*Geotechnical Aspects of Underground Construction in Soft Ground – Viggiani (ed)*
 *ISBN 978-0-415-68367-8*

# Stability of cohesive-frictional soils with square underground openings

K. Yamamoto
*Department of Ocean Civil Engineering, Kagoshima University, Kagoshima, Japan*

A.V. Lyamin, D.W. Wilson, A.J. Abbo & S.W. Sloan
*Centre for Geotechnical and Materials Modelling, University of Newcastle, NSW, Australia*

ABSTRACT: Stability of foundations laying on cohesive-frictional soils with multiple underground openings (tunnels) of square shape has been analyzed under plane strain conditions. Soil domain was loaded with a uniform surface pressure and a series of tunnel size to depth ratios as well as center-to-center distances have been considered. Rigorous lower and upper bound solutions were obtained by applying numerical limit analysis and presented in the form of dimensionless stability charts for the case of dual tunnel. For comparison purposes several rigid block upper bound mechanisms have been also developed and the obtained values of collapse loads were tested against the results of limit analysis computations.

## 1 INTRODUCTION

Rapid increase in the volume of constructions in densely populated urban areas brings the task of accurate estimation of the stability of soils with multiple underground openings to the priority list of geotechnical engineers. On the other hand it appears that there is no generally accepted design or analysis method available to assess the stability of the structures resting on a stratum with shallow underground openings of square or rectangular cross-section. The efficient utilization of underground space dictates that the non-circular openings and tunnels should be preferred in the design as quadrilateral objects like trains and buildings are usually used there. Furthermore, large size non-circular tunnels are quickly becoming a widespread building technology by virtue of the development of advanced tunneling machines. The merits of the construction of non-circular underground openings compared to circular ones would be the reduction of the area of excavated section, the reduction of the surplus soils, thus diminished environmental issues, and the reduction of rented space for the construction of tunnels. Although the top, bottom, right and left spaces of circular tunnels are usually useless, circular tunnels have been predominantly used so far since their construction is easier from the standpoint of excavation and maintenance of the heading.

In practice, it is often the case that the construction of dual square tunnels is a better alternative to a single large square tunnel as in terms of construction cost, as from utilization point of view. In this paper, the ultimate bearing capacity and failure mechanisms of cohesive-frictional soils with dual square tunnels have been theoretically and numerically investigated assuming plane strain conditions. In contrast to the case of a single tunnel, the center-to-center distance of dual tunnels appears as an important factor influencing the stability of the whole system. Drained loading conditions are considered, the internal tunnel pressure is set to zero, and a continuous load is applied to the ground surface. Both smooth and rough interface conditions between the loading and soils are modelled. For a series of tunnel size to depth ratios and material properties, rigorous lower and upper bound solutions for the ultimate bearing capacity of the considered soil mass are obtained by applying advanced limit analysis techniques (Lyamin & Sloan 2002, Krabbenhøft et al. 2005). For practical convenience the results are presented in the form of dimensionless stability charts, with the actual bearing capacities being closely bracketed from above and below. The results of numerical limit analyses are verified through the comparison with the upper bound collapse loads obtained using rigid block mechanisms specifically developed for modeling the collapse of dual square tunnels.

The application of numerical limit analysis to the stability of shallow tunnels has been pioneered by Sloan and Assadi who investigated the undrained stability of a plane strain square tunnel in a cohesive soil (Assadi & Sloan, 1991) and the stability of square and circular tunnels in cohesive soil with shear strength varying linearly with depth (Sloan & Assadi, 1991, 1992) using linear programming techniques. The stability of the tunnel was described conveniently by two load parameters

$(\sigma_s - \sigma_t)/c_{u0}$ and $\gamma D/c_{u0}$. Lyamin & Sloan (2000, 2001) considered stability of plane strain circular and square tunnels in cohesive-frictional soil. The nonlinear programming technique was applied which vastly reduced the CPU time required and also allowed to increase the number of finite elements employed for the analyses, thus resulting in very accurate solutions. The drained stability of the tunnel was described by the load parameter $\sigma_t / c'$. Wilson et al. (2008) investigated the undrained stability of dual square tunnels using numerical limit analysis and rigid block upper bound methods. Stability charts were generated for a variety of tunnel depths, material properties and inter-shaft distances. Recently, Yamamoto et al. (2010a) initiated investigation on the ultimate bearing capacity and the failure mechanism of cohesive-frictional soils with a shallow square tunnel. Several rigid block mechanisms were developed and the obtained upper bounds of collapse loads were compared with the results of numerical limit analysis to verify the validity of both approaches. This paper extends the above mentioned research to the case of dual square tunnels.

