*Theme 4: Safety issues, risk analysis hazard management and control*

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# Research on stochastic seismic analysis of underground pipeline based on physical earthquake model

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ABSTRACT: In this paper, a new approach for the stochastic analysis of finite element modeled underground pipeline system under earthquake excitations is proposed, where a new developed physical random earthquake model and the probability density evolution method are adopted. Based on the physical process of seismic spread, the random earthquake model adopted not only indicates the physical relationship between the random earthquake ground motion and several key random parameters, but also presents the probability configuration of random seismic motions. Associating with the developed probability density evolution method for stochastic structures, the instantaneous probability density and the evolution process for the seismic response of underground pipeline can be analyzed numerically, and then the stochastic response can be obtained easily. With the finite element method, the stochastic response of underground pipeline under seismic excitation can be studied effectively. Using the proposed method, numerical example under different random conditions is investigated, showing that the proposed method is of high application in considering random seismic excitation with random soil parameter at the same time.

## 1 GENERAL INSTRUCTIONS

As a basic and important problem in seismic reliability research for the underground lifelines system, the stochastic seismic response of underground pipelines has for a long time been extensively studied, coming up with a variety of theoretical and numerical methods applicable to engineering practice (Machida & Yoshimura 2002; Nedjara et al. 2007). However, on account of the calculation limitation in previous studies, the stochastic response analysis considering the randomness simultaneously caused by the seismic input and the structural parameters of soil-pipeline system is still an unsolved problem.

For the non-linear stochastic structures, it is difficult to capture the accurate probabilistic information of dynamic performance. The dynamic response of non-linear stochastic structures was analyzed usually by the random simulation, the random perturbation method or by the equivalent linearization technique. In 1986, Liu et al.(1986) started to investigate the problem with extension of the random perturbation technique. In 1980s, the Monte Carlo simulation is also employed (Deodatis & Shinozuka 1988), and some researchers studied the techniques to reduce the considerable computational work (Spanos & Zeldin, 1998). As the alternative approaches, the equivalent linearization technique (Klosner et al. 1992) and the extension methods used for linear stochastic structures such as the orthogonal polynomial expansion (Iwan & Huang 1996), are also investigated. However, for the dynamic response analysis of non-linear stochastic structures, the random perturbation method had great difficulty because of the secular terms problem and the requirement of small coefficient of variation of the random parameters. There are some arguments for the equivalent linearization method because of the misleading results in some occasions. The application of orthogonal polynomials expansion seems also unfeasible for multiple-degree-of-freedom system and non-polynomial-form non-linearity.

Evidently, more investigations are necessary for the dynamic response analysis of the underground pipelines. In resent years, a newly developed probability density evolution method for stochastic structures has been proposed. The instantaneous probability density function and the evolution against the time can be obtained precisely (Li & Chen 2005). For the dynamic response of stochastic structures, the solution can be derived through solving the probability density evolution equation with an initial value condition. On the other hand, different from the models from the power spectral density function, a new physics-based stochastic earthquake model to reflect the fundamental correlation between the critical factors and the random earthquake motions has been proposed by the authors (Li & Ai 2006). Several key random parameters are based on to realize the purpose, and a relational expression with physical background can be constructed considering the propagation process of earthquake motion in an engineering site. Associated with the probability density evolution method, the proposed random earthquake model can be of comparative advance to provide the basis for the stochastic seismic analysis.

In this paper, based on the probability density evolution method and the physics-based stochastic earthquake model, a new approach for the dynamic response analysis of the finite element modeled underground pipeline structures under random earthquake excitations and uncertain soil parameters is proposed and numerical example is investigated.

## 2 PROBABILITY DENSITY EVOLUTION METHOD FOR STOCHASTIC STRUCTURES

### 2.1 *Principle of preservation of probability*

The principle of probability conservation can be described as, during a conservative probability transformation process, within the state space, the increment of the probability within a unit volume equals to the inflow probability that gets across this unit.

If the ordinary differential of a state vector $Y$ can be expressed as,

$$\dot{Y} = G(Y,t) \tag{1}$$

where $Y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}}$, $G = (g_1, g_2, \ldots, g_n)^{\mathrm{T}}$.

