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Elastic-plastic analysis for surrounding rock of pressure tunnel with lining based on material nonlinear softening

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ABSTRACT: The elastic-nonlinear softening-residual plastic surrounding rock mode1 is analyzed. According to the total strain theory, the relationship between equivalent stress and equivalent strain is deduced from uniaxial compression of practical rock. The relational expressions are related with triaxial stress (σ_θ, σ_z, and σ_r) and triaxial strain (ε_θ, ε_z, and ε_r) of surrounding rock. The mechanism of load bearing and acting relation between surrounding rock and support are studied. Stress distribution and displacement of broken zone, plastic zone and elastic zone of tunnel are presented out. The ultimate bearing capacity of surrounding rock is given. The critical pressure leading to yield firstly of surounding rock that caused by inner pressure is introduced. The obtained results are more closely to actual values of surrounding rock. It is pointed out that there are obvious limitations in Kastener formula, which is based on ideal elastic-plastic model. Analysis shows that the ideal plastic model and the brittle model are special cases of the proposed solution.

1 INTRODUCTION

Kastner's solution is often used in elastic-plastic analysis for surrounding rock of circular tunnel. There are obvious limitations in Kastner's formula, which is based on ideal elastic-plastic constitutive model. This leads to the Kastner's solution is far away to coresponding actual values in surounding rock of tunnel. Following along the path of pioneered by Kastner, researchers such as Ma (1995, 1996), Jiang and Zheng (1996, 1997), Ma (1998, 1999), Yu (2002), Fan (2004), Ren and Zhang (2001) and Pan and Wang (2004) published different solutions for surrounding rocks of circular tunnel. However, these solutions are resticted to very simple material models, such as simple linear relationship between stress-strain. They are of limited practical value. This study successfully gets the stress distribution laws of surrounding rock plastic and broken zone of tunnel according to the total strain theory. The relationship between equivalent stress and equivalent strain is deduced from practical rock. The relational expressions are related with triaxial stress (σ_θ, σ_z, and σ_r) and triaxial strain (ε_θ, ε_z, and ε_r) of surrounding rock.

2 ELASTIC-PLASTIC ANALYSIS FOR SURROUNDING ROCK

Fig. 1 shows the geometric model condition of a tunnel in a plane strain state subjected to a pressure difference between its internal and external pressures. Where a and b are respectively the inner and outer radius of tunnel, P_0 and Pa are pressures acting on the inner and outer surfaces of tunnel, and R is the radius of the interface of elastic and plastic zones. Surrounding rock may be generally divided into broken, plastic and elastic zones on the basis of their states, as shown in Fig. 1. The surrounding rock within the elastic zone is in an elastic state, within the plastic zone, in a strain softening state, and within the broken zone, in a residual-strength state. So the surrounding rock within the broken zone is the direct object of tunnel support.

2.1 *Constitutive model of plastic zone*

The tunnel may be simplified as an axisymmetrical plane strain problem. Substituting $r = r_3$, $\varepsilon_z = 0$ and $\varepsilon_i = \varepsilon_c$ into the interface of elastic and plastic zones, the equivalent strain is defined by (Zheng, 1988)

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_\theta-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_r)^2+(\varepsilon_r-\varepsilon_\theta)^2}$$

$$= \varepsilon_c \frac{r_3^2}{r^2} \tag{1}$$

If the volumetric strain of softening zone equals to zero, we can obtain

$$\sigma_z^p = \frac{1}{2}(\sigma_\theta^p + \sigma_r^p) \tag{2}$$

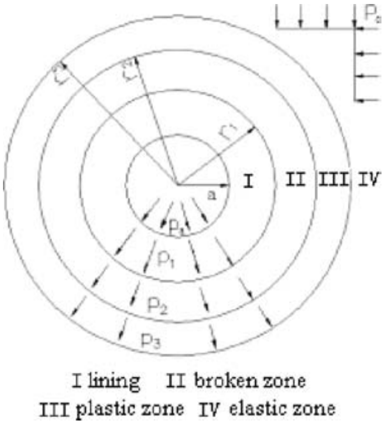

Figure 1. Model of tunnel.

