Estimating displacements associated with deep excavations

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ABSTRACT: Methods for estimating excavation-induced displacements generally fall into three categories: empirical "envelopes" of measurements, structural analyses, and numerical modeling. A few methods combine structural and geotechnical engineering principles but these largely focus on estimating the performance of deep excavations in soft clay. This paper summarizes available design tools, problems in their application, and proposes a new semi-empirical method for this soil structure interaction problem. The proposed method clarifies the distinction between performance of braced and tied-back excavations, isolates several construction variables, and allows estimation of the magnitude and shape of the displacement patterns.

1 INTRODUCTION

Over the last 40 years, methods for estimating displacements of deep excavations have received considerable attention. Available methods generally fall into three categories. Empirical "envelopes" of measured ground displacements, often segregated by soil or wall type, are frequently used but these are somewhat crude. Structural analysis methods are sometimes used for estimating displacements but these suffer from reliance on assumptions of earth pressures that are often greatly simplified and do not address the displacements of the surrounding ground. Numerical modeling is a tool for predicting the performance of excavation support systems, but this comes at the price of increased complexity, cost, and potential for errors or misleading results. A few methods combine structural and geotechnical principles but these focus largely on excavations made in soft clay. This paper summarizes available tools for estimating excavation performance, problems in their application, and proposes a new semi-empirical method for estimating displacement.

2 HISTORICAL BACKGROUND

To compare bending of a model sheet-pile wall with full-scale prototypes, Rowe (1952) used the differential equations for deflection of a simple beam with small angles of rotation at the supports:

$$d\delta/dz = \mathrm{Mz/EI};\ d^2\delta/dz^2 = \mathrm{M/EI}$$
$$d^3\delta/dz^3 = \mathrm{V/EI};\ d^4\delta/dz^4 = q/\mathrm{EI} \qquad (1)$$

where q = distributed load; δ = displacement; z = distance in the vertical plane; M = bending moment; E = modulus of elasticity; and I = internal moment of inertia per unit wall length. He then compared the two systems using:

$$(d\delta/dz)_{\mathrm{model}} = (d\delta/dz)_{\mathrm{prototype}}$$
$$(\mathrm{Mz/EI})_{\mathrm{model}} = (\mathrm{Mz/EI})_{\mathrm{prototype}} \qquad (2)$$

normalized the differential distance, z, by the total height of the wall (including penetration depth), H, for both model and prototype:

$$(\mathrm{z/H})_{\mathrm{model}} = (\mathrm{z/H})_{\mathrm{prototype}} = \eta \qquad (3)$$

and considered that, for given distributed load, relative positions of the top anchor level and penetration depth, the bending moment is proportional to H^3 and thus:

$$(\mathrm{M/H^3})_{\mathrm{model}} = (\mathrm{M/H^3})_{\mathrm{prototype}} = \tau \qquad (4)$$

Assuming that bending moment is proportional to H^3 is contingent upon the load, P, depending upon H (e.g. $P = 1/2\gamma K_a H^2$ for a simple beam subject to a triangular pressure load). By substitution then:

$$[\mathrm{H^3}\eta\mathrm{H}\tau/(\mathrm{EI})]_{\mathrm{model}} = [\mathrm{H^3}\eta\mathrm{H}\tau/(\mathrm{EI})]_{\mathrm{prototype}} =$$
$$\mathrm{H^4/EI} = \text{constant } \rho \qquad (5)$$

though the ρ parameter is dimensionally clumsy.

Caspe (1966) estimated surface settlement based on knowing lateral wall displacements. In this approach, the distance limit of the vertical displacements (zone of influence), D, was defined based on the height of the excavation as follows:

$$\mathrm{D} = (\mathrm{H} + \mathrm{H_d})\tan(45 - \phi'/2) \qquad (6)$$

where $H =$ total excavation depth; and $H_d =$ width of the excavation, B, for cohesive soils, or 0.5B tan(45 – $\phi'/2$) for granular soils. He and Kane (1966) related the total cross section area of the horizontal displacement profile, A_h, to the total area of the vertical displacement profile, A_v, through use of Poisson's ratio, μ, to compare vertical and horizontal volumetric strains, ε_{vert} and ε_{horiz}, respectively, where by:

$$\varepsilon_{vert} = \mu/(1-\mu)\varepsilon_{horiz} \quad (7)$$

Observed magnitudes and patterns of settlements adjacent to excavations supported by soldier-piles and lagging or sheet-piles were illustrated by Peck (1969) in which measurements were generalized by "envelopes" of maximum displacement for different ground types. Goldberg et al. (1976) separated deformation behavior by both soil and wall types. They related deformation to the inverse of the flexibility number using EI/h^4, where h is the average vertical distance between support levels.