If $p_Y(y,t)$ is supposed to be the probability density of $Y(t)$, based on the principle of probability conservation, the probability density evolution equation can be expressed as followed,

$$\frac{\partial}{\partial t} p_Y(y,t) + \sum_{j=1}^{n} \frac{\partial}{\partial y_j}[p_Y(y,t) g_j(y,t)] = 0 \tag{2}$$

### 2.2 *Probability density evolution equation of the seismic response of stochastic structures*

The dynamic equation of the nonlinear structure can be written as,

$$M(\zeta)\ddot{U} + C(\zeta)\dot{U} + f(\zeta,U) = -M(\zeta)\ddot{U}_g \tag{3}$$

where $\boldsymbol{\zeta}$ is the random parameter vector which represents the physical character of a stochastic structure, and its joint probability density function is $p_{\boldsymbol{\zeta}}(\mathbf{x})$; $M$,$C$ are stochastic mass and damping matrices respectively, which include random parameters with the rank of $n \times n$, n is the dynamic freedom degree; $U, \dot{U}, \ddot{U}$ are the displacement, the velocity, and the acceleration vectors, respectively; $f(\zeta, U)$ is the nonlinear restoring force vector; and $\ddot{U}_g$ is the acceleration vector of input earthquake.

Let a response vector $X = (U^{\mathrm{T}}, \dot{U}^{\mathrm{T}})^{\mathrm{T}}$, the dynamic equation is then changed into a format of state equation including random parameters,

$$X = A(X,\zeta,t) \tag{4}$$

$$A = \begin{Bmatrix} \dot{U}_g \\ -M^{-1}C\dot{U} - M^{-1}f - \ddot{U}_g \end{Bmatrix} \tag{5}$$

If $\dot{X} = \frac{\partial}{\partial t}X(\boldsymbol{\zeta},t) = G(\boldsymbol{\zeta},t)$, based on the above probability density evolution equation, the joint probability density function of $X$ and $\zeta$ will satisfy the following equation,

$$\frac{\partial}{\partial t} p_{X\zeta}(x,x_\zeta,t) + \sum_{i=1}^{2n} \frac{\partial}{\partial x_i}[p_{X\zeta}(x,x_\zeta,t) g_i(x,x_\zeta,t)] = 0 \tag{6}$$

Since the velocity component $\dot{X}_i$ is only the function of variable $\zeta$, the above equation can be re-written as

$$\frac{\partial p_{X\zeta}(x,x_\zeta,t)}{\partial t} + \sum_{i=1}^{2n} \dot{X}_i(x_\zeta,t) \frac{\partial p_{X\zeta}(x,x_\zeta,t)}{\partial x_i} = 0 \tag{7}$$

Having integral at both sides with $x_1, \ldots, x_{l-1}, x_{l+1}, \ldots, x_{2n}$, the probability density evolution equation will be decoupled,

$$\frac{\partial p_{X_l\zeta}(x_l,\mathbf{x}_\zeta,t)}{\partial t} + \dot{X}_l(\mathbf{x}_\zeta,t) \frac{\partial p_{X_l\zeta}(x_l,\mathbf{x}_\zeta,t)}{\partial x_l} = 0 \tag{8}$$

where $p_{X_l\boldsymbol{\zeta}}(x_l, x_\zeta, t) = \int p_Y(x,t)\mathrm{d}\boldsymbol{y}_1 \ldots \mathrm{d}\boldsymbol{y}_{l-1}\mathrm{d}\boldsymbol{y}_{l+1} \ldots \mathrm{d}\boldsymbol{y}_{2n}$, is the joint probability density function of $(X_l, \boldsymbol{\zeta}^{\mathrm{T}})^{\mathrm{T}}$.

When the initial displacement and the velocity are independent of the physical parameters of the structure, the corresponding initial condition is described as,

$$p_{X_l\zeta}(x_l,\mathbf{x}_\zeta,t)\big|_{t=0} = \delta(x_l - X_{l,0}) p_\zeta(\mathbf{x}_\zeta) \tag{9}$$

where $X_{l,0}$ is the determinate initial value of $X_l$, for a initial static structure, there is $X_{l,0} = 0$; $\delta(\cdot)$ is the Dirac function; and $p_{\boldsymbol{\zeta}}(\mathbf{x}_\zeta)$ is the joint probability density function of the random vector $\boldsymbol{\zeta}$.