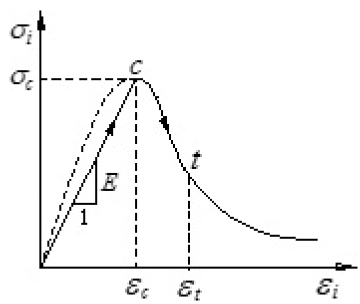

Figure 2. Equivalent stress-equivalent strain.

The equivalent stress is given by

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_\theta^p - \sigma_r^p)^2 + (\sigma_z^p - \sigma_r^p)^2 + (\sigma_z^p - \sigma_\theta^p)^2}$$

$$= \frac{\sqrt{3}}{2}(\sigma_\theta^p - \sigma_r^p) \tag{3}$$

The constitutive model of uniaxial compression is calculated from (Guo, 2004)

$$\sigma = \sigma_c \frac{\varepsilon/\varepsilon_c}{\alpha_c(\varepsilon/\varepsilon_c - 1)^2 + \varepsilon/\varepsilon_c}, (\varepsilon \ge \varepsilon_c) \tag{4}$$

where α_c is a constant, σ_i refers to the stress corresponded to peak strength, ε_i refers to the strain corresponded to peak strength. The constant α_c may be determined by the results of a set of uniaxial tests. The softening section on equivalent $\sigma_i - \varepsilon_i$ curve can be plotted as Fig. 2 shown. We find that the assumed rock mode1 agrees well with practical rocks.

The ultimate bearing capacity of surrounding rock in complex stress state is analyzed in the following parts. We consider that the strain component of surrounding rock keep constant propotion, ie $\varepsilon_z : \varepsilon_\theta : \varepsilon_r = 0{:}1{:}(-1)$. So it may be simplified as simple loading condition. According to the total strain theory (Zheng, 1988), the relationship between equivalent stress and equivalent strain can be deduced from Eq. (4)

$$\sigma_i = \sigma_c \frac{\varepsilon_i/\varepsilon_c}{\alpha_c(\varepsilon_i/\varepsilon_c - 1)^2 + \varepsilon_i/\varepsilon_c}, \ (\varepsilon_i \ge \varepsilon_c) \tag{5}$$

The relational expressions are related with triaxial stress (σ_θ, σ_z, and σ_r) and triaxial strain (ε_θ, ε_z, and ε_r) of surrounding rock.

2.2 *Stresses in the plastic zone*

Its corresponding mechanical equilibrium equation is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{6}$$

Substituting $(\sigma_r)_{r=r_2} = p_2$ into Eq. (3) and Eq. (5) and using the Eq. (6), the stresses in the plastic zone can be obtained as follows

$$\sigma_r^p = p_2 + \frac{2\sigma_c}{\sqrt{3}\sqrt{4\alpha_c - 1}} a\tan\frac{2\alpha_c r_3^2/r_2^2 - (2\alpha_c - 1)}{\sqrt{4\alpha_c - 1}}$$

$$-\frac{2\sigma_c}{\sqrt{3}\sqrt{4\alpha_c - 1}} a\tan\frac{2\alpha_c r_3^2/r^2 - (2\alpha_c - 1)}{\sqrt{4\alpha_c - 1}}$$

$$\sigma_\theta^p = \frac{2\sigma_c}{\sqrt{3}\sqrt{4\alpha_c - 1}} a\tan\frac{2\alpha_c r_3^2/r_2^2 - (2\alpha_c - 1)}{\sqrt{4\alpha_c - 1}}$$

$$-a\tan\frac{2\alpha_c r_3^2/r^2 - (2\alpha_c - 1)}{\sqrt{4\alpha_c - 1}} \tag{7}$$

$$+\frac{2\sigma_c}{\sqrt{3}}\frac{r_3^2/r^2}{\alpha_c(r_3^2/r^2 - 1) + r_3^2/r^2} + p_2$$