Using parametric finite element studies and published measurements, Mana and Clough (1981) evaluated factors influencing the maximum lateral and vertical movements for excavations in soft to medium-stiff clays. The parameters they investigated were: α_w, wall stiffness; α_S, strut stiffness and spacing; α_D, depth to an underlying firm layer; α_B, excavation width; α_P, strut preload; and α_M, elastic modulus of the retained earth. They suggested that an initial estimate of the maximum horizontal wall movements, δ^*_{hmax}, could be modified depending on these factors (see Figure 1):

$$\delta_{hmax} = \delta^*_{hmax}\alpha_W\alpha_S\alpha_D\alpha_B\alpha_P\alpha_M \quad (8)$$

Importantly, they introduced the concept of nondimensional relative stiffness, given the term S_r in this paper. Non-dimensional stiffness is defined as $EI/(h^4\gamma)$, where I is defined as the internal moment of inertia per unit length of the wall. For walls composed of discrete structural elements, such as soldier piles, the moment of inertia must be divided by the horizontal distance between such components. Clough et al. (1989) expanded upon this work to produce a chart relating maximum lateral displacement to the base stability factor of safety and relative wall stiffness (see Figure 1, top left) when all factors in Equation (8) equal 1.0.

Bowles (1988) assumed that the maximum ground surface settlement, δ_{vmax}, would occur next to the wall and that this settlement could be approximated as:

$$\delta_{vmax} = 4V_S/D \quad (9)$$

where V_s is the cross section area (or unit volume) of the surface settlement trough. This method assumed that the surface settlement, δ_v, at any point at a distance d from the wall can then be determined by the parabolic equation:

$$\delta_v = \delta_{vmax}[(D-d)/D]^2 \quad (10)$$

Clough and O'Rourke (1990) summarized methods to estimate maximum ground surface deformations associated with construction of excavations and provided another series of charts illustrating envelopes of ground displacement for different soil types (soft clay, stiff clay, and sand) as well as noting wall types where possible.

Addenbrooke (1994) integrated the simple beam equation to find the displacement, y, over the span distance, h, and defined a "displacement flexibility number", Δ, for use in estimating displacements:

$$\Delta = h^5/(EI) \quad (11)$$

If earth loads are proportional to H and the loading or bending moment equation is integrated to derive deflection, then deflection will indeed be related to h^5. However, when normalizing the complete deflection equation by H (or h) to relate deformation to the height of the wall (or span between supports), the resulting variables should be δ_h/H versus some form of H^4 (or h^4) as utilized by Mana and Clough (1981). The parameter Δ. (per unit length) is also a somewhat counter-intuitive dimensional value with units of $length^2/force$ and there is little intrinsic theoretical or practical benefit to the use of Δ.

Hsieh and Ou (1998) identified two displacement patterns that constitute vertical settlement profiles: spandrel-shaped deformation, A_s, from cantilever bending; and inward bulging or convex bending, A_c. Their method based surface settlement on lateral displacements estimated using finite element or beam on elastic foundation methods. they also stated that "*. . . the deflection of a supported wall during excavation can usually be predicted using the finite element method with good accuracy. However, prediction of the ground surface settlement induced by excavation is not as good as wall deflection*." This begs the question: why can lateral displacement results from finite element analyses by believed if the vertical displacements are unbelievable?

Long (2001), using a large database of published performance results, compared the prediction methods of Clough et al. (1989), Clough and O'Rourke (1990), and Addenbrook (1994) and concluded that the trends illustrated by Clough et al. (1989) were supported by the data slightly better than others, though no approach was judged superior. Clearly, a systematic and rational approach to analytically estimating displacements of supported excavations is necessary.