Solving the above partial differential equation with initial-boundary-values, which include the decoupled probability density evolution equation and the initial condition, the joint probability density function

$p_{X_l\zeta}(x_l, x_\zeta, t)$ can be calculated. After the integral with $x_\zeta$, the probability density function of $X_l(t)$ can also be solved,

$$p_{X_l}(x_l, t) = \int p_{X_l\zeta}(x_l, \mathbf{x}_\zeta, t)\mathrm{d}\mathbf{x}_\zeta \tag{10}$$

The probability density evolution method of the stochastic structures is an effective approach to obtain the instantaneous probability density function and its evolution for the stochastic response.

### 2.3 *Computational algorithm of probability density evolution equation*

Step 1: Disperse the value region that corresponds to $\boldsymbol{\zeta}$, and disperse the initial condition simultaneously;

Step 2: After dispersing, for each determinate value of variable $\zeta$, a determinate dynamic analysis is carried out and the corresponding differential item with time of the target response $\dot{X}_l(x_\zeta, t)$ can be deduced;

Step 3: Using the method of finite difference to solve the probability density evolution equation, then the joint probability density function $p_{X_l\boldsymbol{\zeta}}(x_l, \mathbf{x}_\zeta, t)$ and the probability density function of the target response can be calculated.

## 3 RANDOM MODEL OF EARTHQUAKE GROUND MOTION FOR ENGINEERING SITE

Using a random Fourier function with adherent probability as the modelling form, the proposed model is to reflect the intrinsic relationship between the random seismic motion and the critical parameters.

### 3.1 *The physical relation and modeling*

If searching from the mechanism of the seismic wave propagating through engineering site, earthquake ground motion can be regarded as a physical process, which includes being input from the bedrock base and filtered by the site. Accordingly, it is believed that because of the uncontrollability of the random factors including the afferent wave energy and the soil medium characteristics that the actual earthquake ground motions are observed with significant randomness as a result. The mentioned factors are supposed to be the energy factor of afferent wave, the periodic factor and the dissipative factor of seismic site. Correspondingly, these factors can be practically indicated by the following stochastic variables with operational physical meaning: the basal spectrum parameter, the free angular frequency and the damping ratio of engineering site. Consequently, a physical relation between random earthquake motion and critical factors can be constructed and modeled.

Without loss of generality, the soil layer can be simulated as an equivalent linear single-degree-of-freedom system which is input by a seismic motion from the base, and then the absolute response of this system can be supposed to present the seismic ground motion process. Therefore, if this linear single-degree-of-freedom system is input with one-dimension seismic motion, in frequency domain, the absolute acceleration response is expressed as

$$\ddot{Y}(\omega) = \frac{\omega_0^2 + i2\zeta_0\omega_0\omega}{\omega_0^2 - \omega^2 + i2\zeta_0\omega_0\omega} \cdot \ddot{U}(\omega) \tag{11}$$

where $\omega$ is independent parameter, read as angular-frequency; $\ddot{Y}(\omega)$ is the absolute output acceleration in frequency domain; $\ddot{U}(\omega)$ is the absolute basal input acceleration in frequency domain; $\omega_0$ is the natural angular-frequency of the site; $\zeta_0$ is the damping ratio of the site.

According to the previous statement, the basal input motion and the medium characteristics of soil layer, which lead to the randomness of earthquake acceleration motion, are both of uncertainty. Therefore, the parameters of soil medium–the free frequency $\omega_0$ and the damping ratio $\zeta_0$ are apparently stochastic variables, noted as $X_\omega$ and $X_\zeta$ respectively. Assume random variable vector $X_H = (X_\omega, X_\zeta)^{\mathrm{T}}$ to relate to the soil medium.

