Basal stability of braced excavations in $K_0$-consolidated soft clay by upper bound method

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ABSTRACT: Design of braced excavations in soft clays is usually controlled by the short-term undrained stability. The paper takes into consideration the principal stress axial rotation induced by excavation and undrained anisotropic soil strength. Assuming the Prandtl soil slip failure modes and considering the anisotropic soil strength recommended by Casagrande & Carillo and the non-homogeneous feature of soft soils, a method for evaluating the basal stability is proposed based on the upper bound analysis. Results obtained from the proposed method indicate that the basal stability is significantly influenced by the anisotropy ratio of soil, as well as the plane geometry of the excavation and the thickness of soft soil layer between excavation base and hard stratum. For field case studies, the proposed method is testified by field observations as well as finite element methods.

1 INTRODUCTION

For deep excavations in soft clay, design of the lateral earth support system is often controlled by stability requirements. In current practice, basically there are three methods available for performing stability calculations of braced excavations: (1) limit equilibrium methods; (2) displacement-based elastoplastic finite-element methods; (3) upper and lower bound limit analysis. The limit equilibrium methods are widely used in design practice and include separate calculations of basal stability (based on failure mechanisms proposed by Terzaghi 1948; Bjerrum and Eide 1956) or overall slope stability (using circular or noncircular arc mechanisms) based on well established methods (Morgenstern and Price 1965; Bishop 1966; Spencer 1967). It is often difficult to assess the accuracy of these solutions due to ad hoc assumptions: (1) in selecting the shape of the failure surface; (2) in the search procedures used to locate the critical surface; and (3) in the approximations used to solve the equilibrium calculations (Ukritchon et al. 2003). Further complications arise in analyzing soil structure interactions for embedded supported walls, tieback anchors, etc.

Displacement-based elastoplastic finite-element methods provides a comprehensive framework that can evaluate multiple facets of excavation performance ranging from the design of the wall and support system, to the prediction of ground movements, and the effects of construction activities such as dewatering, ground improvement, etc. They are indispensable for predicting the distribution of ground movement caused by excavations, and for simulating process where there is partial drainage within the soil. Excavation stability is usually assessed by factoring the strength parameters of the soil.


However, the undrained shear strength of natural $K_0$-consolidated clay is anisotropic and has a close relation with the vertical effective stress. The classical rotation of principal stress direction of clays induced by excavation is illustrated in Figure 1 (Clough and Hansen, 1981) and it is well-known that the undrained shear strength of clay varies with the angle of the major principal stress reorientation during the loading. Some researchers (Jiang et al., 1997; Su et al., 1998; Ukritchon et al., 2003) pointed out that without considering the anisotropic and non-homogeneous feature of clays the result of basal stability is not realistic and the safety factor may be much smaller than the real one.

In this paper, an upper bound analysis method for evaluating the basal stability of deep excavations in...
Figure 1. Typical rotation of principal stress.

Soft clay is introduced by taking into consideration of anisotropic feature of $K_0$-consolidated soft clays in the world, especially in Shanghai, as well as the undrained shear strength variation with vertical effective consolidation stress. Practical application of the proposed method is demonstrated through case studies using data published in literature.

2 UNDRAINED ANISOTROPIC SHEAR STRENGTH OF $K_0$-CONSOLIDATED SOFT CLAYS

2.1 Undrained shear strength obtained from triaxial tests

Wei and Huang (2006) presented the detailed formulation of a constitutive model for soft clays which can consider the anisotropic feature of clays based on the bounding surface plasticity. In the case of the shape parameter $R = 2$, the volumetric strain rate can be formulated from the yield function and hardening rule which can be expressed as follows.

\[
\dot{\varepsilon}_v = \frac{\lambda - \kappa}{\lambda} \frac{\dot{p}}{p} - \frac{\lambda - \kappa}{1 + e_0} \frac{2(\eta - \alpha)}{M^2 - \alpha^2 + (\eta - \alpha)^2} \dot{\eta} = -\frac{\dot{p}}{p} \tag{1}
\]

where $\eta = q/p =$ stress ratio; $e_0 =$ initial void ratio; $\lambda$, $\kappa =$ the slopes of virgin consolidation line and swelling line respectively in the $e - \ln p$ space; $M =$ critical stress ratio; and $\alpha =$ slope of yield surface in $p - q$ space.

