Three-dimensional analysis of ground settlements due to tunnelling: Role of $K_0$ and stiffness anisotropy

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ABSTRACT: Ground settlements due to tunnelling are three-dimensional (3D) in nature, especially at the tunnel heading. A series of 3D elasto-plastic coupled-consolidation finite element analyses has been conducted to investigate the effects of $K_0$ and stiffness anisotropy on ground settlements due to tunnelling.

1 INTRODUCTION

In recent years, the demand for tunnels is increasing worldwide. In each tunnelling project, an accurate and appropriate design is very important for excavating the tunnel safely and economically. With the advance of technology, 3D numerical simulations of complex soil-structure interactions become more and more feasible and economical in engineering practice. Over the years, many researchers have studied the influence of different soil models on surface settlements due to tunnelling. Lee & Rowe (1989) conducted a series of two dimensional (2D) elasto-plastic undrained finite element analyses to determine the effect of elastic anisotropy on surface settlements caused by tunnelling. They found out that the ratio of independent shear modulus to vertical modulus ($G_{vh}/E_v$) governs the shape of a settlement trough. By using a range of $G_{vh}/E_v$ values from 0.2 to 0.25, they could produce a reasonable match between their computed results with the centrifuge model tests conducted in kaolin clay by Mair (1979). Theoretically, $G_{vh}/E_v$ should be equal to 0.33 for the centrifuge model tunnel constructed in an isotropic ground under undrained conditions. Moreover, they found that soil stiffness anisotropy in terms of stiffness ratio and Poisson's ratio (i.e., $n = E_h/E_v < 1.5$ and $v_{hh} \neq v_{vh}$) does not greatly affect the ground settlement due to tunnelling. Addenbrooke (1996) conducted a series of comprehensive 2D (plane strain) numerical analyses of tunnelling in stiff clay with different pre-yield constitutive soil models (isotropic linear elastic, anisotropic linear elastic and isotropic non-linear elastic model) and different initial ground conditions (i.e., $K_0 = 0.5$ and $K_0 = 1.5$). Tunnels with 4.146m diameter excavated in London Clay at different depths (i.e., depth of tunnel axis, $z_0 = 20m$, 29.3m and 34m) were modelled. In his undrained analyses, he showed that there was no significant difference between the isotropic and anisotropic cases regarding predicted ground movements around the tunnel with the use of anisotropic soil parameters ($G_{vh}/E_v' = 0.44$) given by Burland & Kalra (1986) for London clay. He also conducted a parametric study to investigate the effects of $G_{vh}$ and $n$ on surface settlements and concluded that by varying $n$ with a constant $G_{vh}$ the surface settlement profile was not greatly affected. On the other hand, a variation of $G_{vh}$ has more significant influences on surface settlement with a constant $n$. The smaller the $G_{vh}$ value, the deeper surface settlement trough will result. Moreover, by assuming a low $K_0$ zone around a 2D tunnel cross-section, he showed that a deep settlement trough could be reproduced by using a nonlinear isotropic soil model and parameters.

Obviously, a tunnel excavation is truly three-dimensional (3D) in nature. In this paper, a series of 3D elasto-plastic coupled-consolidation analyses was conducted using the finite element method (FEM) to investigate the effects of $K_0$ and soil stiffness anisotropy ($n$) on ground surface settlement due to tunnelling. In the 3D numerical simulations, different $K_0$ values (i.e., 0.5 and 1.5) and soil stiffness anisotropy ratio ($n = E_h/E_v' = 1.0$ for isotropic cases and $n = 1.6$ for anisotropic cases) were considered.

2 THREE-DIMENSIONAL NUMERICAL MODELLING

2.1 Finite Element Mesh and Boundary Conditions

In this 3D numerical study, a hypothetical tunnel excavation in a stiff homogenous overconsolidated London clay layer was modelled. The diameter of the tunnel ($D$) was taken as 9m and a constant cover depth ($C$) of 18m was assumed. The finite element
program, ABAQUS (Hibbitt, Karlsson & Sorensen Inc., 1998), was used to model the tunnel excavation with coupled consolidation.

Figure 1 shows the finite element mesh adopted in the analysis. It should be noted that only half of the tunnel was analysed as a plane of symmetry could be readily identified at x = 0. The finite element mesh was 101.25m long, 45m high and 75m wide. It consisted of approximately 5040 elements and 5642 nodes. A section located at the middle of the mesh (i.e. at y = 0m, called monitoring section), was monitored during every stage of excavation and construction. Soil and shotcrete lining was modelled by eight-noded brick and four-noded shell elements, respectively.