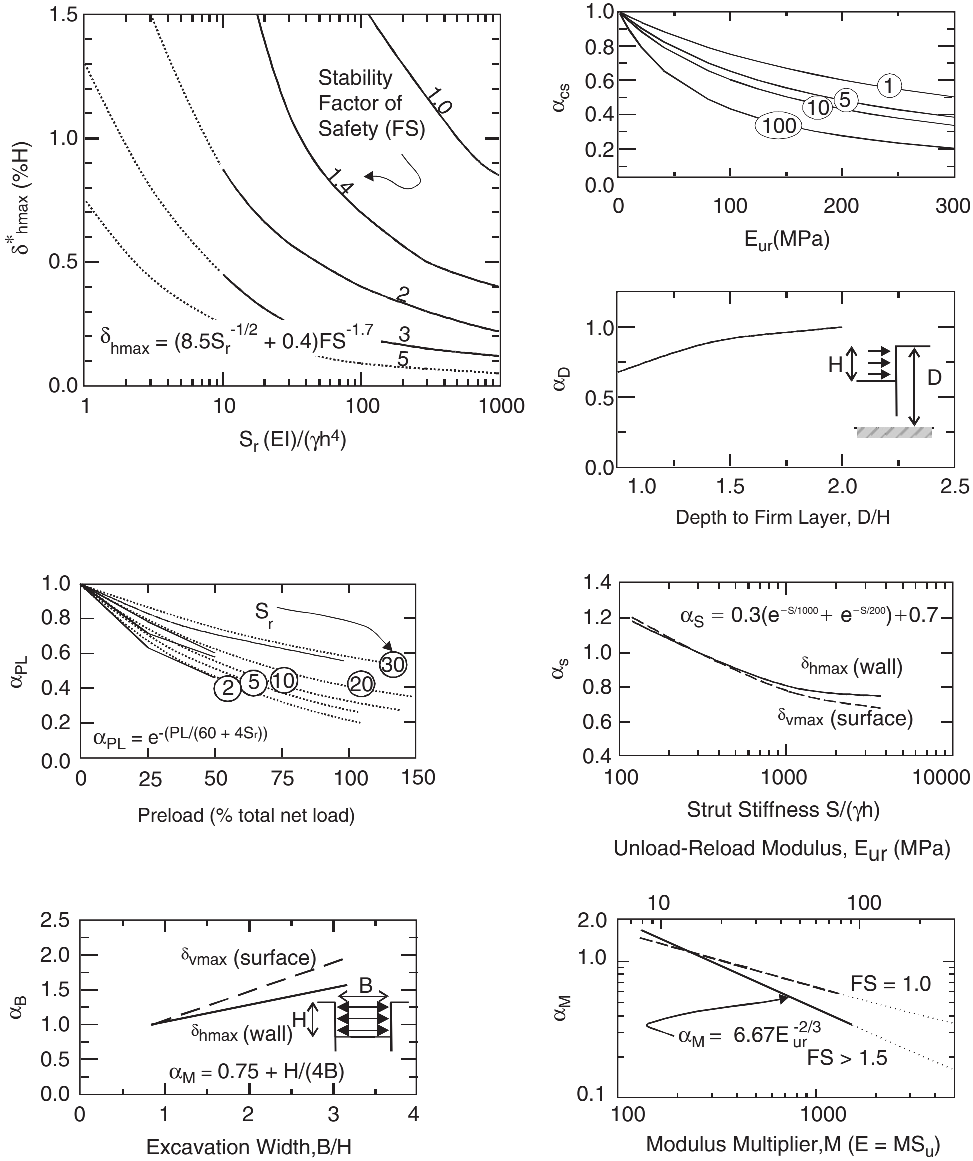

Figure 1. Maximum lateral displacements of excavation support systems and modification factors to account for the influence of support system stiffness and factor of safety, construction stage (removal of supports), depth to a firm layer, degree of preloading, strut stiffness, excavation width, and soil stiffness (after Mana and Clough 1981, Boone et al. 1999, Boone 2003).

3 A NEW APPROACH TO DISPLACEMENT ESTIMATION

Parametric non-linear numerical modeling trials were completed by Boone (2003) to examine excavation support behavior for given design conditions. It was considered that the results might discern patterns that could, through curve fitting, produce closed-form equations for this soilstructure interaction problem.

A non-linear hyperbolic soil stiffness model with a Mohr-Coulomb failure criterion and linear unloading-reloading response (Duncan and Chang 1970) was used to simulate soil behavior. The majority of trials were completed with the ground mass assumed

Table 1. Summary of displacement estimation equations based on curve-fitting of non-linear numerical modeling results.