Under the triaxial undrained condition, i.e. $\dot{\varepsilon}_v = 0$, equation (1) can be rewritten as

\[
\frac{\lambda - \kappa}{\lambda} \frac{2(\eta - \alpha)}{M^2 - \alpha^2 + (\eta - \alpha)^2} \dot{\eta} = -\frac{\dot{p}}{p} \tag{2}
\]

By integrating both sides of equation (2), the undrained stress path can be expressed as

\[
p = \frac{p_c}{\frac{M^2 - \alpha^2}{M^2 - \alpha^2 + (\eta - \alpha)^2}}^{\frac{\lambda - \kappa}{\lambda}} \tag{3}
\]

where $p_c =$ mean effective stress; $\alpha =$ the second invariant of anisotropic tensor: $\alpha_{ij}$. In the analysis, the initial value of $\alpha_{ij}$ is determined by the initial consolidation state of the soil. For the $K_0$-consolidated clay, $p_c$ can be expressed as

\[
p = (1 + 2K_0)\sigma_{v0}'/3 \tag{6}
\]

and

\[
S_{uv} = \frac{1 + 2K_0}{6} M \left( \frac{M + \alpha}{2M} \right)^{\frac{\kappa}{\lambda}} \tag{7}
\]

\[
S_{uh} = \frac{1 + 2K_0}{6} M \left( \frac{M - \alpha}{2M} \right)^{\frac{\kappa}{\lambda}} \tag{8}
\]

where $S_{uv} =$ undrained shear strength obtained from $K_0$-UE triaxial test; and $S_{uh} =$ undrained shear strength obtained from $K_0$UC triaxial test.

Table 1 shows the comparison between the aforementioned formulas and experimental data from Ladd(1973), which testifies that the current expression of undrained strength for soft clays is suitable.

According to Jiang et al. (1997), $1 - \lambda/\kappa = 0.76$. Other input parameters are as follows:

$K_0 = 0.5$, $\phi' = 33^\circ$, $\alpha = 3(1 - K_0)/(1 + 2K_0) = 0.75$, $M = 6\sin \phi'/(3 - \sin \phi') = 1.331$. 

Table 1. Comparison between author's and measured results.

<table>
<thead>
<tr>
<th>Test method</th>
<th>Undrained shear strength expression</th>
<th>Test value (Ladd, 1973)</th>
<th>Theoretical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$UC</td>
<td>eq. (7)</td>
<td>0.330</td>
<td>0.367</td>
</tr>
<tr>
<td>$K_0$UE</td>
<td>eq. (8)</td>
<td>0.155</td>
<td>0.140</td>
</tr>
</tbody>
</table>

The undrained limit shear strength is the intersection of undrained stress path and critical state stress line. If let $\eta = M$, the undrained limit shear strength obtained from compressive test can be expressed as

\[
q_{uc} = M\left[ \frac{M^2 - \alpha^2}{M^2 - \alpha^2 + (\eta - \alpha)^2} \right]^{\frac{\lambda - \kappa}{\lambda}} = M\left[ \frac{M + \alpha}{2M} \right]^{\frac{\lambda - \kappa}{\lambda}} \tag{4}
\]

Similarly, if let $\eta = -M$, the undrained limit shear strength obtained from triaxial extensive test can be expressed as

\[
q_{ue} = M\left[ \frac{M^2 - \alpha^2}{M^2 - \alpha^2 + (\eta - \alpha)^2} \right]^{\frac{\lambda - \kappa}{\lambda}} = M\left[ \frac{M - \alpha}{2M} \right]^{\frac{\lambda - \kappa}{\lambda}} \tag{5}
\]
2.2 Undrained shear strength considering the rotation of principal stress direction caused by excavations