Roller supports were applied on all vertical sides of the mesh while pin supports were assigned to the base of the mesh. Therefore, the movement in normal direction to all vertical sides of the mesh and the movements in all directions at the base of the mesh were restrained. The water table was assumed to be located at the ground surface.

2.2 Constitutive Model, Model Parameters and Simulation Procedures

An elastic-perfectly-plastic soil model governed by the Drucker-Prager failure criterion with a non-associated flow rule was used in this study. In all cases, the effective angle of friction (\(\phi'\)) was assumed to be 22° and the angle of dilation (\(\psi\)) was assumed to be 11°. The effective cohesion (\(c'\)) was 5kPa. The soil stiffness parameters published by Burland & Kalra (1986) for the London clay at the New Queen Elizabeth II Conference Centre were adopted to simulate the clay layer. The variations of \(E_v'\) and \(E_h'\) were assumed to increase linearly with depth. The ratio of the independent shear modulus (\(G_{shh}\)) to the vertical effective Young's Modulus (\(E_v'\)) was assumed to be 0.44 in all cases. The initial \(K_0\) conditions were assumed to be either 0.5 or 1.5. All the soil parameters used in this study are summarised in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_v') (kPa)</td>
<td>(12000 + 6240z)</td>
<td></td>
</tr>
<tr>
<td>(E_h') (kPa)</td>
<td>(12000 + 6240z)</td>
<td></td>
</tr>
<tr>
<td>(\nu_{sh} = \nu_{sh}^h)</td>
<td>(0.125)</td>
<td>0.125</td>
</tr>
<tr>
<td>(G_{sh} / E_v')</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>(\rho_s ) (kN/m³)</td>
<td>15*</td>
<td>15*</td>
</tr>
<tr>
<td>(e)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(k (m/s))</td>
<td>(1 \times 10^{-9})</td>
<td></td>
</tr>
<tr>
<td>(c' ) (kPa)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>(\phi') (°)</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>(\psi) (°)</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

Note: * equivalent to saturated unit weight of 20 kN/m³

3 COMPUTED RESULTS

3.1 Transverse Ground Settlements

As suggested by Peck (1969), measured transverse surface settlements in the field can be represented by a normal distribution or Gaussian distribution as follows:

\[
S_x = S_{max} \exp\left(-x^2 / 2i^2\right)
\]

where \(S_x\) is the transverse surface settlement; \(S_{max}\) is the maximum transverse surface settlement on the tunnel centreline; \(x\) is the transverse distance from the tunnel centreline and \(i\) is the point of inflection of the settlement trough. New and Bowers (1994) have reported measured values of \(S_{max}\) and \(i\) for oval-shaped tunnels excavated by the New Austrian Tunnelling Method (NATM) in London clay. The tunnel had a face area of about 59m² with an equivalent tunnel diameter of 8.7m and the depth of the tunnel axis (\(z_0\)) was located at approximately 21m below ground. The measured values of \(S_{max}/D\) and \(i/z_0\) ranged from 0.24% to 0.46% and from 0.35\(z_0\) to 0.48\(z_0\), respectively and volume losses lied between 1.1% and 1.4%. For shield tunnels constructed in London clay, Lake et al. (1992) reported that the measured values of \(S_{max}/D\) ranged from 0.10% to
0.17% and $i/z_0$ lied between 0.45 and 0.52. The volume losses were ranging from 0.19% to 1.38% (based on reported $S_{\text{max}}$ and $i$).

Figure 2 shows the normalised immediate transverse surface settlements induced at the monitoring section (at $y = 0$) when the tunnel face just reaches it. For the $K_0 = 0.5$ and $n = 1.6$ case, it gives the deepest settlement trough. The $S_{\text{max}}$ is 0.18%D with $i/z_0 = 0.44$ (Note that the $i$ value was deduced from the numerical analysis). For the case with $K_0 = 0.5$ and $n = 1.0$, the $S_{\text{max}}$ reduces to 0.09%D and $i$ increases to 0.51$z_0$ (see Table 2a). Obviously, $n$ has significant influence on the magnitude and the shape of surface settlements. Since stress change due to tunnelling is governed by $K_0$ values only, irrespective $n$ value used in the analysis. Thus, for a given $K_0$, the higher the $n$ value, the smaller the ratio of the horizontal to vertical strain. This leads to a deeper settlement trough. Similar trends are also obtained for the $K_0 = 1.5$ cases.

