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An analytical method to control tunnel lining settlements

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ABSTRACT: A method to control tunnel lining settlements, particularly indicated in urban areas for soft soils and shallow bored tunnels, is presented. The method is based on the classical strength of material virtual works theorem and uses both the displacements and the forces forms. It is noticeable that this approach also permits to follow analytically the internal forces behaviour along the multi supported circular lining beam. Moreover the method, from this point of view, overwhelms the known F.E.M. hyper-static reaction method as this latter requires the definition of the loads acting on the supports. In fact this analytical solution analogously to the static and seismic methods recently illustrated by the author for flexible anchored retaining walls permits to impose the internally calculated support reactions to eliminate or limit the settlements, so rendering about neutral the underground tunnel intervention on the urban environment, by an opportune choice of tensioned anchors.

1 INTRODUCTION

1.1 *Underground urban environment*

Underground works in soft soils urban areas are undoubtedly a very difficult environment for the engineers to dominate a design process and, besides the important dig-related aspects of preserving underground services, the main problem is the settlement controlled operations, because even displacements of few millimetres may damage important monuments or produce considerable buildings economic losses.

On the other hand, nowadays there are numerous techniques to face these difficulties; besides the traditional temporary interventions such as bolts, centres, shotcrete and jet-grouted umbrellas, it is now possible to use closed shield tunnelling with TBM/EPBS machines and especially the compensation grouting techniques, either by soil fracture, for fine grained soils, or by compaction grouting for granular soils, all of which effective in terms of the hole stiffness reinforcement while the engineering analysis is faced via a continuum stress-strain behaviour with as output the tunnel convergences.

1.2 *Technical approach*

In this paper, considering whatever intervention technique, we are focusing on pre-stressed anchors to limit the tunnel convergence, immediately after having bored the tunnel. In the paper a strength of materials analytic approach is proposed to avoid important settlements of the tunnel ring and so of the surrounding soil and building environment. This approach suggests containing the radial reactions of an arch with pre-stressed anchors so imposing in those points even no settlements at all.

The main related phenomenological aspects are the viscous–plastic soil deformations behaviour, which may be encountered in squeezing soils or similar soils, that in general produce long term—movements.

To focus on the maximum dimensions of the phenomenon, in case of middle squeezing poor rock overburden of about 330 m they experienced convergences of up to 2 m in the very noticeable case of the Saint Martin la Porte access adit linked to the 53 km long Lyon–Turin Base Tunnel (Russo *et al*. 2009).

This entails that, in case of more limited dimensions of the phenomenon, like shallower soil overburden, as in urban areas, the settlement problems on the definitive lining may be a complex and complicated matter to deal with.

This new analytical procedure is similar to that proposed for the case of multi–anchored vertical walls in multilayered soils as Garini (2008) has provided as a consequence of the analytical method with the limit equilibrium approach to calculate these kinds of walls, the possibility to control the lateral settlement behaviour by imposing the right reaction force by the anchorages.

Indeed for tunnel calculation too exist numerical methods such that of hyper-static reactions or other more complex that has the limit of not having the tie-back force as a calculated reaction (Oreste, 2002). Actually, in this case these forces can be deduced from the convergence-confinement method, but with this method, suitable for deep tunnels, it's impossible to consider a precise particular load distribution on the lining circle.

Moreover, if one considers a particular load distribution the approach presented in this paper has shown to be fit even in seismic areas, for vertical walls (Garini, 2009), and so with an opportune choice of the seismic loads, may applied also for circular tunnels.

2 THE METHOD: CIRCULAR ARCHES MULTI-SUPPORTS SYSTEM

2.1 *Strength of materials approach*

As known from the strength of materials, a system of beams can be solved via the theorem of virtual works either in the form of the forces through a compatibility equation or in the form of the displacements as an equilibrium equation.

So if we consider an entire circular arch of 180° we can divide it in several little arches, each of them with a centre angle less than 90°.

Once the external load combination of each arch *ij* between node *i* and node *j* is known, it is possible to calculate the entire arch internal forces M_{io}, T_{io} and N_{io} for node *i* (and the same can be done for node *j*) via the forces method for the particular external loads.

In the same way it is automatically possible to know the little arch internal forces, angular, tangential and radial displacements via the forces method for a single node *j* rotation φ_j or a single node *j* tangential displacement w_j or at last for a node *j* single radial displacement v_j.

