Geotechnical Aspects of Underground Construction in Soft Ground, Kastner, Emeriault, Dias, Guilloux (eds)
© 2002 Spécifique, Lyon. ISBN 2-9510416-3-2

Numerical implementation of plastic hinge in the Subgrade Reaction Method for steel sheet pile design

B. Bornarel, F. Emeriault & R. Kastner
URGC Géotechnique, INSA Lyon, Villeurbanne, France

ABSTRACT: The development of a steel sheet pile design software based on the SubGrade Reaction Method is presented in this paper with particular attention on the numerical implementation of plastic hinges. Several plastic behaviours of the steel sheet pile section (moment - rotation curves) are considered in Section 2: perfectly plastic, hardening or softening behaviour. Their numerical implementation is detailed in Section 3 together with the results of several validation tests proving that the chosen numerical scheme is stable.
In Section 4, typical design cases have then been considered including: single anchored sheet piles in sand or double anchored sheet piles in both sand and clay. In each case, several pile profiles are also analyzed. Depending on the geometry of the sheet pile wall, the number and position of supports and the plastic hinge M - ϕ curve, up to three plastic hinges have been obtained. The resulting maximum excavation depth increase is quantified and compared to the value obtained with traditionnal SGRM assuming an elastic behaviour of the wall.

1 INTRODUCTION

According to the Eurocode EC3 - Part 5 (CEN 1997), the sections of steel sheet piles (SSP) can be designed for Ultimate Limit States considering large deflection due to the plastic behaviour (with the possible development of plastic hinges). From a geotechnical point of view, the SubGrade Reaction Method (SGRM) is one the most popular design method used for large excavations and particularly steel sheet piles (especially in France).

2 BACKGROUND

2.1 Subgrade Reaction Method

The Subgrade Reaction Method (SGRM) has been developed in order to enhance the design of flexible retaining structure (Fages et al. 1971, Houy 1986, Monnet 1994, Schmitt 1998). The retaining structure (steel sheet pile for instance) is considered as a vertical elastic beam interacting with the surrounding soil (either on the active or passive sides) via an infinite set of uncoupled horizontal linear springs (corresponding to Winkler's approach). The pressure resulting from a horizontal displacement of the wall is nevertheless limited to the active or passive earth pressures. The reaction modulus K_h relating the horizontal displacement to the soil pressure in the linear range can be assessed based on the results of in-situ or laboratory tests.

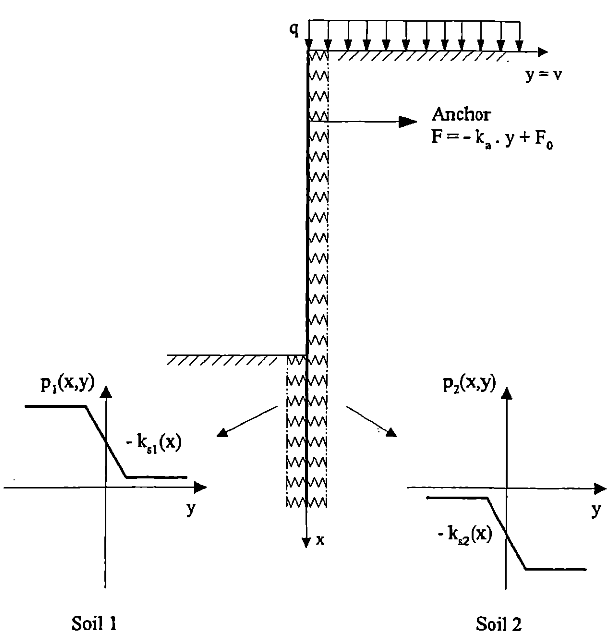


Figure 1: Subgrade Reaction Method

The SGRM enables the design engineer to take into account the different construction stages: excavation, water table lowering or installation of several levels of struts or anchors.

