected in a database in an equivalent frame model program. The novelty of the method relies on the computation of the strength domains for piers and spandrels assembled in irregular texture by means of limit analysis and their implementation into a frame analysis software. Due to the geometrical dimension of the piers, the very irregular texture of the walls and the small dimension of the stones/blocks if compared to the overall width of the piers, a considerable number of heterogeneous limit analysis problems with several optimization variables have been solved.

In the first step, isolated masonry piers and spandrels have been analyzed. In both cases, shear and bending strength have been uncoupled, assuming that the ultimate shear depends exclusively on vertical pre-compression and that the ultimate bending is a function of the axial load applied to the spandrel and the relative end rotations of the beam, which characterize the type of loading to which the beam is subjected. For spandrels it is assumed that the deformation of the large zones where the beam and pier axes cross, does not significantly affect the capacity of the spandrel. This assumption is also corroborated by numerical results reported in 0. When carrying out the limit analysis, two main assumptions were necessary: (i) It was assumed that the ratio of rotational velocities applied in limit analysis can be set equal to the ratio of the end rotations of the structural element, which is relevant for the frame analysis. Additionally, (ii) it was postulated that the ratio of end moments $\eta = M_2 / M_1$ applied for the evaluation of ultimate bending moments, which are both increased by the plastic multiplier α during the limit analysis, can be estimated from the moment distribution of an elastic Timoshenko beam subjected to end rotations with $\rho' = \dot{\vartheta}_1 / \dot{\vartheta}_2$. Limit analysis results obtained both for piers and spandrels have been then stored in a database, which have been used to analyze with a non commercial software a mid-size masonry wall (two bays, two stories) arranged in irregular texture and loaded with an equivalent static seismic load increased until the failure of wall.

Structural pushover curves obtained with the model proposed have been finally compared with those provide by commercial codes and a discussion on the differences found has been reported in order to evaluate the limitations and the capabilities of simplified formulas provided by the Italian code. It is found that particular care should be used by practitioners in the utilization of formulas tailored for regular texture in case of ancient structures built in irregular texture and mortar with poor mechanical properties.

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