On the other hand, for a given constant $n = 1.6$, the $S_{\text{max}}$ reduces from 0.18%D ($K_0 = 0.5$ case) to 0.06%D ($K_0 = 1.5$ case) and $i$ increases from 0.44$z_0$ ($K_0 = 0.5$ case) to 0.58$z_0$ ($K_0 = 1.5$ case). Similar trends are also obtained for $n = 1.0$ cases. It is clear that at a given $K_0$, the ground settlement trough becomes shallower and wider as $n$ reduces from 1.6 to 1.0 whereas at a constant $n$, the ground settlement becomes shallower and wider as $K_0$ increases from 0.5 to 1.5. Although the effect of $n$ on the magnitude of maximum surface settlements is more significant in the low $K_0$ case than that in the high $K_0$ case, the ratio of the maximum surface settlement computed in the $n = 1.6$ case to that in the $n = 1.0$ case is 2.65 and 1.96 for $K_0 = 1.5$ case and $K_0 = 0.5$ case, respectively.

Based on the $S_{\text{max}}$ and $i$ deduced from each numerical analysis, a Gaussian distribution is fitted to the transverse surface settlement trough and plotted in Figure 2 for comparisons. It can be seen from the figure that immediate surface settlements can be represented by Gaussian distributions for the two $K_0 = 0.5$ cases (i.e., $n = 1.6$ and 1.0). Parameters obtained to describe the Gaussian distributions are given Table 2a. As $K_0$ increases, the discrepancy between the Gaussian distribution and the numerical prediction appears to be more significant at the remote boundary (away from the tunnel). For $K_0 = 1.5$ and $n = 1.6$ case, the numerical result gives a much wider settlement trough than the Gaussian distribution.

In addition for the $K_0 = 1.5$ and $n = 1.0$ case, numerical predicted settlements do not show the form of a Gaussian distribution as the $S_{\text{max}}$ does not occur at the tunnel centreline (i.e. at $x/D = 0$).

Table 2a – Summary of $S_{\text{max}}$, $i$ and $V_L$ for the immediate surface settlement troughs obtained from the numerical analyses.

<table>
<thead>
<tr>
<th>$K_0$</th>
<th>0.5</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>$S_{\text{max}}/D$ (%)</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>$i/z_0$</td>
<td>0.44</td>
<td>0.51</td>
</tr>
<tr>
<td>$V_L$ (%)</td>
<td>0.64</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Table 2b – Summary of $S_{\text{max}}$, $i$ and $V_L$ for the plane strain surface settlement troughs obtained from the numerical analyses.

<table>
<thead>
<tr>
<th>$K_0$</th>
<th>0.5</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>$S_{\text{max}}/D$ (%)</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td>$i/z_0$</td>
<td>0.47</td>
<td>0.80</td>
</tr>
<tr>
<td>$V_L$ (%)</td>
<td>0.84</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Figure 3 shows the normalised transverse surface settlements at the monitoring section when the section reaches the plane strain condition (i.e., when the tunnel face advances to a distance at 3 times the tunnel diameter or more away from the monitoring section). Similar to the immediate surface settlements directly above the tunnel face, for a given $K_0$, the magnitude of plane strain surface settlements decreases with the $n$ value. On the other hand, for a given $n$ value, the magnitude of plane strain surface settlement decreases as $K_0$ increases.

The deduced $S_{\text{max}}/D$ and $i/z_0$ from the numerical analyses for the plane strain surface settlements are summarized in Table 2b. By comparing the deduced values and the field observations from shield tunnels reported by Lake et al. (1992), it is clear that computed results from the two analyses using $K_0 = 0.5$ are generally consistent with field measurements. However, this is not the case for predictions obtained from analyses using $K_0 = 1.5$.

Previous studies have revealed that $G_{vh}$ has a significant effect on the depth of plane strain surface settlement (Simpson & Ng, 1995 and Addenbrooke, 1996). Using a smaller value of $G_{vh}$ in an analysis...
will predict a deeper settlement trough. In the current study, \( G_{vh} \) is assumed to be a function of \( E' \) (i.e., \( G_{vh}/E' = 0.44 \)). Since \( E' \) used in the \( n = 1.6 \) analysis is smaller (see Table 1) than that in \( n = 1.0 \) one, this results in a smaller value of \( G_{vh} \) in the former than that in the latter analysis. Therefore, for a given constant \( K_0 \), the plane strain surface settlements for \( n = 1.6 \) are deeper than those in the \( n = 1.0 \) cases (see in Figure 3). The same explanations are likely applicable to the immediate settlements shown in Figure 2.