2.2 Plastic behavior of steel sheet piles

Depending on the geometry, length, thickness and steel mechanical properties, the plastic behavior of

SSP can lead to complex results when tested in 3 or 4 points bending tests. The plastic hinge behavior also depends on the applied loading (sagging or hogging). The occurrence of plastic hinges is expressed by moment – rotation (M-ϕ) curves (cf. Figure 2) presenting the following features:

- Type A perfectly plastic hinge: the M-ϕ curve is only defined by the ultimate moment value M_{pl}
- Type B plastic hinge presenting a hardening behavior: the M-ϕ curve is derived from the following analytical expression:

$$M = M_{el} + (M_l - M_{el}) \frac{\exp\left[\dfrac{2k_h}{M_l - M_{el}}(-\varphi^p)\right] - 1}{\exp\left[\dfrac{2k_h}{M_l - M_{el}}(-\varphi^p)\right] + 1} \qquad (1)$$

- Type C plastic hinge presenting a softening behavior: the corresponding M-ϕ curve is derived from experimental results. This approach can be applied regardless of the class of SSP cross-section. Furthermore it does not require a verification of the rotation capacity of the SSP.

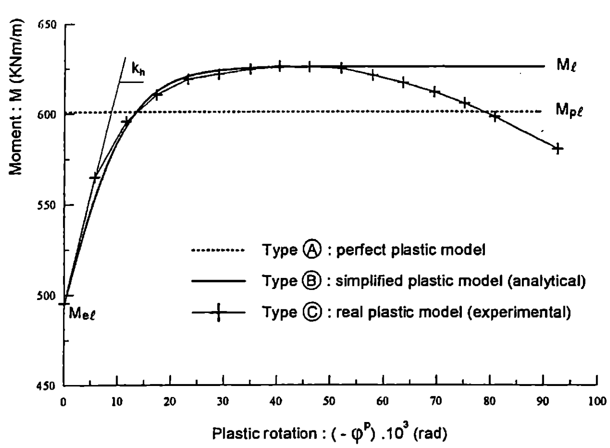


Figure 2: Moment – rotation curves implemented in the design software.

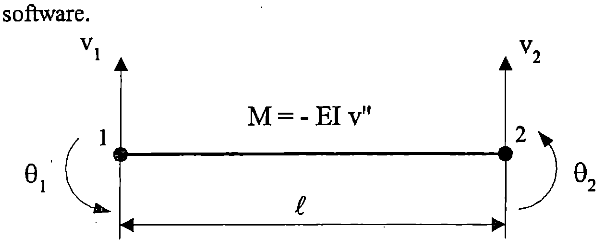


Figure 3: Linear elastic beam element

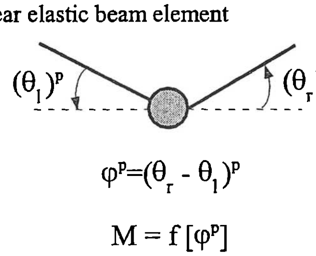


Figure 4: Plastic hinge element

3 NUMERICAL IMPLEMENTATION

3.1 Numerical implementation

Classical two nodes flexural beam elements are considered in the modelling of the steel sheet pile (cf. Figure 3). Since the SGRM only considers the horizontal behavior of the retaining structure, two degrees of freedom are required at each node (horizontal displacement v and rotation θ), these unknowns are approximated by cubic shape functions. The soil actions on both sides of the beam are derived from the corresponding subgrade reaction moduli (varying with the depth). The elastic stiffness matrix is computed numerically using four Gaussian integration points.

Plastic hinges are modeled by nodal elements with two degrees of freedom: the left and right plastic rotations $(\theta_l)^p$ and $(\theta_r)^p$ (cf. Figure 4). Their plastic behavior is simulated by enforcing a M-ϕ curve (Type A, B or C). Potential plastic hinges are located at each node and connected to elastic beam elements. One hinge is activated when the moment reaches the elastic limit, provided that this corresponds to a local extremum of the bending moment.