Finally, for a generic double clamped circular truss with *n* intermediate hinged supports, in a particular node *i* there is only the influence of nodes $i-1$ and $i+1$ and so, in the node *i* where the internal force R_i is present, the equilibrium can be imposed, expressed by the equation (Baldacci, 1979):

$$R_i = R_{io} + \sum_{j=1}^{n} R_{ij}\,\xi_j \tag{1}$$

For *n* intermediate hinged supports, there are $n + 2$ total supports and a double clamped arch will have $n + 3$ hyper-static degrees.

In practice, a double clamped arch with *n* hinged intermediate supports will have *n* unknown rotations and a system of *n* equations with *n* unknown rotations will have to be solved.

Notice only that we need to have a method to evaluate the internal forces in isostatic arches (*e.g.* cantilever arches).

Solving the linear system expressed by equation (1) will bring to the total knowledge of the internal forces and especially of the reactions of the supports that are the pre-tension forces to give to the tie back anchor strands.

2.2 *Isostatic little arches calculation method*

In general we have a little circular arch with a central angle $\underline{\beta}$, less than 90° with a $q_x(x)$ and $q_y(y)$ load distributed respectively along the horizontal x and vertical y dimensions, but directly applied on the circular arch at the point (x, y) of central angle β; hence $x = R\,(1 - \cos(\beta))$, and $y = R \sin(\beta)$, and the single distributed load contributions can be added in function of β, as β varies.

So, according to the Taylor series $dx = -\sin(\beta)\, dl$ and $dy = \cos(\underline{\beta})\, dl$ we'll have a radial and tangential distributed load of this type:

$$q_r = -(\sin(\beta))^2\, q_x + (\cos(\beta))^2\, q_y \tag{2}$$

$$q_t = -\sin(\beta)\cos(\beta)\, q_x - \cos(\beta)\sin(\beta)\, q_y \tag{3}$$

where β is the central angle in the load application point, and regarding the distributed load q_t positive when in compression, and the radial load q_r positive when centripetal.

The shear forces can be evaluated, analogously to the rectilinear beams, for an arch of central angle $\underline{\beta}$, considering the various contributions of $q_r(\beta)$ and $q_t(\beta)$, as β varies, eventually making an integral operation on β (Fig. 1).

It follows that:

$$\begin{aligned} T(\underline{\beta}) = {} & H_0 \cos(\underline{\beta}) - V_0 \sin(\underline{\beta}) \\ & + \int_0^{\underline{\beta}} [q_r(\beta)\cos(\underline{\beta} - \beta) \\ & + q_t(\beta)\sin(\underline{\beta} - \beta)]\, R\, d\beta \end{aligned} \tag{4}$$

where R is the little arch radius and we apply the ordinary beam sign convention namely T and H_0 are positive when we begin the calculation from the left end of the beam (arch in this case) and this is acted upon by an upward force.

In the same manner the normal force can be expressed as:

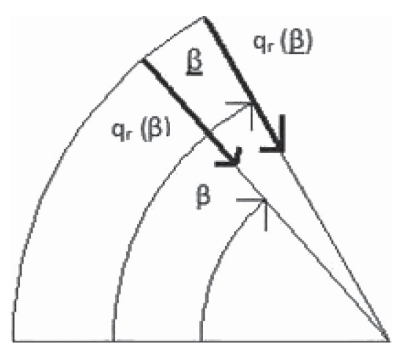

Figure 1. Evaluation of the q_r contributions to Shear Force as β varies for an arch of central angle $\underline{\beta}$. Analogous considerations are done for the q_t contributions and hence for the Normal Force.

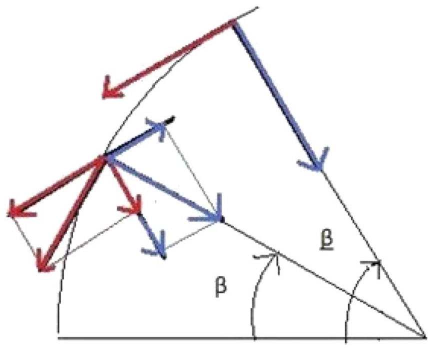

Figure 2. Evaluation of normal and shear forces contributions to the bending moment as β varies for an arch of central angle $\underline{\beta}$. Notice that T is here drawn negative according to the ordinary beam left end conventions, while N is positive as in traction.

$$N(\underline{\beta}) = H_0 \sin(\underline{\beta}) + V_0 \cos(\underline{\beta}) - \int_0^{\underline{\beta}} [q_r(\beta) \sin(\underline{\beta} - \beta) - q_t(\beta) \cos(\underline{\beta} - \beta)] R \, d\beta \quad (5)$$

Here also, as in ordinary convention rules, V_0 and N are positive when in traction.