## 2 PROBLEM DESCRIPTION

The problem description is given in Figure 1. The ground is modelled as a uniform Mohr-Coulomb material with a cohesion $c'$, friction angle $\phi'$ and unit weight $\gamma$, assuming drained loading conditions. The tunnel is of dimension $B$, depth $H$ and the center-to-center distance $S$. The internal tunnel pressure is set to zero ($\sigma_t = 0$). The ultimate bearing capacity of cohesive-frictional soils with the inclusion of dual square tunnels is described conveniently by the dimensionless load parameter $\sigma_s / c'$ which is a function of $\phi'$, $\gamma B / c'$, $H/B$, $S/B$ and $L/B$, as shown in Eq. (1).

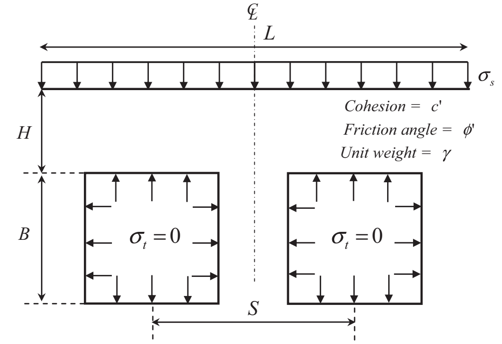



Figure 1. Plane strain dual square tunnels in cohesive-frictional soil.

$$\sigma_s / c' = f(\phi', \gamma B / c', H / B, S / B, L / B) \quad (1)$$

As continuous loading is applied to the ground surface the $L/B$ parameter is eliminated from equation (1). Formulating the problem in this manner permits a compact set of stability charts to be constructed, which are useful for design purposes. The problem parameters considered in this paper are $H/B = 1.0$–$5.0$, $\phi' = 0^\circ - 20^\circ$ and $\gamma B / c' = 0 - 3$. Both smooth and rough interface conditions between the loading and soil are considered.

## 3 NUMERICAL LIMIT ANALYSIS

Limit analysis utilizes the power of lower and upper bound theorems of plasticity theory to provide rigorous bounds on collapse loads from both below and above. The theorems themselves are based on the principle of maximum energy dissipation, which is valid for soils following an associated flow rule. The use of finite element discretization of the soil combined with mathematical optimization to maximize the lower bound and minimize the upper bound has now made it possible to handle efficiently the problems with complex geometries and loading conditions. The formulations of numerical limit analysis used in this paper originate from those given by Sloan (1988, 1989) and Sloan & Kleeman (1995) who employed active set linear programming and discontinuous stress and velocity fields to solve variety of stability problems. Since then, numerical limit analysis has evolved significantly and the computational tools used for this study are those described in Lyamin & Sloan (2002) and Krabbenhøft et al. (2005).