If the basal input Fourier spectrum $\ddot{U}(\omega)$ is the function of stochastic variables $X_1, \ldots, X_n$, $\ddot{U}(\omega)$ is defined with

$$\ddot{U}(\omega) = \mathrm{G}(X_1, \cdots, X_n, \omega) \tag{12}$$

Assume random variable vector presenting the basal input Fourier spectrum to be $\mathbf{X}_G = (X_1, \ldots, X_n)^{\mathrm{T}}$, and assign random variable vector $\mathbf{X} = (\mathbf{X}_H^{\mathrm{T}}, \mathbf{X}_G^{\mathrm{T}})^{\mathrm{T}}$ with the joint probability $p_{\mathbf{X}}(x)$. Then the random earthquake ground process described with acceleration can be expressed by a random Fourier spectrum function $\mathrm{F}(\mathbf{X}, \omega)$, which is expressed as

$$\mathrm{F}(\mathbf{X}, \omega) = \mathrm{H}(\mathbf{X}_H, \omega) \cdot \mathrm{G}(\mathbf{X}_G, \omega) \tag{13}$$

where $\mathrm{H}(\mathbf{X}_H, \omega) = \dfrac{X_\omega^2 + i2X_\zeta X_\omega\omega}{X_\omega^2 - \omega^2 + i2X_\zeta X_\omega\omega}$.

### 3.2 *The basal input spectrum*

If presented by power spectral density function, the bedrock seismic responses are generally assumed to be a band-limited white noise spectrum. However, a large amount of observed acceleration records have shown that the bedrock spectrum presents filtering and limited band character. Moreover, relevant analysis indicates that the frequency range greater than

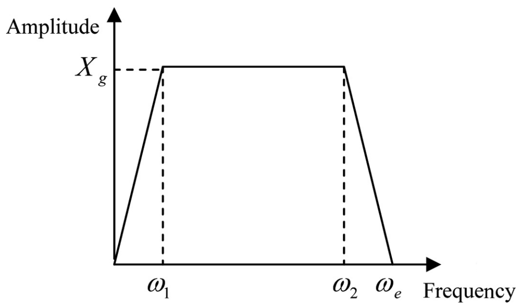
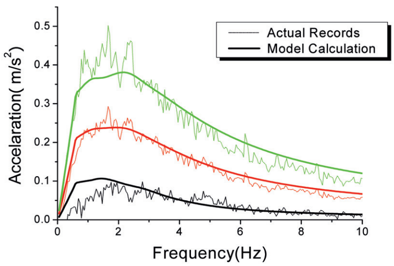
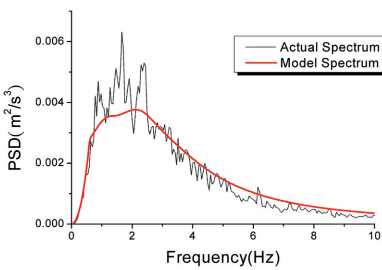



Figure 1. Basal Input Spectrum.




(a) Mean and Plus-Minus Single Standard Deviation of Fourier Amplitude Spectrum




(b) Power Spectral Density (PSD) Curves

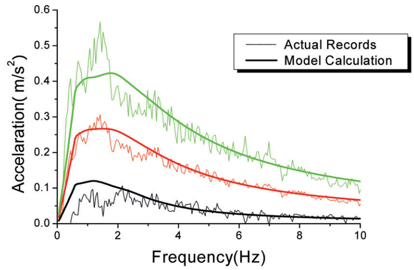
Figure 2. Contrast of the proposed random earthquake model and real records for Soil Type I.

94 Rad/s (15 Hz) can be neglected because of corresponding minute amplitude.

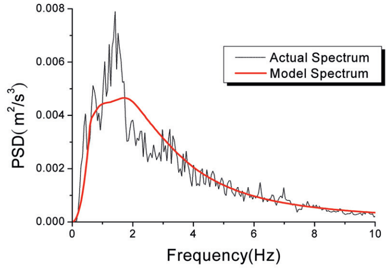
According to the preceding thoughts, a basal input spectrum is constructed (Li & Ai 2006), which is shown in Figure 1, where, $\omega_1$, $\omega_2$ are the control angular-frequency; $\omega_e$ is the truncated angular-frequency; $X_u$ is the random variable defining the amplitude of the basal input spectrum.