In Figure 3, the results from a relevant two dimensional plane strain numerical study of tunnelling at 20m below ground (\( z_0 = 20m \)) in London clay reported by Addenbrooke (1996) are also included for comparisons. The surface settlements shown are obtained from isotropic and anisotropic linear elastic analyses. Since the tunnel diameter and the depth of tunnel axis in the Addenbrooke’s study (\( D = 4.146m, z_0 = 20m \)) are different from this current study (\( D = 9m, z_0 = 22.5m \)), only qualitative comparisons are possible. It can be seen from the figure that the general trends of the predicted settlement troughs from the Addenbrooke’s and this study are consistent. Both sets of analyses show the importance of \( K_0 \) in predicting ground settlement profiles due to tunnelling. However, for a given \( K_0 \), it appears that the effects of \( n \) value on the depth of surface settlement trough in this study are far more significant than that in Addenbrooke’s one. In his study, surface settlements were mainly governed by \( K_0 \), but were very little influenced by \( n \) value. This may be attributed to the undrained assumptions (i.e., constant volume) used in his study.

Moreover, the wider settlement troughs (for \( K_0 = 0.5 \) cases) obtained from the Addenbrooke’s study than those from this study, may be attributed to different modelling techniques adopted in these two types of analyses. In the 2D plane strain analyses conducted by Addenbrooke, a prescribed volume loss was imposed in his analyses. The tunnel lining was applied to the tunnel wall until the prescribed volume loss (i.e., \( V_s = 1.3\% \)) was reached. However, in this study, actual tunnel excavation sequence was closely simulated three-dimensionally. Soil elements were removed in the tunnel and lining was applied to the tunnel wall at a specific unsupported span behind the tunnel face. Therefore, it is not surprising to obtain different settlement profiles from different modelling techniques.

Due to the assumption of linear elastic soil stiffness within the yielding surface, the predicted volume losses in this study range from 0.38% to 0.84% (see Table 2b), which fall between measured values of 0.19% to 1.4% for shield tunnels constructed in London clay. However, the computed values are smaller than the measured ranges from 1.1% to 1.4% for tunnels constructed using the NATM in London clay. It is believed that the predicted volume loss will increase if soil stiffness is allowed to vary with strains within the yield surface. Also Tang (2001) demonstrated that volume loss in a 3D linear elastic analysis is proportional to the length of unsupported span. The longer the unsupported span assumed in an analysis, the larger the volume loss will be.

Figure 3 shows the comparisons between normalised immediate and plane strain surface settlements obtained in the 3D analyses. For \( K_0 = 0.5 \), there is a significant increase in surface settlements at the monitoring section when the tunnel face advances from the monitoring section to a distance at three times of the tunnel diameter away from the monitoring section (i.e., the monitoring section reaches the plane strain conditions). The increased maximum settlement is about 22% and 56% for \( n = 1.6 \) and 1.0, respectively (refer to Table 2a and 2b). On the con-
trary, there is no major difference between the computed immediate and plane strain settlements for $K_0 = 1.5$. Due to the low water permeability in the clay, the increase in settlement for the low $K_0$ case cannot be mainly attributed to the dissipations of excess pore water pressure. In fact, the increase in settlement in the plane strain section is attributed to the significant increase in vertical effective stress beneath the invert. Relatively speaking, there is only a small increase in the vertical stress for the high $K_0$ case. Details of the 3D stress transfer mechanisms during the tunnel advancement are given by Ng & Lee (2003).

### 3.2 Longitudinal Surface Settlements

Attewell and Woodman (1982) studied a number of case histories of tunnel excavation in clays and they suggested a cumulative probability function for estimating longitudinal settlement profile due to tunnelling as follows:

$$S_y = \frac{V_s}{\sqrt{2\pi i}} \exp\left(-\frac{x^2}{2i^2}\right) G\left(\frac{y - y_i}{i}\right) G\left(\frac{y - y_f}{i}\right)$$

where $S_y$ is the longitudinal surface settlements; $V_s$ is the volume of the transverse settlement trough per unit distance of tunnel advancement; $x$ is the transverse distance from the tunnel centreline; $i$ is the point of inflection of the settlement trough; $y_i$ is the start point of tunnel; $y_f$ is the final position of tunnel face, and probability function $G$ is defined as

$$G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{\beta^2}{2}\right) d\beta$$

the result of the above probability function can be obtained from standard probability tables.