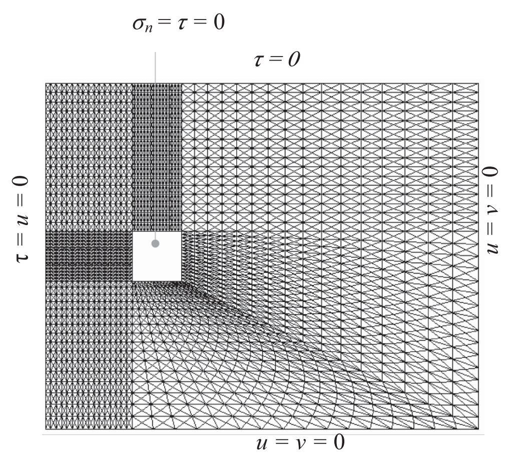


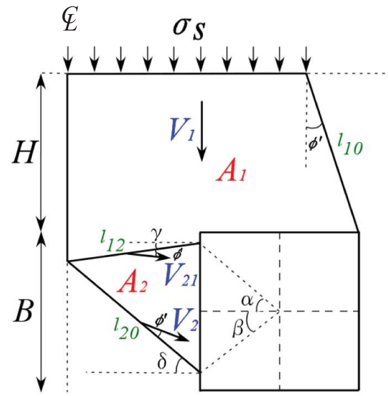

Figure 2. Finite element mesh for $H/B$ = 3 and $S/B$ = 4.5 showing boundary conditions for numerical limit analysis.

Figure 2 show the lower bound and upper bound half-mesh for $H/B = 3$ and $S/B = 4.5$ with smooth interfaces. The mesh is symmetric, and similar meshes are used for both lower and upper bound analyses. Typical lower/upper bound mesh has 7168 triangular elements and 10640 stress/velocity discontinuities. In the lower bound analysis, extension elements are included along the soil domain boundaries to represent a semi-infinite material. The loading is modelled as an infinite width loading, thus it is applied to the whole extend of the surface of soil domain. Careful mesh refinement around the tunnel is required to get accurate solutions. The mesh is particularly dense around the tunnel and is smoothly connected from the tunnel face toward the boundary.

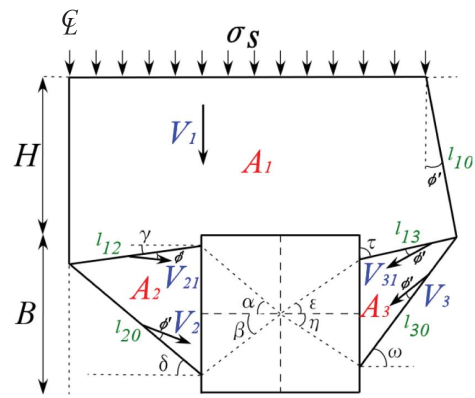
## 4 UPPER BOUND RIGID BLOCK ANALYSIS

Together with numerical limit analysis techniques the semi-analytical rigid block methods were employed in this study to find the upper bound solutions for considered problem. The solutions obtained provide an additional check on the finite element analysis results and the mechanisms developed will be able to serve as simple design tools for practicing engineers. Three types of rigid block mechanisms were constructed as shown in Figure 3. In this Figure, $A_i$ is the area of the rigid block; $V_i$ is the kinematically admissible velocity of block $i$; $V_{ij}$ is the velocity jump along discontinuity and $l_{ij}$ is the length of the discontinuity itself between blocks $i$ and $j$; $\alpha$, $\beta$ and $\delta$ are the angular parameters which determine the geometry of the rigid block mechanisms for mechanism 1 ($\gamma$ is dependent parameter for mechanism 1); $\phi'$ is the dilatancy angle. Mechanism 1 is a simple roof and left (one) side collapse mechanism suitable for shallow tunnels and the mechanisms 2–3 are more complex, roof and both sides collapse mechanisms. The total numbers of angular parameters of mechanisms 1–3 are 3, 7 and 8, respectively. The soil mass was assumed to be governed by Mohr-Coulomb failure criterion and an associated flow rule. The geometry of the blocks is allowed to vary while being constrained such that their areas and boundary segments lengths stay positive. The details of rigid block analysis can be found in Chen (1975).

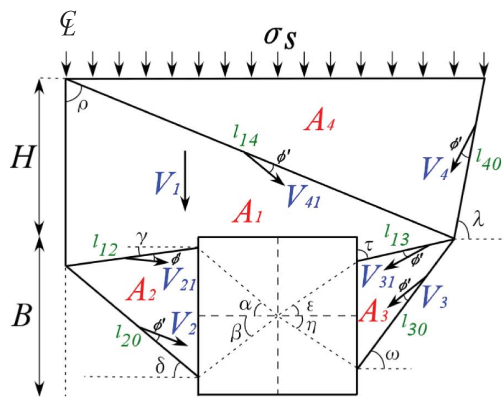
The minimum upper bound solution for each mechanism was obtained by optimizing its geometry using the Hooke & Jeeves algorithm with discrete steps (Bunday, 1984). This method works by performing two different types of searches, an exploratory search and a pattern search. The rigid block analyses are extremely quick taking of the




(a) mechanism 1




(b) mechanism 2




(c) mechanism 3

Figure 3. Upper bound rigid block mechanisms for dual square tunnels.

order of just one second. Provided an appropriate mechanism is chosen, this technique gives a fairly accurate upper bound estimate which can be used to check the finite element solutions and does not require significant computational efforts.