### 3.3 *Random earthquake model basing on physical process*

In order to determine the physical variables in the proposed random earthquake model, according to




(a) Mean and Plus-Minus Single Standard Deviation of Fourier Amplitude Spectrum




(b) Power Spectral Density (PSD) Curves

Figure 3. Contrast of the proposed random earthquake model and real records for Soil Type II.

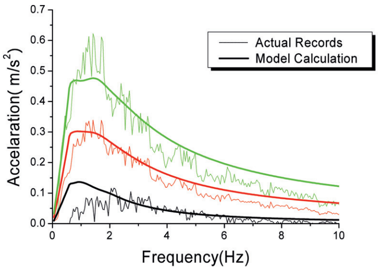
stochastic modeling theory, actual strong seismic records are regarded as a target sample aggregation and numerical methods are adopted to identify different distribution parameters of the variables corresponding to the proposed model (Li & Ai 2006). According to different site class, a collection of acceleration records, which chiefly come from western American strong seismic records, are collected and reorganized to establish different equivalent random earthquake model.

For different site class, the contrasts between actual seismic records and the proposed model are shown in Figures 2–5. The contrasts imply that the proposed model is shown of definite physical concept and propriety to reflect the variation of random seismic motion.

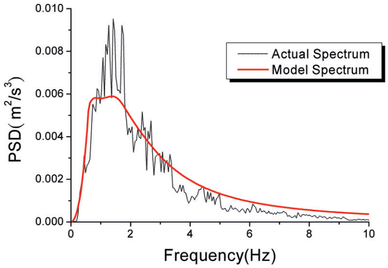
## 4 STOCHASTIC SEISMIC ANALYSIS OF UNDERGROUND PIPELINE

### 4.1 *Computational example*

The site is a $50 \times 10$ m uniform saturated sandy soil space and the ground water is set at the ground surface. And the pipe made of cast iron is buried at the depth of 1m with the diameter is 0.4 m.

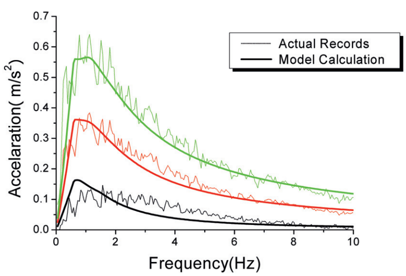



(a) Mean and Plus-Minus Single Standard Deviation of Fourier Amplitude Spectrum

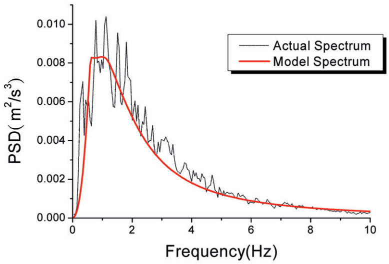



(b) Power Spectral Density (PSD) Curves

Figure 4. Contrast of the proposed random earthquake model and real records for Soil Type III.




(a) Mean and Plus-Minus Single Standard Deviation of Fourier Amplitude Spectrum




(b) Power Spectral Density (PSD) Curves

Figure 5. Contrast of the proposed random earthquake model and real records for Soil Type IV.

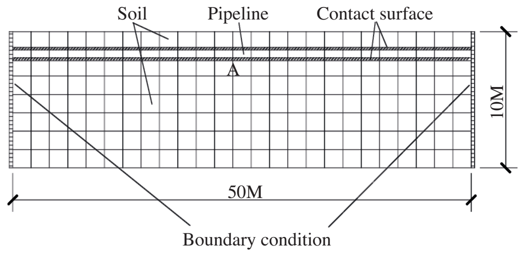



Figure 6. The Structural Mode.

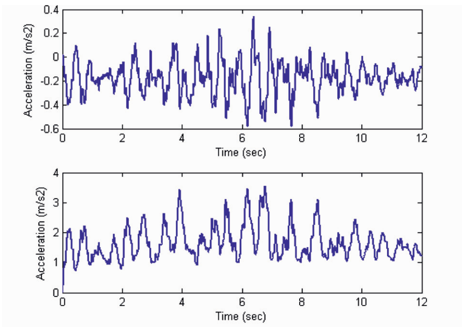
Figure 6 shows the finite element mode of the computational example. The structure is subjected to earthquake excitation in the shape of the conditional random earthquake function which is defined subsequently. The earthquake is input from the bottom horizontally with the propagation velocity 250 m/s and the duration is 20 s. The joint elements adopt the same form with the pipeline element, while different Young's module and density are defined.