Since the numerical problem is solved using a constant stiffness matrix approach, an elastic stiffness k_h is allocated to all the potential plastic hinges. k_h is initially set to a very high value (greater than 10^{10} KNm/m) in order to observe an elastic behavior. When the moment reaches the elastic limit M_{el} in one node, the corresponding stiffness is relaxed to its real value and plastic moment can be developed at that node. This approach implies an accurate determination of the external loading leading to the onset of plastic behavior.

The associated plastic M-ϕ curve is derived from experimental data for Type C plastic hinge or analytical expressions for Type A and B hinges. The curve generally depends on the moment distribution shape (sagging or hogging) and on the distance between the points of zero moment. After the plastic hinge is activated, this distance may evolve during further loading and the bending moment near the hinge may overcome the elastic limit and the moment value at the hinge level.

3.2 Validation tests

In order to validate the implementation of plastic hinges in the numerical code, three simple tests have been performed and confronted when available with theoretical or experimental results.

When a beam restrained at its both ends is subjected to a uniform surface load q and Type A hinges are considered, two hinges first appear at both ends when q and the deflection at mid-span v reach:

$$q_{el} = \frac{12 M_{pl}}{l^2} \qquad v_{el} = \frac{3 l^2 M_{pl}}{96 EI} \qquad (2)$$

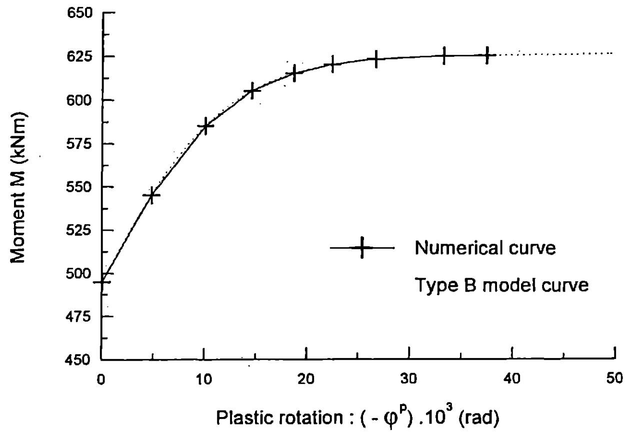


Figure 5: Validation test for Type B plastic hinge

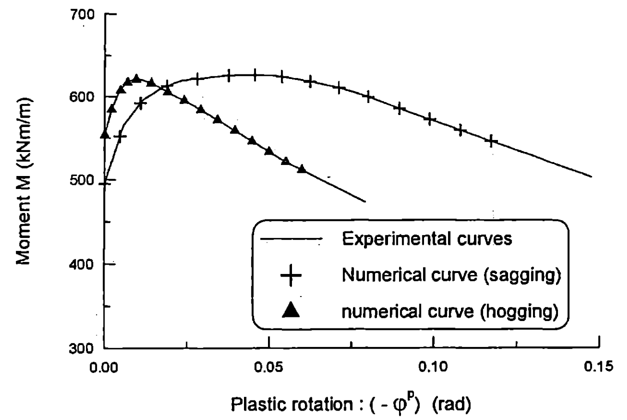


Figure 6: Comparison of validation tests results for Type C plastic hinge with experimental results