## 5 RESULTS AND DISCUSSION

Figures 4–6a, b, c show the plastic zones, power dissipations and rigid block mechanisms for dual square tunnels. The plastic zones and power dissipations are obtained from the numerical lower and upper bound limit analyses, respectively, and shown by grey shading. The dimensionless load parameter $\sigma_s / c'$ obtained from all of the methods used is included in each figure. It can be observed from Figures 4a, b that for the case of shallow depth, small friction angles and close center-to-center spacing between the tunnels, the collapse mechanism consists of two distinctive zones. Firstly, the wall separating the tunnels is collapsing by virtue of classical "cross" shaped slip surface. Secondly, the failure zone extends to the ground surface from the outside top corners of the tunnels. As can be noticed from Figure 4c the failure mechanism of the rigid block techniques agree well with those observed in the plastic zones and power dissipations (Figs. 4a, b). Moreover, in this case, the upper bound solution from the rigid block technique is superior to the solution obtained by numerical limit analysis. For the case of moderate depth, small friction angles and moderate center-to-center spacing presented in Figures 5a, b, the collapse mechanism enlargers forming the dual slip band between the tunnels and extending now to the ground surface from both lower and upper corners on the outside of the tunnels. The ultimate bearing capacity in this case is high than that shown in Figure 4, due to increase of *H*/*B* and *S*/*B*. The upper bound solution (Fig. 5c) obtained from the rigid block technique is also in good agreement with that from the numerical limit analysis (Fig. 5b). Figure 6 shows the case of deep depth, small friction angles and moderate center-to-center distance between the tunnels. In this case, two failure areas join each other, forming continuous collapse mechanism which encompasses the entire tunnel. Not surprisingly, the rigid block results (Fig. 6c) tend to be larger than the results of numerical limit analysis (Fig. 6b) as it becomes harder to provide accurate solutions due to increased depth and moderate center-to-center distance. Concluding the discussion on the collapse patterns presented in Figures 4–6, it can be stated that when *H*/*B* and *S*/*B* increases the failure mechanism slowly extends vertically and transversely, and shows the multiple modes to eventually encompass the entire tunnel. The *S*/*B* parameter plays, therefore, a key in the ongoing failure pattern and the increase in bearing capacity due to the effect of interaction.

Of the several rigid block mechanisms developed in this study (Figure 3), the best upper bound solutions were almost always obtained from mechanisms 1 and 3, which consist of three and eight angular parameters, respectively. In general, mechanism 1 is suitable for the cases of shallow tunnels with low friction angles and mechanism 3 for the case of deeper tunnels, high friction angles and wider center-to-center distance. In general for the cases of shallow tunnels, small friction angles and close center-to-center spacing between the tunnels the upper bound solutions obtained from the rigid block and numerical limit analyses are in good agreement (Figs. 4–5b, c), with the rigid block results tending to be larger than the limit analysis when *H*/*B* or $\phi'$ or *S*/*B* increases. This is due to the assumption that failure mechanism extends from the ground surface into the depth with the inclination angle equal to the friction angle $\phi'$. Also, tunnels with a deeper and wider center-to-center spacing have a more complex collapse pattern, therefore, the simple rigid block mechanisms proposed in this study are generally less accurate for such configurations. Furthermore, it is more difficult to propose an efficient rigid block mechanism in the case of cohesive-frictional soils than in the case of purely cohesive material. As shown in Figure 6, the collapse mechanism for deeper tunnels is quite wide at the surface as well as it extends further around the bottom of the tunnel. Even using mechanism 3, the feasible solutions could not be easily obtained for high values of *H*/*B*, $\phi'$ or *S*/*B*.