2.3 *Stresses in the elastic zone*

The stresses in the elastic zone may be expressed by

$$\sigma_\theta^e = p_0(1 + \frac{r_3^2}{r^2}) - p_3\frac{r_3^2}{r^2}$$

$$\sigma_r^e = p_0(1 - \frac{r_3^2}{r^2}) + p_3\frac{r_3^2}{r^2} \tag{8}$$

Substituting the bounding condition on the interface of elastic and plastic zones $(\sigma_\theta^e)_{r=r_3} + (\sigma_r^e)_{r=r_3} = 2p_0 = (\sigma_\theta^p)_{r=r_3} + (\sigma_r^p)_{r=r_3}$ into Eq. (7) and Eq. (8), we get

$$p_0 - p_2 = \frac{\sigma_c}{\sqrt{3}} - \frac{2\sigma_c}{\sqrt{3}\sqrt{4\alpha_c - 1}} a\tan\frac{1}{\sqrt{4\alpha_c - 1}}$$

$$+\frac{2\sigma_c}{\sqrt{3}\sqrt{4\alpha_c - 1}} a\tan\frac{2\alpha_c r_3^2/r_2^2 - (2\alpha_c - 1)}{\sqrt{4\alpha_c - 1}} \tag{9}$$

The plastic zone has the support effect to surrounding rock. Eq. (9) shows that for a given p_0, surrounding rock can be balanced by itself through adjusting the plastic zone. So it is also named the equilibrium equation of surrounding rock. Surrounding rock without support has ultimate bearing capacity. If $r_3 \to \infty$, we get the ultimate bearing capacity of surrounding rock. In practical, the surrounding rock is collaped as $r_3 \to \infty$. However, it gives us the theoretical result. The ultimate bearing capacity of surrounding rock in practical can not be larger than the theoretical result of surrounding rock.

2.4 *Deformation in the plastic zone*

According to the elastic-plastic theory, the toatal strain of plastic zone can be calculated by the following formula (Zheng, 1988)

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \end{Bmatrix} = \begin{Bmatrix} \varepsilon_r^e \\ \varepsilon_\theta^e \end{Bmatrix} + \begin{Bmatrix} \varepsilon_r^p \\ \varepsilon_\theta^p \end{Bmatrix} \tag{10}$$

The elastic strain of plastic zone is defined by

$$\begin{aligned} \varepsilon_r^e &= \frac{1}{E_c}\left[\left(1-\frac{1}{2}\mu_c\right)\sigma_r - \frac{3}{2}\mu_c\sigma_\theta\right] \\ \varepsilon_\theta^e &= \frac{1}{E_c}\left[\left(1-\frac{1}{2}\mu_c\right)\sigma_\theta - \frac{3}{2}\mu_c\sigma_r\right] \end{aligned} \tag{11}$$

The plastic strain of plastic zone is defined by

$$\begin{aligned} \varepsilon_r^p &= \frac{\varphi}{4G_c}(\sigma_r - \sigma_\theta) \\ \varepsilon_\theta^p &= \frac{\varphi}{4G_c}(\sigma_\theta - \sigma_r) \end{aligned} \tag{12}$$

The total strain of plastic zone can be expressed by

$$\begin{aligned} \varepsilon_r &= \frac{du}{dr} = \frac{1}{E_c}\left[\left(1-\frac{1}{2}\mu_c\right)\sigma_r - \frac{3}{2}\mu_c\sigma_\theta\right] - \frac{\varphi}{4G_c}(\sigma_\theta - \sigma_r) \\ \varepsilon_\theta &= \frac{u}{r} = \frac{1}{E_c}\left[\left(1-\frac{1}{2}\mu_c\right)\sigma_\theta - \frac{3}{2}\mu_c\sigma_r\right] + \frac{\varphi}{4G_c}(\sigma_\theta - \sigma_r) \end{aligned} \tag{13}$$

where E_c refers to elastic modulus of surrounding rock, G_c refers to shear modulus, μ_c refers to Poission's radio, φ refers to plastic function. The plastic functio φ is zero in elastic deformation. Using Eq. (13), the deformation on the interface of elastic and plastic zones can be calculated from

$$\begin{aligned} u_{r_3} &= \frac{r}{E_c}\left[\left(1-\frac{1}{2}\mu_c\right)\sigma_\theta - \frac{3}{2}\mu_c\sigma_r\right] - \frac{r_3(1+\mu_c)(1-2\mu_c)}{E_c}p_0 \\ &= \frac{2\sigma_c r_3(1-2\mu_c)}{\sqrt{3}E_c\sqrt{4\alpha-1}}\left[a\tan\frac{\frac{2\alpha r_3^2}{r_2^2}-2\alpha+1}{\sqrt{4\alpha-1}} - a\tan\frac{1}{\sqrt{4\alpha-1}}\right. \\ &\quad + \frac{2\sigma_c r_3(1-\mu_c/2)}{\sqrt{3}E_c} + \frac{r_3(1-2\mu_c)}{E_c}\left[p_2-(1+\mu_c)p_0\right] \end{aligned} \tag{14}$$