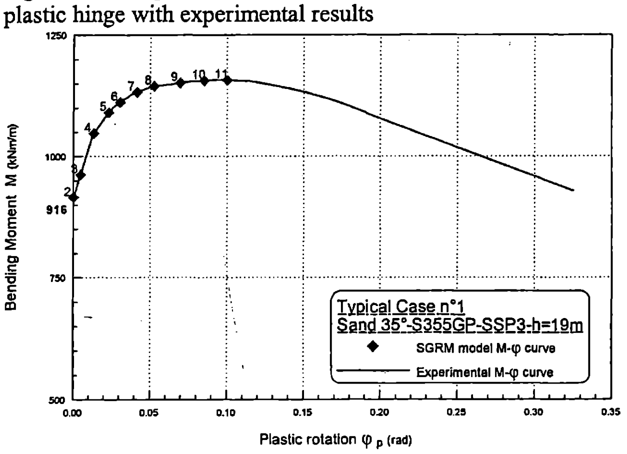


Figure 7: Moment – rotation curve corresponding to Append. I

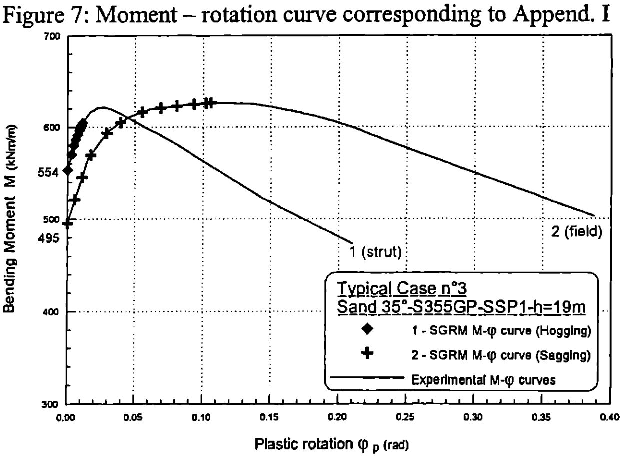


Figure 8: Moment – rotation curve corresponding to Append. II

The structure fails when the third plastic hinge occurs at mid-span with:

$$q_{pl} = \frac{16 M_{pl}}{l^2} \qquad v_{el} = \frac{8 l^2 M_{pl}}{96 EI} \qquad (3)$$

If a beam is restrained at its left end and submitted to a load F at its right end, a plastic hinge occurs at the left end and can follow the Type B model presenting hardening.

This validation test is modified to analyse the case of Type C hinges presenting softening. Figure 6 presents the results of the corresponding validation test in which a linear spring is attached to the right end of the beam used in the preceeding test. It shows that the developed software correctly reproduces the softening behavior of plastic hinges observed in laboratory tests (either sagging or hogging).

4 TYPICAL DESIGN CASES

The developed numerical software is applied to the design of several SSP walls. The single and double anchored cases are analysed with different soil conditions (sand or stiff clay) and pile profiles. An elastic predesign has been performed in order to determine the length of the wall. The maximum excavation depth is determined for both elastic and elasto-plastic behaviour of the SSP. The displacement, bending moment, shear force and earth pressure distribution along the sheet pile at various stages of excavation are presented in Appendix.

4.1 Single anchored sheet piles

For single anchored sheet piles, up to two plastic hinges can occur depending on the sheet pile section and steel grade.

For SSP with high bending stiffness, only one plastic hinge occur, generally at mid-span (cf. Appendix I) and close to the maximum of the M - φ curve (cf. Figure 7). The wall behaves like a non-anchored wall and failure is due to a lack of embedment length. Figure 7 shows the case of a 19 m long sheet pile in sand (φ' = 35° and c' = 0 kPa). The maximum excavation depth represents 73.0 % of the total length of the pile for an elastic design and 76.1 % when a plastic hinge is considered (increase in excavation depth of approximately 0.60 m). This represents a decrease of 11.5 % of the embedded length.

For SSP with low or medium bending stiffness, two plastic hinges are created (cf. Appendix II). The first one appears at the anchor level, the second approximately at mid-span. Soil can be further excavated and failure occurs when the second hinge reaches its maximum moment value whereas the first hinge is still in the hardening part of the M - φ curve (cf. Figure 8). The maximum excavation depth increases from 75.5 % of the total length of the pile (for elastic design) to 79.7 % with the development of plastic hinges. The 0.80 m extra excavation represents a 17.3 % decrease in embedded length.

These results are obtained assuming that the characteristics of the SSP and the possible related plastic hinges are not affected by the technology used for the support. For example, the reduction of mechanical characteristics due to the hole in the SSP necessary when an anchor is used is not accounted for.

4.2 Double anchored sheet piles

For double anchored sheet piles, up to three plastic hinges can occur depending on the SSP section and steel grade.