Figures 7–8 show the comparison of stability numbers obtained from both numerical limit and rigid block analyses for smooth interface conditions. The lower and upper bound solutions of the limit analysis for each $\gamma B / c'$ are plotted using broken and solid lines, respectively. It can be observed from Figures 7–8 that for all considered cases the stability numbers decrease when $\gamma B / c'$ increases and the ultimate bearing capacity increases monotonically with the increase of *S*/*B*, except for the cases of close proximity, $S / B \leq 2.0$ (Fig. 8a) and $S / B \leq 3.0$ (Fig. 8b). For the cases of 1) *H*/*B* = 1 and $\phi' = 10^\circ$, $S / B \leq 3.5$, 2) *H*/*B* = 1 and $\phi' = 20^\circ$, $S / B \leq 2.5$, 3) *H*/*B* = 3 and $\phi' = 10^\circ$, $S / B \leq 4.5$ and 4) *H*/*B* = 4 and $\phi' = 10^\circ$, $S / B \leq 4.0$, it is found that the upper bound solutions from the rigid block method have relatively good agreement with those obtained from the numerical limit analysis. But, with an increase of *S*/*B* the accuracy of considered rigid block mechanisms becomes poor. In most cases of $\gamma B / c' = 3$ (Fig. 8a, b), feasible solutions could not be obtained because the tunnel collapses under soil self weight. It is important to mention

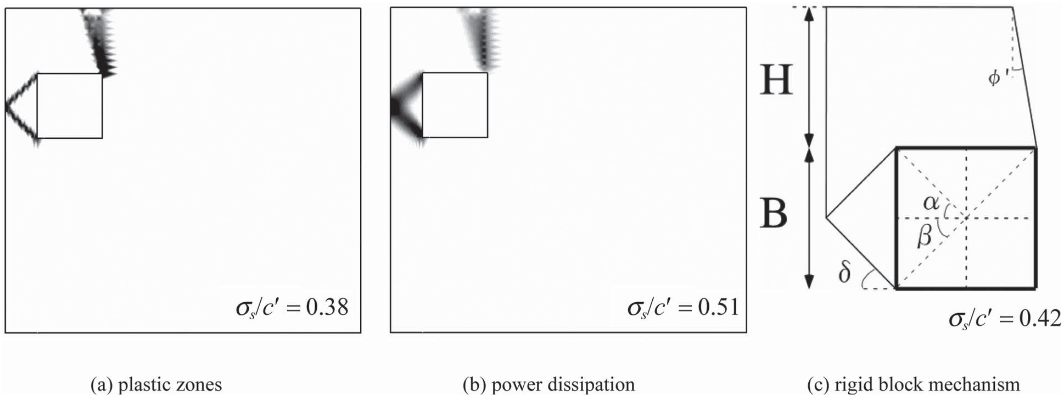



Figure 4. Comparison of numerical limit analysis with rigid block mechanism ($H/B$=1, $\phi'$= 10°, $\gamma B/c'$= 1, $S/B$ = 2.0, smooth interface).