For the deterministic analysis of the seismic response of buried pipelines, the finite element method is adopted to study the seismic response of buried pipelines and the surrounding soil. The soil that surrounds the pipeline is regarded as a solid-liquid two-phase medium. The effective stress method and nonlinear constitutive model of soil are used to study the increase and the dissipation of pore water pressure during the seismic process. At the same time, the contact interface between the pipeline and the surrounding soil is also included. Detailed techniques and the dynamic parameters of the sandy soil and the contact surface can be referred to the related paper of the authors (Ai & Li 2004).

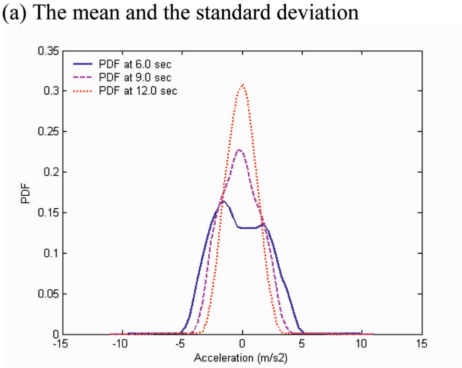
### 4.2 *Stochastic seismic analysis of underground pipeline*

In this paper, the random inputs and the uncertain soil parameter are both considered and defined according to the reference (Li and Ai 2006): associated with the seismic risk analysis, in the future definite time range, the conditional random earthquake function is defined as the case of the transcendental probability of the earthquake motion being $p = 3\%$ and the peak value of the seismic acceleration is $0.2\,g$; The internal friction angle $\phi$, which is defined as the random soil parameter, has a logarithmic normal distribution with the mean $15^0$ and the standard deviation 0.30.

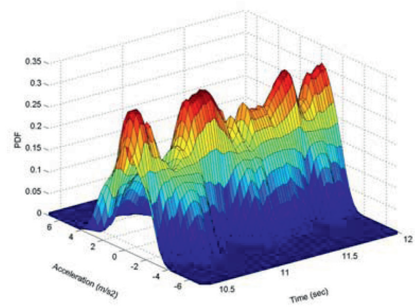
Under the proposed stochastic conditions, the probability density function (PDF) of the response for joint E is presented in Figure 7, including the mean and the standard deviation, typical instantaneous PDFs at certain instants of time, evolution of PDF against time and contour to the PDF surface. These indicate the



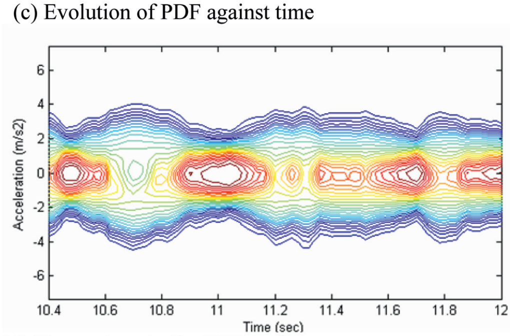
(a) The mean and the standard deviation



(b) Typical instantaneous PDFs at certain instants of time



(c) Evolution of PDF against time



(d) The contour to the PDF surface

Figure 7. The PDF of the response for joint E.

stochastic fluctuation character of a nonlinear random response.

## 5 CONCLUSIONS

A new approach is proposed for the stochastic seismic response analysis of finite element modeled underground pipeline structures under earthquake excitations and uncertain soil parameters. The approach is established based on the thoughts of the newly developed probability density evolution method. Associated with the physics-based stochastic earthquake model and the finite element method, the instantaneous probability density and the evolution process for the seismic response of underground pipeline can be studied effectively. A computational example is investigated with stochastic input and random soil parameters. Some features of the responses are observed and discussed. It is found that the proposed method is of high application in analyzing the stochastic response of underground pipeline and the stochastic seismic response for different structural forms requires further investigation. Furthermore, the seismic reliability problem can be investigated easily by imposing the failure criterion of the first passage problem reliability theory.

## ACKNOWLEDGEMENTS

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