Substituting Eq. (7) into Eq. (13), we have

$$\begin{aligned} \frac{du}{dr}+\frac{u}{r} &= \frac{2(1-2\mu_c)}{E_c}\left[p_2+\frac{\sigma_c}{\sqrt{3}}\frac{r_3^2/r^2}{\alpha_c(r_3^2/r^2-1)+r_3^2/r^2}\right] \\ &\quad + \frac{4\sigma_c(1-2\mu_c)}{\sqrt{3}E_c\sqrt{4\alpha_c-1}}\left[a\tan\frac{2\alpha_c r_3^2/r_2^2-(2\alpha_c-1)}{\sqrt{4\alpha_c-1}}\right] \\ &\quad - \frac{4\sigma_c(1-2\mu_c)}{\sqrt{3}E_c\sqrt{4\alpha_c-1}}a\tan\frac{2\alpha_c r_3^2/r^2-(2\alpha_c-1)}{\sqrt{4\alpha_c-1}} \end{aligned} \tag{15}$$

Combining Eq. (14), we can solve Eq. (15) and get the following formula

$$\begin{aligned} u_r &= C_1 + \frac{gp_2 r}{2} + \frac{gsr}{2}a\tan\left(\frac{2\alpha_c r_3^2 - Ar_2^2}{r_2^2\sqrt{4\alpha_c-1}}\right) \\ &\quad - \frac{ghr_3^2}{2\alpha_c r}\ln\left[(\alpha_c+1)r_3^2-\alpha_c r^2\right] + \frac{gsr}{2}a\tan\left(\frac{A-\frac{2\alpha_c r_3^2}{r^2}}{B}\right) \\ &\quad + \frac{2\alpha_c r_3^2 Bgs}{r(B^2+A^2)}\left[\ln\left(\frac{1}{r}\right)+\ln\left(B^2+A^2+\frac{4\alpha_c^2 r_3^4}{r^4}-\frac{4A\alpha_c r_3^2}{r^2}\right)\right] \\ &\quad \frac{gs}{r}\left\{\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan\left(\frac{B}{A}\right)+\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan\left[\frac{A}{B}-\frac{r^2(A^2+B)}{2B\alpha_c r_3^2}\right]\right\} \end{aligned} \tag{16}$$

where

$$A = 2\alpha_c - 1,\ B = \sqrt{4\alpha_c-1},\ g = \frac{2(1-2\mu_c)}{E_c},\ h = \frac{\sigma_c}{\sqrt{3}},$$

$$s = \frac{2h}{\sqrt{4\alpha-1}},\ n = \frac{gp_0(1+\mu_c)}{2},\quad w = \frac{2h(1-\mu_c/2)}{E_c},$$

$$\begin{aligned} C_1 &= \frac{sgr_3}{2}\left[a\tan\left(\frac{2\alpha_c r_3^2/r_2^2-A}{\sqrt{4\alpha_c-1}}\right)-a\tan\left(\frac{1}{\sqrt{4\alpha_c-1}}\right)\right]+ \\ &\quad -gsr_3\left\{\frac{2\alpha_c B}{B^2+A^2}\left[\ln\left(\frac{1}{r_3}\right)+\ln(B^2+A^2+4\alpha_c^2-4A\alpha_c)\right]\right\} \\ &\quad -gsr_3\left\{\frac{A\alpha_c}{B^2+A^2}a\tan\left(\frac{B}{A}\right)+\frac{A\alpha_c}{B^2+A^2}a\tan\left(\frac{A}{B}-\frac{A^2+B}{2B\alpha_c}\right)\right\} \\ &\quad -\frac{gsr_3}{2}a\tan\left(\frac{2\alpha_c r_3^2-Ar_2^2}{r_2^2\sqrt{4\alpha_c-1}}\right)+\frac{ghr_3}{2\alpha_c}\ln(-r_3^2) \\ &\quad -gsr_3\left[\frac{1}{2}a\tan\left(\frac{-1}{B}\right)\right]+wr_3-nr_3+ \end{aligned}$$