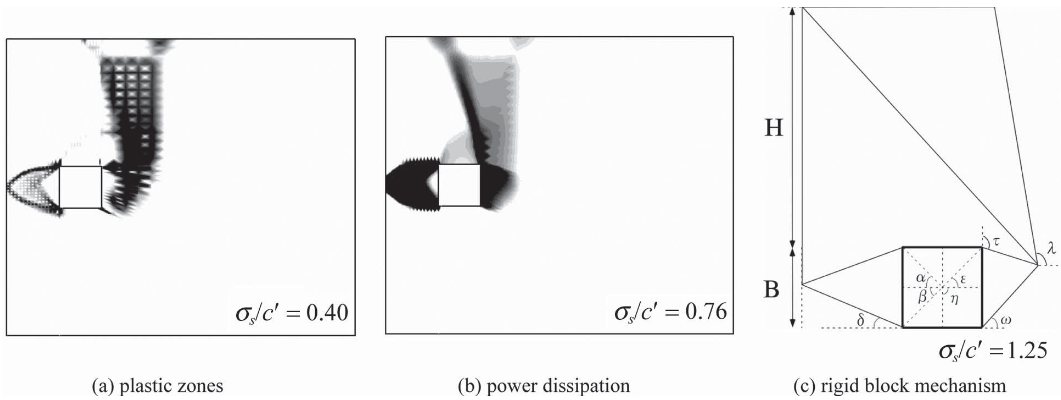



Figure 5. Comparison of numerical limit analysis with rigid block mechanism ($H/B$ = 3, $\phi'$= 10°, $\gamma B/c'$= 1, $S/B$ = 3.5, smooth interface).

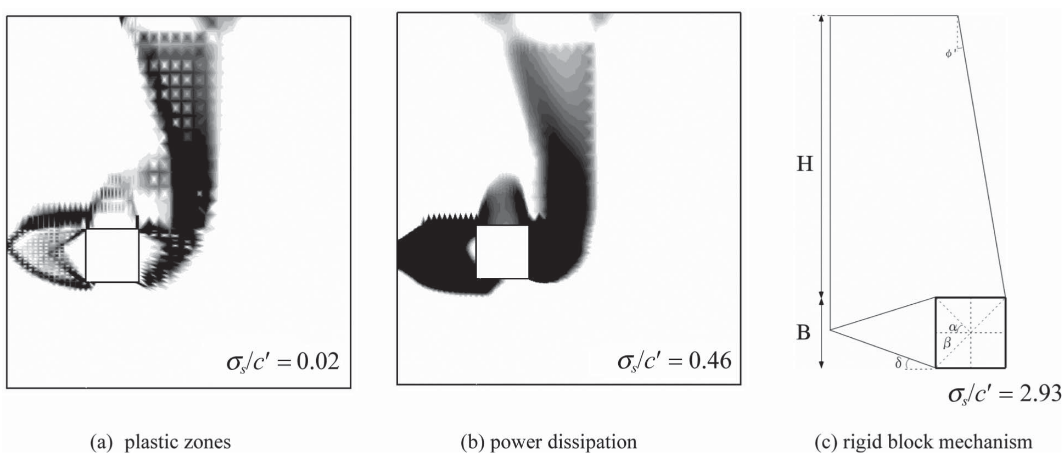



Figure 6. Comparison of numerical limit analysis with rigid block mechanism ($H/B$ = 4, $\phi'$= 10°, $\gamma B/c'$= 1, $S/B$ = 4.0, smooth interface).

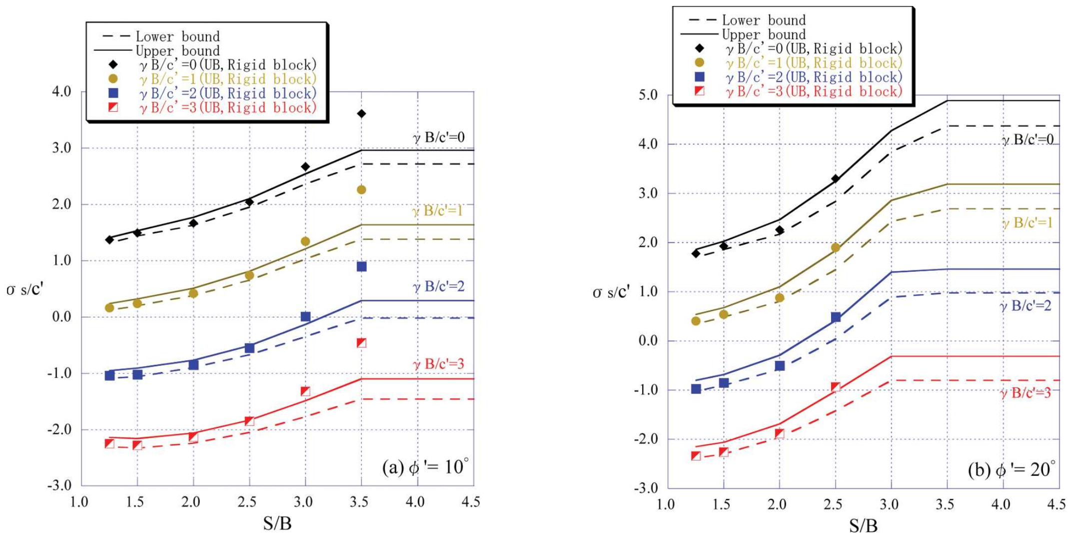



Figure 7. Stability bounds for dual square tunnels ($H/B$ = 1, $\phi'$ = 10°, 20°, smooth interface).