Characteristic	Condition	Equation
Maximum lateral displacement		
maximum unfactored lateral displacement, δ^*_{hmax}	support installation and removal	$\delta^*_{hmax} = (8.5S_r + 0.4)FS^{-1.7}$
construction Stage, α_{cs}	supports removed	$\alpha_{CS} = 1$
	tiebacks remaining stressed	$\alpha_{CS} = 1 - \frac{(E_{ur}/p_a)}{3000/S_r^{0.3} + (E_{ur}/p_a)}$
preloading, α_{PL}	percent of preload maintained	$\alpha_{PL} = e^{-\{PL/(60+4S_r)\}}$
excavation width, α_B		$\alpha_B = 0.75 + H/(4B)$
strut stiffness, α_S		$\alpha_S = 0.3(e^{S_r/1000} + e^{S_r/200}) + 0.7$
soil modulus, α_M		$\alpha_M = 6.67E_{ur}^{-2/3}$
max. lateral displacement, δ_{hmax}		$\delta_{hmax} = \delta^*_{hmax}\alpha_M\alpha_S\alpha_{PL}\alpha_D\alpha_B\alpha_{CS}$
Ground surface displacements		
maximum lateral displacement at surfacce, $\delta_{hsurface}$	Supports remain in place	$\frac{\delta_{hsurfacce}}{\delta_{hmax}} = \frac{(E_{ur}/p_a)}{500 + (E_{ur}/p_a)S_r^{0.2}}$
	supports removed	$\frac{\delta_{hsurfacce}}{\delta_{hmax}} = \frac{(E_{ur}/p_a)}{700} \leq 1.0$
Lateral displacement areas		
Area of lateral spandrel displacement, A_{hs}		$A_{hs} = \delta_{hsurfacce}(H + H_p)/2$
Ratio of spandrel displacement to total displacement area, A_{hs}/A_{ht}	End of excavation stage	$\frac{A_{hs}}{A_{ht}} = \frac{(E_{ur}/p_a)}{1,600 + (E_{ur}/p_a)S_r^{0.35}}$
	After support removal	$\frac{A_{hs}}{A_{ht}} = \frac{(E_{ur}/p_a)}{300 + (E_{ur}/p_a)}$
Area of convex displacement, A_{hc}		$A_{hc} = 1 - A_{hs}/A_{ht}$
Ratios of vertical and lateral displacement areas		
Ratio of vertical and horizontal displacement areas	Cantilever walls	$A_{vs}/A_{hs} = A_{vc}/A_{hc} = A_{vt}/A_{ht} = 1$
	Supports remain in place	$A_{vs}/A_{hs} = A_{vc}/A_{hc} = A_{vt}/A_{ht} = 0.85$
	Supports removed	$A_{vs}/A_{hs} = A_{vc}/A_{hc} = A_v/A_h = 1.1$ (no dilation)
Spandrel portion of settlement trough		
Maximum settlement, δ_{vsmax}		$\delta_{vsmax} = 3A_{vs}/D_s$; $D_s \approx 1.2H$ to $1.5H$
Settlement at any point, δ_{vs}		$\delta_{vs} = \delta_{vsmax}3[(D_s - d)/D_s]^2$; $D_s \approx 1.2H$ to $1.5H$
Concave settlement portion of settlement trough		
Maximum settlement, δ_{vcmax}		$\delta_{vcmax} = \frac{A_{vc}}{[1 - \Phi(0, d_{min}, i)]\sqrt{2\pi\, i}}$
Settlement at any point, δ_{vc}		$\delta_{vc} = e^{\frac{(d - d_{min})^2}{2i^2}}$
	D_c = twice the distance from the wall top to the position of the load resultant i = inflection point, defined as $(D_c - d_{min})$/constant, where the constant $\approx$ 4 to 5 Φ = area of standard normal distribution function, with random variable = 0 (wall position), mean = d_{min}, and standard deviation = i	
Complete settlement profile		
Total settlement at any point		$\delta_v = \delta_{vc} + \delta_{vs}$

to consist of granular soils. Trials were also completed assuming cohesive soils similar to those used by Mana and Clough (1981) as a check on the results.

One of the more important conclusions was that displacement behavior associated with deep excavations is governed in large measure by the unload-reload stiffness properties of the soil mass, whether or not the soil was predominantly granular or cohesive in nature. Vertical displacements are not often well-represented by finite element models because, when using