In the case of two plastic hinges (cf. Append. III), the first one appears at the lower anchor level, the second between the lower anchor level and the excavation level. Soil can be further excavated and failure occurs when the second hinge reaches its maximum moment value while the first hinge is already in the softening part of the M - ϕ curve (cf. Figure 9). The maximum excavation depth increases from 79.6 % of the total length of the pile (for elastic design) to 83.0 % with the development of plastic hinges. This increase represents a 0.81 m extra excavation for a 24 m long SSP or a 16.5 % decrease in embedded length.

When three plastic hinges develop before failure (cf. Append. IV), the first one appears between the lower anchor level and the excavation level, the second at the lower anchor level and the last one below the excavation level. Soil can be slightly further excavated: failure occurs when the first hinge reaches its maximum moment value while the second hinge

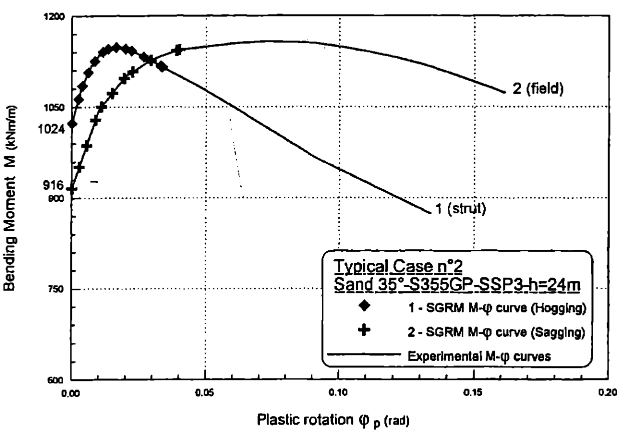


Figure 9: Moment – rotation curve corresponding to Appendix III

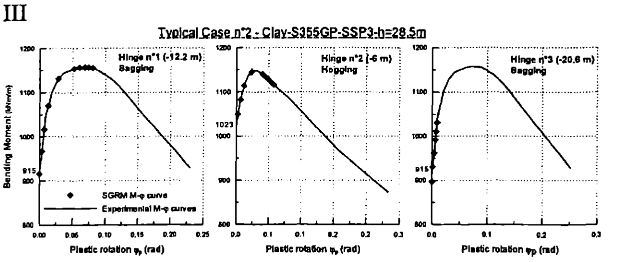


Figure 10: Moment – rotation curve corresponding to Appendix IV

is already in the softening part of the M - ϕ curve and the last hinge is still in the hardening part (cf. Figure 9). Figure 13 corresponds to the case of a 28.5 m long SSP in stiff clay (ϕ' = 23° and c' = 5kPa). The maximum excavation depth increases from 47.7 % of the total length for elastic design to 51.2 % with plastic hinges. The 1.0 m extra excavation represents a 6.7 % decrease in embedded length.
It is reminded that the simulation results presented above do not take into account the support technology and the possible implication in the SSP mechanical characteristics (cf. Section 4.1).

5 CONCLUSION

The possible development of plastic hinges in steel sheet pile walls has been numerically implemented in the SGRM design method. Different plastic behavior can be reproduced: perfect plasticity, plastic behavior with hardening and softening.

The results obtained for typical design cases, including single anchored or double anchored SSP walls with various soil conditions, show that up to three plastic hinges can develop in the wall at different levels (anchor, mid-span or below the excavation level). The decrease in embedded length ranges from 6.7 to 17.3 % between elastic and plastic design.

ACKNOWLEDGEMENTS

This document presents partial results of a European Research Program funded by the steel sheet pile producers Profil Arbed ISPC, Hoesch Spundwand und Profile and British Steel - Corus. This support is greatly acknowledged by the authors.

This research program also involved the following research partners: Imperial College London, RWTH Aachen, BU Wuppertal and RU Bochum. The experimental data of bending tests have been provided by RWTH Aachen. The fruitful discussions with all the partners have been much appreciated.

REFERENCES

CEN European Committee for Standardization. 1997. *Eurocode 3: Design of Steel Structures, Part 5: Piling.* ENV 1993-5.