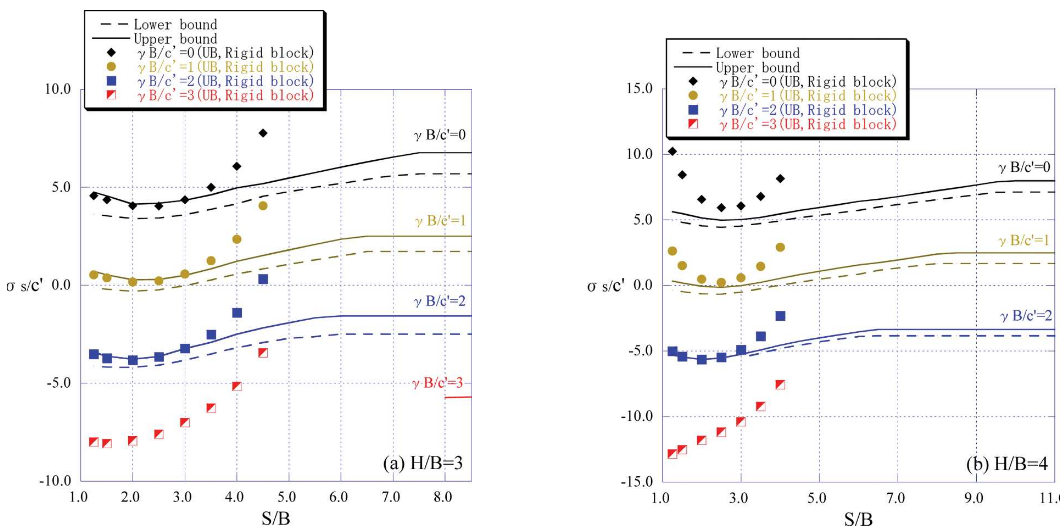



Figure 8. Stability bounds for dual square tunnels ($H/B$ = 3,4, $\phi'$ = 10°, smooth interface).

the sign convention used for stability numbers presentation. A positive value of stability number implies that a compressive normal stress can be applied to the ground surface up to this value, while a negative stability number means that we can only apply a tensile normal stress to the soil surface (no bearing capacity in normal sense). The negative range of stability numbers is likely to be of less practical interest than the positive one. When $S/B$ increases further, the lower and upper bound limit analysis solutions tend to stay constant after a certain value of $S/B$ has been reached (e.g. $S/B = 3.5$ for $\gamma B/c' = 0-3$ (Fig.7a)). The plots of plastic zones and power dissipations show no interaction between dual square tunnels at these points and after that the failure mechanism becomes that of two individual single tunnels. Thus, such points are regarded as the no interaction points for dual square tun nels and this information would be important for engineering practice. The no interaction spacing decreases when $\gamma B/c'$ increases for each $H/B$ and $\phi'$. Comparing to the case of dual circular tunnels (Yamamoto et al., 2010b), it can be mentioned that the bearing capacity of cohesive-frictional soils with dual square tunnels is obviously smaller due to the singular points (corners) of non-smooth tunnel shape.

## 6 CONCLUSIONS

The stability problem of soil domains under continuous uniform surface load with the inclusion of dual square tunnels has been investigated analytically and numerically under plane strain conditions. The results of conducted analyses have been presented in the form of dimensionless stability charts. Several rigid block mechanisms have been developed to check the validity of the numerical limit analysis and serve as a handy practical means as well. Comparison of upper bound solutions obtained from the rigid block analysis with those of the numerical limit analysis shows good agreement when $H/B$, $\phi'$ and $S/B$ are small. In general, the accuracy of the rigid block technique for dual square tunnels was found to be better than that for dual circular tunnels.