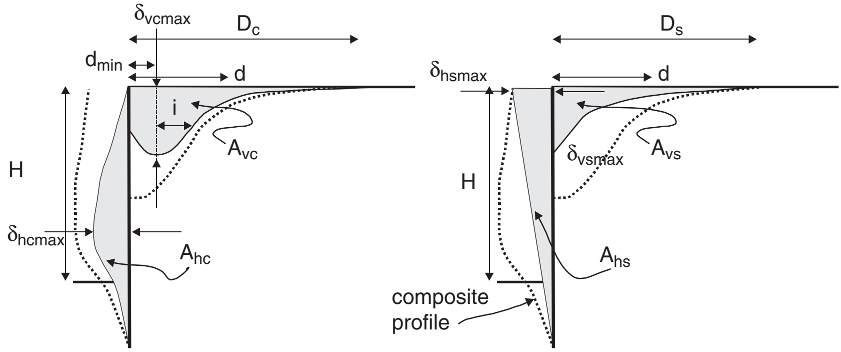

Figure 2. Definitions of lateral and vertical displacement parameters: concave on left, spandrel on right (after Boone 2003).

linear-elastic constitutive models, the ground mass in and beyond the excavation rebounds upward disproportionately in comparison with lateral displacements. By using an unload-reload modulus that is larger than the virgin-loading modulus, vertical rebound is restricted and vertical settlement caused by lateral shoring displacements become evident and more consistent with real observations.

The parametric modeling results were combined with those of Mana and Clough (1981), Clough et al. (1989), and Boone et al. (1999). The resulting plots of excavation support behavior influence factors are illustrated in Figure 1. Through curve-fitting, a series of equations were developed that could be implemented in spreadsheets without resort to use of charts (see Table 1) for estimating displacements. All parameters are defined in Figure 2.

Prior to estimating displacements for an excavation support system, it is necessary to first choose a trial value of relative stiffness. A gauge of an appropriate value can be based on Figure 1. It is then necessary to estimate the excavation factor of safety as an index of the shear strength and deformation modulus mobilized within the retained soil mass. Estimating this factor of safety is best accomplished using limit equilibrium methods (Sabatini et al. 1998, Boone 2003). Based on the relative stiffness and stability factor of safety, the maximum horizontal displacement for the basic conditions (e.g. zero pre-load, removal of supports during backfilling, etc.) may be defined by use of Figure 1 or Table 1. Then, using displacement modification factors provided in Table 1, apply these to estimate the maximum horizontal displacement for the subject analysis conditions. Once the maximum lateral displacement is determined, the equations in Table 1 can be used to define the lateral displacement at the ground surface (at the wall face) and the remaining characteristics of the vertical displacement profiles through superposition of the spandrel and concave profiles as illustrated in Figure 2.

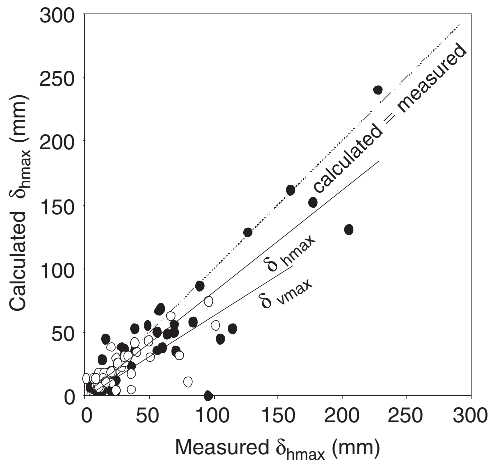

Figure 3. Comparison of measured and calculated lateral (solid) and vertical (open) displacements using a limited database (after Boone 2003).

4 DISCUSSION & CONCLUSIONS

The method of estimating displacements proposed in this paper provides a logical and step-wise approach to account for a variety of the most important factors that govern the movement of excavation support structures. Results obtained using this method are consistent with both observations and empirical charts, as well as with prior numerical studies (Boone 2003). Comparisons of measured and back-calculated displacements for 69 and 38 cases of lateral and vertical displacements, respectively, are illustrated in Figure 3. This figure demonstrates some reasonable agreement between the estimation method and field performance where regression coefficients for the trend lines were calculated to be about 0.89 and 0.65 for the lateral and vertical displacements, respectively.

Figure 3 suggests that the method may under-predict vertical settlements; however, the following issues must be considered:

- published cases in which all required data were included were limited;
- published information regarding when the measurements were taken in the construction sequence (e.g., end of excavation, after struts removed, etc.) was sometimes unclear; and
- other construction factors may have influenced performance.

Future comparisons will be made between preconstruction predictions and construction monitoring results, and to a more refined database of case histories in order to gauge the reliability of this approach.

The proposed method can serve as either a check on numerical results, or provide a more rigorous design approach than empirical charts and clarifies the distinction between braced and tiedback excavations, isolates a number of construction variables, and allows direct estimation of both the magnitude and shape of the displacement patterns.

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