Finally (see Fig. 2), the bending moment can be expressed as:

$$M(\underline{\beta}) = M_0 + H_0 R \sin(\underline{\beta}) - V_0 R (1 - \cos(\underline{\beta})) + \int_0^{\underline{\beta}} [T(\beta) \cos(\underline{\beta} - \beta) - N(\beta) \sin(\underline{\beta} - \beta)] R \sin(\underline{\beta} - \beta) \, d\beta + \int_0^{\underline{\beta}} [N(\beta) \cos(\underline{\beta} - \beta) + T(\beta) \sin(\underline{\beta} - \beta)] \times R [1 - \cos(\underline{\beta} - \beta)] \, d\beta \quad (6)$$

Obviously H_0,V_0 and M_0 are the reaction forces in the cantilever clamp, namely for $\beta = 0$ and $T(\beta)$, $N(\beta)$ must be calculated in Eqs. 4 and 5 respectively with the position $T(\beta) = T(\underline{\beta}, \beta) = T(\beta, \zeta)$ and $N(\beta) = N(\underline{\beta}, \beta) = N(\beta, \zeta)$, where ζ is the integrand variable. Finally as usual M is regarded as positive when is tending the intrados fibres.

3 SETTLEMENT PRONE URBAN SOFT SOILS ENVIROMENTS APPLICATION EXAMPLES

3.1 *Compensation grouting approach and its limits*

Compensation grouting (Essler *et al.*, 2000) either the Soil Fracture Grouting (Soilfrac) process in fine grained soils or the Compaction Grouting process in granular soils are nowadays used in these urban soft soil environments.

Compaction grouting has however the limit of high pressure of injection, that permits its application only in close proximity to the tunnel face (Essler *et al.*, 2000).

Actually, soil fracture, as the grouting injection propagate as much as greater is the overburden, that in general is heterogeneous, does not permit to know exactly the mechanical properties of the new *in situ* realized material and so any FEM analysis to model the soil improvement will result in a lack of good numerical quantitative reliability for the long term settlement to precisely figure out.

3.2 *Analytical controlled settlement approach application example*

When on the other hand we apply the method herein presented, the design features, instead of longitudinal and transverse grouting height, taking in account the urbanized complex environment close to probable building piled foundations, are thought in terms of lining thickness and pre–stressed anchoring distances.

We can now suppose to have the situation depicted in Figure 3 with the load q = 150 KN/m^2, representative of an average urban subsoil, where we focus on the upper semi circle of the tunnel lining, that for sub–urban areas purposes is of greater concern. Besides we consider the upper semicircle divided in 3 equal arches, so each with centre angle of 60 degrees.

Just for sake of simplicity we will use the noticeable elastic weight strength of materials formulas to figure out the rotations in nodes 2 and 3, by the artifice to consider the parabolic arch equivalent to the particular 60 degree circular arch, imposing the point passage for the keystone and the imposts, or in a simpler way by considering the rise of $f = R(1 - \sqrt{3}/2) = 6 \text{ m} \times 0.13397 = 5.866$ m with reference to the arch chord $l_{chord} = R$.

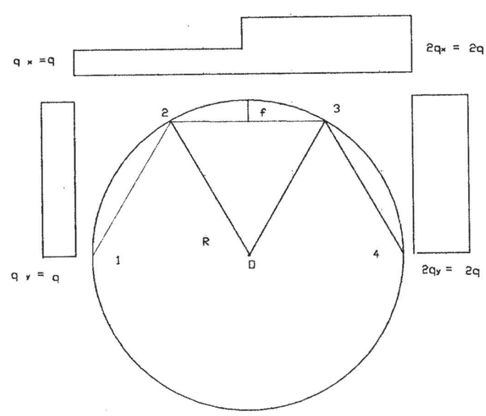

Figure 3. Tunnel of radius R = 6 m in which are imposed no convergences at all in the node 2 and 3 by the application of two pre stressed tie back in the nodes 2 and 3, while, in this example, in the nodes 1 and 4 invert arch is supposed sufficiently stiff and the building loads sufficiently reduced to avoid significant convergences.