For the cases of deeper and closely spaced tunnels, it has been observed that the bearing capacity exhibits a slight decrease due to the interaction between the tunnels. This interaction tends to disappear when the center-to-center distance exceeds a certain value (no interaction point). As expected, bearing capacity of soils with dual square tunnels has been found to be inferior comparing to case of dual circular tunnels. For future work, it is proposed to develop suitable for practical use rigid block mechanisms which are efficient also for high frictional angles and moderate center-to-center distance.

## REFERENCES


Assadi, A. & Sloan, S.W. 1991. Undrained stability of shallow square tunnel. *Journal of Geotechnical Engineering*, ASCE, **117**(8): 1152–1173.

Bunday, B.D. 1984. Basic optimisation methods. Edward Arnold.

Chen, W.F. 1975. Limit analysis and soil plasticity. Elsevier, Amsterdam.

Krabbenhøft, K., Lyamin, A.V., Hjiaj, M. & Sloan, S.W. 2005. A new discontinuous upper bound limit analysis formulation. *International Journal for Numerical Methods in Engineering*, 63: 1069–1088.

Lyamin, A.V. & Sloan, S.W. 2000. Stability of a plane strain circular tunnel in a cohesive-frictional soil. In D.W. Smith & J.P. Carter (eds), *Developments in theoretical geomechanics*, Balkema, Rotterdam, 139–153.

Lyamin, A.V., Jack, D.L., & Sloan, S.W. 2001. Collapse analysis of square tunnels in cohesive-frictional soils. In S. Valliappan & N. Khalili (eds), *Proc. of the First Asian-Pacific Congress on Computational Mechanics*, Sydney, 20–23 November 2001. Elsevier, 405–414.

Lyamin, A.V. & Sloan, S.W. 2002. Lower bound limit analysis using non-linear programming. *Int. J. Numer. Meth. Engng.*, 55: 573–611.

Sloan, S.W. 1988. Lower bound limit analysis using finite elements and linear programming. *Int. J. Numer. Analyt. Meth. Geomech.*, 12: 61–77.

Sloan, S.W. 1989. Upper bound limit analysis using finite elements and linear programming. *Int. J. Numer. Analyt. Meth. Geomech.*, 13: 263–282.

Sloan, S.W. & Assadi, A. 1991. Undrained stability of a square tunnel in a soil whose strength increases linearly with depth. *Computers and Geotechnics*, 12: 321–346.

Sloan, S.W. & Assadi, A. 1992. Stability of shallow tunnels in soft ground. In G.T. Houlsby & A.N. Schofield (eds), *Predictive soil mechanics*, Thomas Telford, London, 644–663.

Sloan, S.W. & Kleeman, P.W. 1995. Upper bound limit analysis using discontinuous velocity fields. *Comp. Methods in Appl. Mech. and Engng.*, 127: 293–314.

Wilson, D.W., Abbo, A.J., Sloan, S.W. & Lyamin, A.V. 2008. Undrained stability of dual square tunnels. In D.N. Singh (ed.), *Proc. of the 12th Int. Conf. of Int. Assoc. for Comp. Methods and Advances in Geomechanics (IACMAG)*, Goa, India.

Yamamoto, K., Lyamin, A.V., Wilson, D.W., Abbo, A.J. & Sloan, S.W. 2010a. Bearing capacity analysis of cohesive-frictional soils with a shallow square tunnel. In John Chien-Chung. Li & Meei-Ling. Lin (eds), *Proc. of 17th Southeast Asian Geotechnical Conference*, Taipei, 10–13 May 2010. Taiwan Geotechnical Society, 219–222.

Yamamoto, K., Lyamin, A.V., Wilson, D.W., Abbo, A.J. & Sloan, S.W. 2010b. Bearing capacity analysis of cohesive- frictional soils with dual circular tunnel. In R. Kastner & S. Aversa (eds), *Proc. of 7th Int. Sym. on Geotechnical Aspects of Underground Construction in Soft Ground*, Roma, 16–18 May 2011. CRC Press, submitted.