In order to consider the rotation of principal stress direction caused by excavation in clays, the classical anisotropic strength formula for soft clays recommended by Casagrande & Carillo (1944) is adopted here. In the vertical plane, the undrained shear strength under any principal stress direction is expressed as

\[ S_{ui} = S_{uh} + (S_{uv} - S_{uh}) \cos^2 i \]

where \( S_{ui} \) = undrained shear strength of clays when the angle between principal stress direction and vertical direction is \( i \). \( S_{uh} \) and \( S_{uv} \) can be obtained from general undrained triaxial tests.

According to the geometric relationship in Figure 2, \( i \) can be express as

\[ i = \theta - \psi \]

where \( \theta \) = angle between the direction of soil failure surface and vertical direction; \( \psi \) = angle between the direction of soil failure surface and the principal stress direction.

According to the test result from Lo (1965) \( \psi \) is a constant which does not vary with the rotation of the principal stress direction. For an undrained analysis, \( \psi = \pi/4 \) is preferred. In addition, the anisotropic strength ratio is defined as \( k = S_{uh}/S_{uv} \). For an isotropic clay \( S_{uv} = S_{uh} \), \( k = 1 \). Substituting the expressions of \( k \) and \( \psi \) results in the following expression,

\[ S_{ui} = S_{uv} [k + (1-k) \cos^2 (\theta - \pi / 4)] \quad (11) \]

By substituting equation (4) to equation (11), the anisotropic strength considering both the vertical effective stress and the rotation of principal stress caused by excavation can then be described as follows:

\[ S_{ui} = [k + (1-k) \cos^2 (\theta - \pi / 4)]^{1+2K_h/6} \left( \frac{M + \alpha}{2M} \right)^{\frac{1}{2}} \sigma'_{oi} \quad (12) \]

Taking advantage of the definition of safety factor by strength reduction finite element method, we can define the basal stability factor as

\[ F_s = \frac{S_u(z)}{S_u(z)_{critical}} \quad (13) \]

where \( S_u(z) \) = real undrained shear strength of clays; and \( S_u(z)_{critical} \) = critical undrained shear strength.

3 UPPER BOUND ANALYSIS OF BASAL STABILITY OF EXCAVATION CONSIDERING UNDRAINED ANISOTROPIC SHEAR STRENGTH OF CLAYS

3.1 Soil slide failure mechanism

Chen (1975) strengthened that the assumed soil slide failure mode is crucially important to the final result of the limit analysis. Chang (2000) and Faheem et al. (2003) suggested that the Prandtl slide failure is close to the real basal failure mode of excavations. In the paper, the Prandtl's failure mode is adopted.

According to the distance between the hard stratum under the base of excavation and the excavation base, the slide failure mode can be classified into two typical modes as shown in Figure 3. Based on the strength reduction finite element method, Goh (1990) and Faheem et al. (2003) found that the shear strength of clay-wall interface has slight influence on the base stability. Here, we simply assume that the adhesion of clay-wall interface is neglectable, which leads to the simplicity in the upper bound analysis. The sliding surface consists of a 90° circular arc sandwiched between two 45° isosceles wedges, an elastic wedge \( gh \) and plastic \( jik \). The soil column \( efji \) acts as a surcharge.

According to the upper bound limit theorem, the rate of external work should be equal to the rate of internal energy dissipation in the system in a stability problem. If a realistic, kinematically admissible sliding mechanism is assumed, a reasonable collapse load can be evaluated.

3.2 Calculation of basal stability

Assume the clay has a unit weight \( \gamma \) and a uniform surcharge \( q \) is present adjacent to the excavation.