Substituting $r=r_2$ into Eq. (16), we have

$$u_{r_2}=\frac{gsr_2}{2}a\tan(\frac{2\alpha_c r_3^2-Ar_2^2}{r_2^2\sqrt{4\alpha_c-1}})-\frac{ghr_3^2}{2\alpha_c r_2}\ln\left[(\alpha_c+1)r_3^2-\alpha_c r_2^{\,2}\right]+$$
$$\frac{gs}{r_2}\left\{\frac{2\alpha_c r_3^2 B}{B^2+A^2}\left[\ln(\frac{1}{r_2})+\ln(B^2+A^2+\frac{4\alpha_c^2 r_3^4}{r_2^{\,4}}-\frac{4A\alpha_c r_3^2}{r_2^{\,2}})\right]\right\}+$$
$$\frac{gs}{r_2}\left\{\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan(\frac{B}{A})+\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan(\frac{A}{B}-\frac{A^2r_2^{\,2}+Br_2^{\,2}}{2B\alpha_c r_3^2})\right\}+$$
$$\frac{gs}{r_2}\left\{\frac{r_2^{\,2}}{2}a\tan(\frac{-\frac{2\alpha_c r_3^2}{r_2^{\,2}}+A}{B})\right\}+C_1+\frac{gp_2r_2}{2}+ \tag{17}$$

2.5 *Stresses and deformation in the broken zone*

The broken zone cannot bear the tangential stress, so the tangential stress is zero. Its corresponding mechanical equilibrium equation is

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r}{r}=0 \tag{18}$$

$$\sigma_r=\frac{p_1r_1}{r} \tag{19}$$

$$p_2=\frac{p_1r_1}{r_2} \tag{20}$$

The deformation in the broken zone may be expressed by

$$du_r=\varepsilon_r dr=\frac{1-\mu_0^2}{E_0}\sigma_r dr=\frac{(1-\mu_0^2)p_1r_1}{E_0 r}dr \tag{21}$$

Combining Eq. (17), we can solve Eq. (21) and get the deformation formula

$$u_r=\frac{gsr_2}{2}a\tan(\frac{2\alpha_c r_3^2-Ar_2^2}{r_2^2\sqrt{4\alpha_c-1}})-\frac{ghr_3^2}{2\alpha_c r_2}\ln\left[(\alpha_c+1)r_3^2-\alpha_c r_2^{\,2}\right]+$$
$$\frac{gs}{r_2}\left\{\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan(\frac{B}{A})+\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan(\frac{A}{B}-\frac{A^2r_2^{\,2}+Br_2^{\,2}}{2B\alpha_c r_3^2})\right\}+$$
$$\frac{gs}{r_2}\left\{\frac{2\alpha_c r_3^2 B}{B^2+A^2}\left[\ln(\frac{1}{r_2})+\ln(B^2+A^2+\frac{4\alpha_c^2 r_3^4}{r_2^{\,4}}-\frac{4A\alpha_c r_3^2}{r_2^{\,2}})\right]\right\}+$$
$$\frac{gs}{r_2}\left\{\frac{r_2^{\,2}}{2}a\tan\left(\frac{-\frac{2\alpha_c r_3^2}{r_2^{\,2}}+A}{B}\right)\right\}+fp_1\ln(\frac{r}{r_2})+C_1+\frac{gp_2r_2}{2} \tag{22}$$

where $f=\frac{(1-\mu_0^2)r_1}{E_0}$.