So it is:

$$M_2 = M_{2C}^{o} + 4/G\varphi_2 + 2/G\varphi_3 + M_{2L}^{o} + 4/G\varphi_2 \quad (7)$$

$$M_3 = M_{3C}^{o} + 4/G\varphi_3 + 2/G\varphi_2 + M_{3L}^{o} + 4/G\varphi_3 \quad (8)$$

where $G = l_{chord}/EJ = R/EJ = 5.86\ E\text{-}6\ (KNm)^{-1}$ for $h_{lining} = 0.7$ m, is the beam elastic weight and:

$$M_{2C}^{o} = +\, q\, l_{chord}^2/192\ (3 + 11\upsilon)/(1 + \upsilon) - 2\, q\, l_{chord}^{\ 2}/192\ (3–5\upsilon)/(1 + \upsilon) + 51/280\ q\, f^2 - 38/280\ q\, f^2 = 200.600\ KNm \quad (9)$$

and:

$$M_{3C}^{o} = q\, l_{chord}^2/192\ (3 - 5\upsilon)/(1 + \upsilon) - 2\, q\, l_{chord}^2/192 \times (3 + 11\upsilon)/(1 + \upsilon) + 19/280\ q\, f^2 - 102/280\ q\, f^2 = -393.582\ KNm \quad (10)$$

for the central arch (suffix C), and $\upsilon = 45/(4\ f^2)$ J/A(=45 $h^2/(48\ f^2)$ for a rectangular lining section per unit length), while:

$$M_{2L}^{o} = -\, q\, l_{chord}^2/12\ \upsilon/(1+ \upsilon) = -186.983\ KNm \quad (11)$$

$$M_{3L}^{o} = +\, q\, l_{chord}^2/6\ \upsilon/(1+ \upsilon) = +373.967\ KNm \quad (12)$$

for the lateral arches (suffix L).

So, solving the linear system with 2 equations and 2 unknowns, we have:

$\varphi_2 = -0.0000145$ rad; and $\varphi_3 = 0.000018$ rad;

hence the vertical forces V are:

$$V_2 = V_2^{o} - 6/(G\, l_{chord})(2\varphi_2 + \varphi_3) \quad (13)$$

$$V_3 = V_3^{o} - 6/(G\, l_{chord})(2\varphi_3 + \varphi_2) \quad (14)$$

where:

$$V_2^{o} = V_{2C}^{o} + V_{2L}^{o} = (13 + 6)\ q\, l_{chord}^2/32 + q\, l_{chord}^2/4 + q\, f^2/(4\, l_{chord}) - q\, l_{chord}^2/[(8\ f) \times (1 + \upsilon)]\ (\sqrt{3}/2) = 4135.247\ KN/m \quad (15)$$

$$V_3^{o} = V_{3C}^{o} + V_{3L}^{o} = (26 + 3)\ q\, l_{chord}^2/32 + q\, l_{chord}^2/2 - q\, f^2/(4\, l_{chord}) - q\, l_{chord}^2/[(4\ f) \times (1 + \upsilon)]\ (\sqrt{3}/2) = 6739.628\ KN/m \quad (16)$$

While, if we suppose EA = ∞, we have the horizontal forces Z:

$$Z_2 = Z_2^{o} = Z_{2C}^{o} + Z_{2L}^{o} = 3\ q\, l_{chord}^2/[(16\ f) \times (1 + \upsilon)] + (11 + 6)/14\ q\, f - q\, l_{chord}\ (\sqrt{3}/4) - q\, l_{chord}^2/[(16\ f)\ (1 + \upsilon)] = 247.499\ KN/m \quad (17)$$

$$Z_3 = Z_3^{o} = Z_{3C}^{o} + Z_{3L}^{o} = 3\ q\, l_{chord}^2/[(16\ f) \times (1 + \upsilon)] + (3 + 22)/14\ q\, f - q\, l_{chord}\ (\sqrt{3}/2) - q\, l_{chord}^2/[(8\ f)\ (1 + \upsilon)] = -318.709\ KN/m \quad (18)$$

And, finally, the reaction forces, that will equal the tie- back pre - stressed forces are:

$$R_2 = (\sqrt{Z_2^2 + V_2^2}) = 4144.513\ KN/m \quad (19)$$

$$R_3 = (\sqrt{Z_3^2 + V_3^2}) = 6743.495\ KN/m \quad (20)$$

These reactions will permit to have, once the tunnel loads be correctly evaluated, for any long term condition, to have no settlement at all in the nodes 2 and 3, whilst a good choice of the lining section will give a limited flexural displacement according to the above shown elastic line equation. In this way it is possible to limit the typical trough like cross surface settlement feature due to the tunnel boring (Essler et al. 2000).