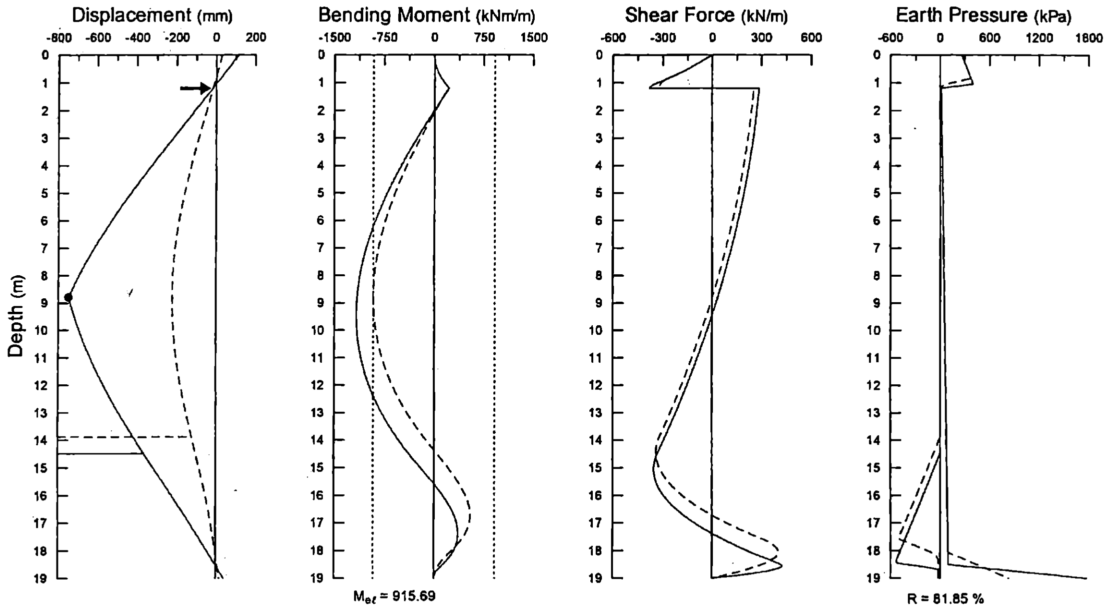
Fages, R. & Bouyat C. 1971, Modèle mathématique integrant le comportement irreversible du sol en état élastoplastique. Exemple d'application. Etude de l'influence des paramètres, *Travaux*, 441 ; pp. 38-46.

Houy, A. 1986. *Dimensionnement des ouvrages en palplanches en acier.* Nancy: Imp. Gueblez.

Monnet A. 1994. Module de réaction, coefficient de décompression, au sujet des paramètres utilisés dans la méthode de calcul élasto-plastique des soutènements. *Revue Francaise de Géotechnique*, 65 : pp. 67-72.

Schmitt P. 1998. De l'élasticité linéaire au coefficient de réaction: théories, observations et ordres de grandeur. *Revue Francaise de Géotechnique*, 85 : pp. 79-87.