Combining Eq. (22) and Eq. (20), we get

$$u_{r_1}=fp_1\ln(\frac{r_1}{r_2})+\frac{gp_1r_1}{2}+C_2 \tag{23}$$

$$C_2=\frac{gsr_2}{2}a\tan\left(\frac{\frac{2\alpha_c r_3^2}{r_2^2}-A}{\sqrt{4\alpha_c-1}}\right)-\frac{ghr_3^2}{2\alpha_c r_2}\ln\left[(\alpha_c+1)r_3^2-\alpha_c r_2^{\,2}\right]+$$
$$\frac{gs}{r_2}\left\{\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan(\frac{B}{A})+\frac{A\alpha_c r_3^2}{B^2+A^2}a\tan(\frac{A}{B}-\frac{A^2r_2^{\,2}+Br_2^{\,2}}{2B\alpha_c r_3^2})\right\}+$$
$$\frac{gs}{r_2}\left\{\frac{2\alpha_c r_3^2 B}{B^2+A^2}\left[\ln(\frac{1}{r_2})+\ln(B^2+A^2+\frac{4\alpha_c^2 r_3^4}{r_2^{\,4}}-\frac{4A\alpha_c r_3^2}{r_2^{\,2}})\right]\right\}+$$
$$\frac{gs}{r_2}\left\{\frac{r_2^{\,2}}{2}a\tan(\frac{-\frac{2\alpha_c r_3^2}{r_2^{\,2}}+A}{B})\right\}+C_1$$

3 SUBMISSION OF MATERIAL TO THE EDITOR

The lining can be considered as thick-wall cylinder in inner pressure p_a and outer pressure p_1. The deformation of lining may be expressed by

$$u_{r1}=\frac{(1+\mu_d)r_1}{E_d(r_1^2-a^2)}\left[2(1-\mu_d)a^2p_a\right]-\frac{(1+\mu_d)r_1}{E_d(r_1^2-a^2)}\left[(1-2\mu_d)r_1^2+a^2\right]p_1 \tag{24}$$

Combining Eq. (23) and Eq. (24), we get

$$p_1=\frac{kp_a-C_2}{m+f\ln(r_1/r_2)+gr_1/2} \tag{25}$$

where $m=\frac{r_1(1+\mu_d)\left[(1-2\mu_d)r_1^2+a^2\right]}{E_d(r_1^2-a^2)}$,

$k=\frac{2(1-\mu_d^2)a^2r_1}{E_d(r_1^2-a^2)}$.

Combining Eq. (25), Eq. (20) and Eq. (9), we get

$$p_a=C_2+\frac{r_2\left[m+\frac{gr_1}{2}+f\ln(\frac{r_1}{r_2})\right]}{kr_1}\left(p_0-\frac{\sigma_c}{\sqrt{3}}\right)-\frac{2\sigma_c r_2\left[m+\frac{gr_1}{2}+f\ln(\frac{r_1}{r_2})\right]}{kr_1\sqrt{3}\sqrt{4\alpha_c-1}}\left[a\tan(\frac{\frac{2\alpha_c r_3^2}{r_2^{\,2}}-A}{\sqrt{4\alpha_c-1}})-a\tan(\frac{1}{\sqrt{4\alpha_c-1}})\right] \tag{26}$$

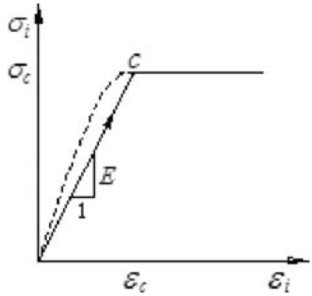

Figure 3. Ideal plastic model of rock.

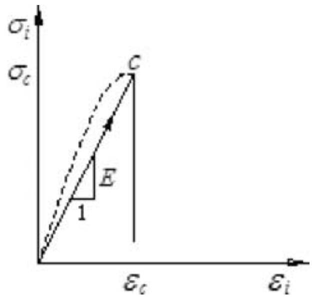

Figure 4. Brittle model of rock.

Using Eq. (26), we may get r_3. Substituting r_3 into Eq. (25), we find the stress formula of lining as follows

$$\sigma_{rc} = \frac{p_1 r_1^2 - p_a a^2}{r_1^2 - a^2} + \frac{r_1^2 a^2 (p_a - p_1)}{(r_1^2 - a^2) r^2}$$

$$\sigma_{\theta c} = \frac{p_1 r_1^2 - p_a a^2}{r_1^2 - a^2} - \frac{r_1^2 a^2 (p_a - p_1)}{(r_1^2 - a^2) r^2} \quad (27)$$

4 DISCUSSION OF RESULTS

(1) According to Eq. (5), we know

If $\alpha_c = 0$, we get $\sigma = \sigma_c$. The curve of strength after yield changes into horizon line, as shown in Fig. 3. Which means the equation stands for ideal plastic material. This suggests that the ideal plastic model, such as Fenner's solution and Kastner's solution is special cases of the proposed solution.