Attewell & Woodman (1982) found that the immediate surface settlement directly above the tunnel face corresponds to about 0.5$S_{\text{max}}$ (at the plane strain conditions) for tunnel constructed in stiff clays without face support. However, for tunnels excavated in soft clay with full-face support (e.g., shield tunneling), the surface settlements are mainly caused by movements at the tail void. The immediate surface settlement directly above the tunnel face is considerably less than 0.5$S_{\text{max}}$. Therefore, a translation of the cumulative probability function is required in order to match the field observation.

Figure 5 compares computed longitudinal settlements along the centreline of the tunnel ($x = 0$) by the numerical analyses and predictions using the cumulative probability function. For the $K_0 = 0.5$ cases, the ground surface behind the tunnel face slightly heaves when the tunnel face reaches the monitoring section. Therefore, the longitudinal settlement troughs cannot be represented by the cumulative probability function.

For the $K_0 = 1.5$ cases, the ground surface behind the tunnel face slightly heaves when the tunnel face reaches the monitoring section. Therefore, the longitudinal settlement troughs cannot be represented by the cumulative probability function.

### 4 CONCLUSIONS

Based on the 3D numerical simulations of the tunnel advancement at 2.25m/day (unsupported length), it is clear that surface ground settlements including immediate and plane strain settlements are governed by the combined effects of $K_0$ and anisotropic stiff-
ness ratio, \( n \). A combination of low \( K_0 \) condition with high degree of stiffness anisotropy will produce the deepest settlement trough. For the cases considered in this paper, it appears that the effects of \( K_0 \) are relatively more important than stiffness anisotropy on the calculations of ground settlements. Only under the low \( K_0 \) ground conditions, computed transverse and longitudinal settlements can be fitted well by empirical Gaussian distributions and Attewell and Woodman’s cumulative probability functions, respectively.

By comparing the computed transverse immediate and plane strain settlements, it is found that there is a significant increase in surface settlements at the monitoring section under the low \( K_0 = 0.5 \) ground conditions. The increased maximum settlement at the monitoring section as the tunnel face advances from the monitoring section to a distance three times the tunnel diameter away are about 22% and 56% for \( n = 1.6 \) and 1.0, respectively. On the contrary, there is no major difference between the computed immediate and plane strain settlements for \( K_0 : A 1.5 \).

ACKNOWLEDGEMENTS

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REFERENCES


APPENDIX I. NOTATIONS

\begin{align*}
C & \quad \text{Cover depth} \\
c' & \quad \text{Effective cohesion} \\
D & \quad \text{Tunnel diameter} \\
E_h & \quad \text{Horizontal effective Young's modulus of soil} \\
E_v & \quad \text{Vertical effective Young's modulus of soil} \\
G_{sh} & \quad \text{Independent shear modulus} \\
i & \quad \text{Point of inflection of the settlement trough} \\
k & \quad \text{Coefficient permeability of soil} \\
K_0 & \quad \text{Coefficient of earth pressure at rest} \\
n & \quad \text{Ratio of horizontal effective Young's modulus to vertical effective Young's modulus of soil} \\
S_{max} & \quad \text{Maximum surface settlement} \\
S_x & \quad \text{Transverse surface settlement} \\
S_y & \quad \text{Longitudinal ground settlement} \\
V_L & \quad \text{Volume of ground loss expressed as a fraction of the tunnel area} \\
V_s & \quad \text{Volume of the transverse settlement trough per unit distance of tunnel advancement} \\
x & \quad \text{Horizontal distance from tunnel centreline} \\
y & \quad \text{Horizontal distance measured in the longitudinal direction} \\
y_i & \quad \text{Start point of tunnel} \\
y_f & \quad \text{Final position of tunnel face} \\
z & \quad \text{Depth measured from ground surface} \\
Z_0 & \quad \text{Depth of tunnel axis} \\
\phi' & \quad \text{Effective angle of friction} \\
\nu_{vh} & \quad \text{Effective Poisson's ratio for the effect of vertical stress on horizontal strain} \\
\nu_{hh} & \quad \text{Effective Poisson's ratio for the effect of horizontal stress on horizontal strain} \\
p_d & \quad \text{Dry density} \\
\psi & \quad \text{Angle of dilation}
\end{align*}