APPENDIX I: SINGLE ANCHORED WALL WITH ONE PLASTIC HINGE

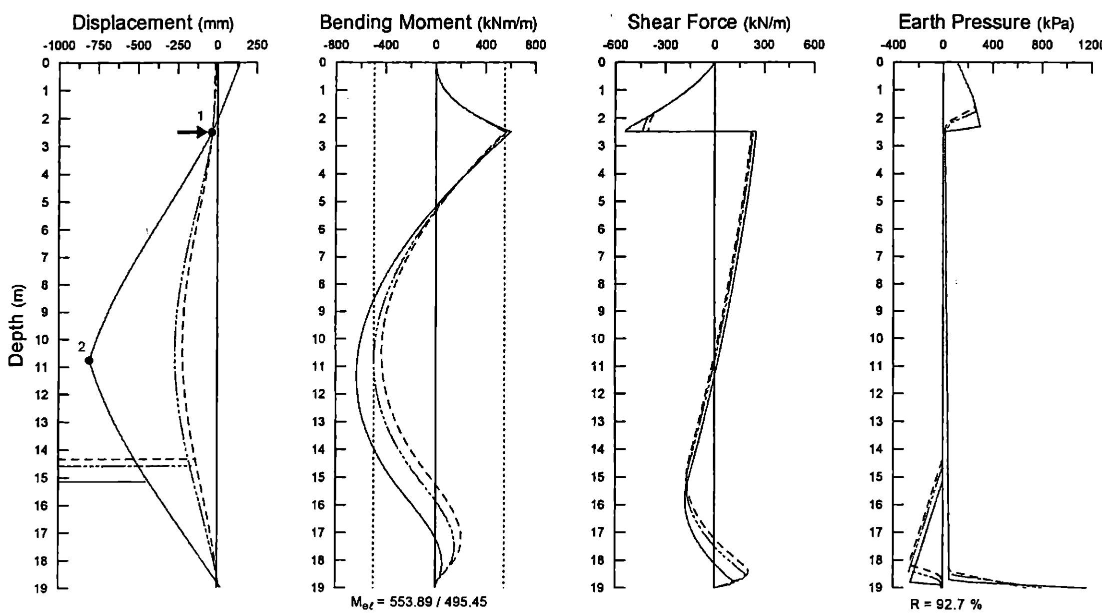


Typical Case n°1
Sand 35°
S 355 GP - SSP3
h = 19 m
Plastic Pile

c1s3g3p3h19

- - - - - - - - SGRM - Excavation - 13.878 m - h_{exc} / h = 73 % (Elastic limit)
———— SGRM - Excavation - 14.470 m - h_{exc} / h = 76.1 % (Failure)

APPENDIX II: SINGLE ANCHORED WALL WITH TWO PLASTIC HINGES

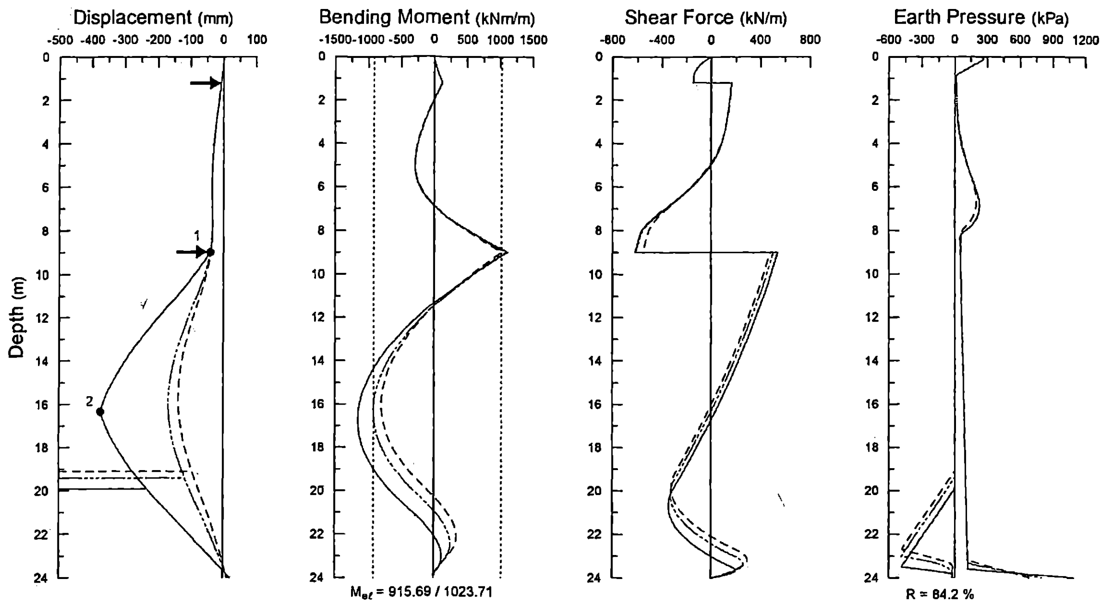


Typical Case n°3
Sand 35°
S 355 GP - SSP1
h=19 m
Plastic Pile

c3s3g3p1h19

- - - - - - - - SGRM - Excavation - 14.340 m - h_{exc} / h = 75.5 % (First plastic hinge)
—·—·—·— SGRM - Excavation - 14.590 m - h_{exc} / h = 76.8 % (Second plastic hinge)
———— SGRM - Excavation - 15.147 m - h_{exc} / h = 79.7 % (Failure)