If $\alpha_c \to \infty$, we get $\sigma = 0$. The curve of strength after yield changes into zero, as shown in Fig. 4. Which means the equation stands for brittle material. This shows that the brittle model is special case of the proposed solution.

If $0 < \alpha_c < \infty$, the equation stands for the behavior of post-failure strength. Different integration constant α_c means different behavior of strength after yield, as shown in Fig. 2. The equation betterly describes the characteristic of the strength dropping after softening of surrounding rock's plasticity. It is suitable for analyzing the stress of plastic zone of wall rock openings.

(2) If the rock does not enter plastic state, but has broken zone, substituting $r_3 = r_2$ into Eq. (25), we find

$$p_1 = \frac{k p_a + n r_2 - w r_2}{m + f \ln(r_1/r_2) + g r_1/2} \quad (28)$$

If the rock does not enter broken state, but has plastic zone, substituting $r_2 = r_1$ into Eq. (25), we find

$$p_1 = \frac{k p_a + n r_3 - w r_3}{m + g r_1/2} - \frac{\frac{s g r_3}{2}\left[a\tan\left(\frac{2\alpha_c r_3^2/r_1^2 - A}{\sqrt{4\alpha_c - 1}}\right) - a\tan\left(\frac{1}{\sqrt{4\alpha_c - 1}}\right)\right]}{m + g r_1/2} \quad (29)$$

If the rock does not have plastic zone and broken zone, substituting $r_3 = r_2, r_2 = r_1$ into Eq. (25), we find

$$p_1 = \frac{k p_a + n r_1 - w r_1}{m + g r_1/2} \quad (30)$$

(3) Substituting $r_3 = r_2$ into Eq. (26), we get the formula of critical pressure leading to yield firstly for suronding rock that caused by inner pressure

$$p_a^{cr} = \frac{r_2\left(p_0 - \frac{\sigma_c}{\sqrt{3}}\right)\left[m + \frac{g r_1}{2} + f \ln\left(\frac{r_1}{r_2}\right)\right]}{k r_1} - n r_2 + w r_2 \quad (31)$$

5 CALCULATION EXAMPLE AND EXTENSION OF ITS APPLICATION

Typical cross-section of a pressure tunnel is shown in Fig. 1. The design length of tunnel is 150 m with inner diameter of $a = 3.0$ m, outer diameter of $r_1 = 3.5$ m. The mechanics parameters of rock and lining can be gotten by test, $E_0 = E_c = 30$ MPa, $E_d = 60$ MPa, $\mu_c = u_0 = u_d = 0.25$, $\sigma_d = 50$ MPa, $r_2 = 3.5$ m, $p_0 = 1.3$ MPa, $p_a = 2$ MPa.

According to Eq. (31), we get the critical pressure $p_a^{cr} = 1.67$ MPa. As the inner pressure p_a is greater than the critical pressure p_a^{cr}, plastic zone occurs. Then we get $r_3 = 5.01$ m from Eq. (26). The loosen range of surrounding rock is obtained by Ultrasonic tests to be 5.15 m. It is very close to the theory result to be 5.29 m from Eq. (26). Table 1 shows stresses of different positon of lining from Eq. (29).

Table 1. Stresses of different position of lining.

r/m	3.00	3.25	3.50
σ_{rc} (kPa)	1.96	1.74	1.47
$\sigma_{\theta c}^{\Delta}$ (kPa)	−1.92	−1.62	−1.37

6 CONCLUSIONS

Here we may draw the following conclusions.

1 The elastic-nonlinear softening-residual plastic surrounding rock model is analyzed. According to the total strain theory, the relationship between equivalent stress and equivalent strain is deduced from uniaxial compression of practical rock, which is related with triaxial stress (σ_θ, σ_z, and σ_r) and triaxial strain (ε_θ, ε_z, and ε_r) of surrounding rock. Stress distribution laws of different position of surrounding rock, the mechanism of load bearing and acting relation between surounding rock and support are studied. Analysis shows that the ideal plastic model and the brittle model are special cases of the proposed solution.

2 Different radial stresses of the interface under different conditions, such as elstic-plastic condition, elastic-broken conditions are obtained. The ultimate bearing capacity of surrounding rock is given. The critical pressure leading to yield firstly for surounding rock caused by inner pressure is also obtained.

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