**Case One:** \( T_e \geq B/\sqrt{2} \)

From Figure 3(a), the rate of external work done by (1) the weight of the soil columns \( efji \) and \( mnjg \), (2) the
Figure 3. Velocity field based on assumed Prandtl sliding mechanism.

weight of soil in the two isosceles wedges \( jik \) and \( gjh \), and (3) the weight of soil in the radial zone \( jhk \) is

\[ dW = rHBv + qHV \]  

The total rate of internal energy dissipation from (1) sliding along \( fi \) (2) sliding along \( ik \) and \( gh \), and (3) the radial shear combined with arc sliding in the radial shear zone \( jhk \) is

\[ dE_r = \sum_{n=1}^{5} dE_r^i \]  

\[ dE_r^1 = \int_{H+D}^{(H+D)+B/2} vS_\omega(z)[k + (1-k)\cos^2(\pi/4)]dz \]

\[ dE_r^2 = \int_{H+D}^{(H+D)+B/2} \sqrt{2vS_\omega(z)[k + (1-k)\cos^2(\pi/4)]}dz \]

\[ dE_r^3 = \int_{H+D}^{(H+D)+B/2} \sqrt{2vS_\omega(z)[k + (1-k)\cos^2(\pi/4)]}dz \]

\[ dE_r^4 = \int_{H+D}^{(H+D)+B/2} \sqrt{2vS_\omega(z)[k + (1-k)\cos^2(\pi/4)]}dz \]

\[ dE_r^5 = \int_{H+D}^{(H+D)+B/2} \sqrt{2vS_\omega(z)[k + (1-k)\cos^2(\pi/4)]}dz \]

\[ dE_r = \frac{\sqrt{2}}{4} \int_{H+D}^{(H+D)+B/2} S_\omega(z)[k + (1-k)\cos^2(\theta/4)]\sqrt{2v} \frac{dz}{\sin\theta} \]

\[ dE_r = \frac{\sqrt{2}}{4} \int_{H+D}^{(H+D)+B/2} S_\omega(z)[k + (1-k)\cos^2(\theta/4)]\sqrt{2v}d\theta \]

\[ dE_r = \int_{H+D}^{(H+D)+B/2} 2vS_\omega(z)dz \]

By equating \( dW \) to \( dE \), the basal stability of excavation is

\[ F_s = \frac{\sqrt{2}}{4} \int_{H+D}^{(H+D)+B/2} S_\omega(z)[k + (1-k)\cos^2(\theta/4)]\frac{dz}{\sin\theta} \]

When the clay is homogeneous and the undrained shear doesn’t vary with the soil profile.

\[ F_s = \frac{(1+k)(H+D+2B) + [\pi - 1 + k(\pi + 1)]B}{2(rH + q)B} S_\omega \]  

Case Two: \( T < T_c \)

Similarly,

\[ F_s = \frac{\sqrt{2}}{4} \int_{H+D}^{(H+D)+B/2} S_\omega(z)[k + (1-k)\cos^2(\theta/4)]\frac{dz}{\sin\theta} \]

When the clay is homogeneous and the undrained shear doesn’t vary with the soil profile.

\[ F_s = \frac{(1+k)(H+D+2\sqrt{2T}) + [\pi - 1 + k(\pi + 1)]\sqrt{2T}S_\omega}{2\sqrt{2(rH + q)T}} \]

4 Parametric Studies

The current analysis of basal stability can consider the rotation of principal stress direction and anisotropic
feature of clays. To study the effect of the strength anisotropy, \( D/H \) and \( T/T_c \) to the basal stability of deep excavation in clays, an infinitely long vertical deep excavation in soft clays is analyzed using the aforementioned method. The unit weight of the clay is \( \gamma = 18 \text{kN/m}^3 \), the undrained shear strength is \( S_{uv}(z) = 0.33\sigma'_{v} \), the width of the excavation is \( B = 15 \text{m} \), the depth of the excavation is \( H = 12 \text{m} \), the penetration of the diaphragm is 6 m, 8 m, 10 m and 12 m, respectively.

4.1 Effects of strength anisotropy

Figure 4 shows the factor of safety of the basal stability of deep excavation varies linearly with the degree of strength anisotropy of clay. In addition, the larger degree of anisotropy, the smaller of safety factor. With the same anisotropic ratio, increasing the penetration of the diaphragm can not increase the basal safety factor greatly.