APPENDIX III: DOUBLE ANCHORED WALL WITH TWO PLASTIC HINGES

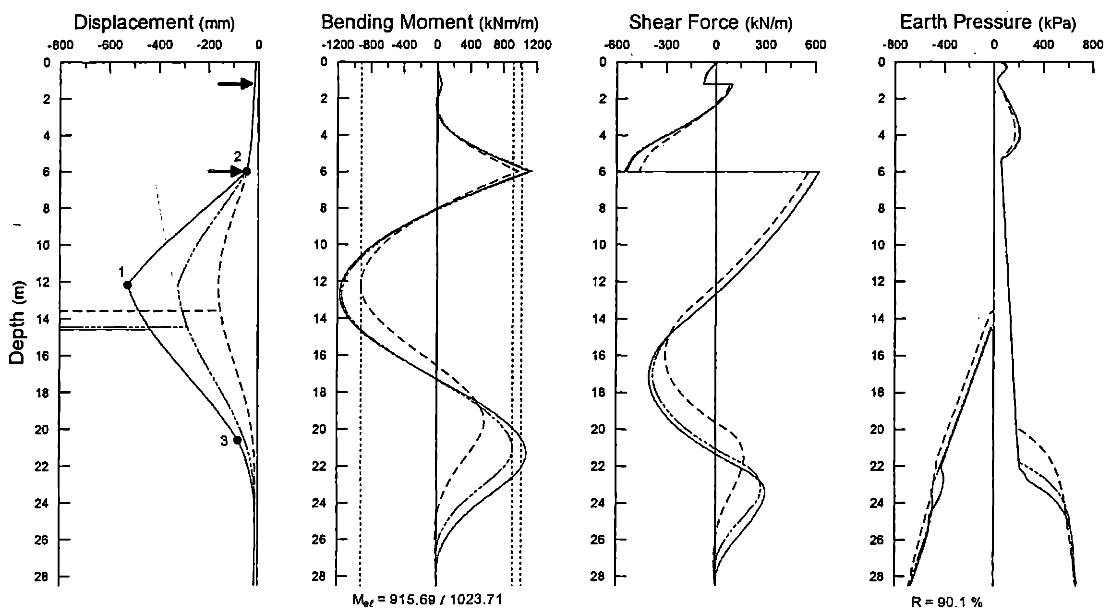


Typical Case n°2
Sand 35°
S 355 GP - SSP3
h=24 m
Plastic Pile

c2s3g3p3h24

- - - - - - - SGRM - Excavation - 19.114 m - h_{exc} / h = 79.6 % (First plastic hinge)
-·-·-·-·- SGRM - Excavation - 19.417 m - h_{exc} / h = 80.1 % (Second plastic hinge)
———— SGRM - Excavation - 19.922 m - h_{exc} / h = 83.0 % (Failure)

APPENDIX IV: DOUBLE ANCHORED WALL WITH THREE PLASTIC HINGES



Typical Case n°2
Stiff clay
S 355 GP - SSP3
h=28.5 m
Plastic Pile

c2cg3p3h285

- - - - - - - SGRM - Excavation - 13.590 m - h_{exc} / h = 47.7 % (First and second plastic hinge)
-·-·-·-·- SGRM - Excavation - 14.455 m - h_{exc} / h = 50.7 % (Third plastic hinge)
———— SGRM - Excavation - 14.590 m - h_{exc} / h = 51.2 % (Failure)