4.2 Effects of \( D/H \)

Figure 5 indicates the safety factor of basal stability increases with the increasing of \( D/H \), which shows the contribution of \( D/H \) to \( F_s \). However, with the same \( D/H \), the anisotropic ratio is more influential than \( D/H \) to the safety factor.

4.3 Effects of \( T/T_c \)

Figure 6 indicates that presence of the bedrock close to the base of the excavation increases \( F_s \). This can be explained that the size of the yielding zone is affected since the displacement of the soil beneath and around the excavation is restrained when the rigid stratum is close to the base of excavation. However, with the same \( T/T_c \), the factor of basal safety is decreased clearly with the decreasing of anisotropic ratio.

5 FIELD CASE STUDIES

5.1 Boston case

Hashash and Whittle (1996) analyzed an deep excavation in Boston Blue Clays through finite element method incorporating an advanced effective stress soil model, MIT-E3 (Whittle and Kavvadas, 1994). The study focus on an idealized (symmetric) plain strain excavation geometry with half-width, \( B/2 = 20 \text{m} \); the depth of excavation is 10 m, 15 m, 22.5 m and 30 m respectively; the penetration of the diaphragm is 2.5 m, 5 m, 17.5 m and 30 m respectively. The vertical effective stress is expressed as \( \sigma'_{v0} = 8.19z + 24.5 \), the gravity of the clay is \( \gamma = 18.0 \text{kPa} \). When the OCR is 1.0, 2.0 and 4.0, the strength anisotropy ratio is 0.5, 0.48 and 0.43 respectively. In the current analysis the author assumes the strength anisotropy ratio is about \( k = 0.5 \) for BBC. According to the proposed method, the factor of basal stability is 1.16 when the depth of
excavation is 10 m and penetration of diaphragm is 2.5 which quite agrees with the result (Hashash et al. 1996) based on the finite element method incorporating with the complicated MIT-E3 model.

5.2 Shanghai case

One failure case of excavation for a railway station in Shanghai was reported by Jiang et al. (1997). The width of the excavation is 23 m, the depth and the length is 16.7 m and 600 m. The penetration of the diaphragm is 13.3 m and 0.8 m thick. Soil parameters concerned are $K_0 = 0.65$, $1 - k/\lambda = 0.91$, $M = 0.818$. According to the formula for calculating the basal stability recommended by the Shanghai Foundation Design Code and the formula recommended by the Shanghai Tunnel Engineering Design and Research Institute, the safety factors are 2.4 and 1.4, respectively. However, the safety factor will reduce to 0.97 when the undrained shear strength is considered as anisotropic and non-homogeneous.

5.3 Taipei case

Su et al. (1998) analyzed the base failure of an excavation in Taipei using an anisotropic strength formula introduced by themselves. The dimension of this excavation is about 100 m long, 17.5 m to 25.8 m wide, and 13.45 m deep. The diaphragm wall retaining the excavation is 24.0 m deep and 0.7 m thick. Failure occurred about two and a half hours after completion of the last stage excavation and only two minutes were needed before the entire internal bracing system collapsed. The excavation site was located in a reclaimed land along the Keelung River, which meanders through the Taipei Basin. The top 8.7 m of the subsoil profile was backfill and hydraulic fill materials. Underlying the fills are a 2.0 m thick silty sand layer, a thick soft clay layer ranging from Ground Level $-10.7$ to $-44.7$ m, and a silty sand layer ranging from GL $-44.7$ m to the bed rock located at GL $-55.0$ m. The distribution of $S_{uc}$ and $S_{tc}$ with depth are $S_{uc} = 0.271\sigma'_{vc}$ and $S_{tc} = 0.189\sigma'_{vc}$, respectively, i.e. $k = 0.7$. The calculated safety factor is 1.10 which shows a possibility of basal failure.

REFERENCES