From this example is evident that to have no settlement at all, the tie-back forces are huge and so a correct design process must consider the use of multi–anchoring points along with a design controlled convergence threshold.

The alternative solution of a protective compensation grouting canopy could not be cost effective while could be less reliable from a stress-strain modelling point of view.

Indeed this example has been developed using most important parabolic arch beam particular loading cases noticeable formulas.

If we have to consider a generic load distribution we would develop the force method as follow, according to the sketch depicted in Figure 4, to find out T_2, N_2 and T_3, N_3 and as above calculate R_2 and R_3:

$$\eta_1 = \eta_{10} + \eta_{11}\, X_1 + \eta_{12}\, X_2 + \eta_{13}\, X_3 \quad (21)$$

$$\eta_2 = \eta_{20} + \eta_{21}\, X_1 + \eta_{22}\, X_2 + \eta_{23}\, X_3 \quad (22)$$

$$\eta_3 = \eta_{30} + \eta_{31}\, X_1 + \eta_{32}\, X_2 + \eta_{33}\, X_3 \quad (23)$$

where obviously:

$$\eta_{i0} = \int_l (N_0 N_i/EA + T_0 T_i/GK + M_0 M_i/EJ)\, dl \quad (24)$$

$$\eta_{ij} = \int_l (N_i N_j/EA + T_i T_j/GK + M_i M_j/EJ)\, dl \quad (25)$$

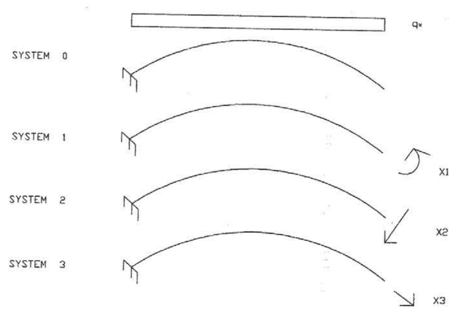

Figure 4. Typical example to calculate a generic circular arch internal forces distribution by the virtual works theorem in the forces form.

Of course equations (24) and (25) must be used along with equations (4), (5) and (6).

Solving the system in X_1, X_2 and X_3, permits to figure out node *i* internal forces for the displacement to use as perfect or sinking clamp values:

$$N_i = N_o + \sum_{j=1}^{3} N_j X_j \text{ (i = 2 or 3)} \quad (26)$$

$$T_i = T_o + \sum_{j=1}^{3} T_j X_j \text{ (i = 2 or 3)} \quad (27)$$

$$M_i = M_o + \sum_{j=1}^{3} M_j X_j \text{ (i = 2 or 3)} \quad (28)$$

4 CONCLUSIONS

An analytical approach, particularly suitable for shallow soft soils, to calculate multi-supported circular arches using known theories of strength of materials, but also implementing the basic isostatic little arches analytical solution has been illustrated.

This methodology is particularly versatile for sub soils, which tend to behave with viscous and even squeezing constitutive laws so for the long term settlement behaviour that typically shows a transverse trough like settlement feature, when the tunnel had been bored without an alternative technique like compensation grouting, that anyway is not easy to contain costs, to control the geo mechanical reliability, and the operative construction aspects.

This method also permits as in other analogue calculations for vertical multi-anchored walls, developed by the author, to calculate the generally non radial supports reactions, imposing certain maximum supports convergences or no supports settlements at all by figuring out the pre-stressed tie back forces.

Only notice that, as a consequence, the maximum convergence, for a particular truss length, will depend from the number of intermediate supports and on the lining flexural rigidity and especially can be calculated on a target base, so deciding the maximum allowable convergence.

REFERENCES

Baldacci, R. 1983. *Scienza delle Costruzioni.* Torino: UTET, vol. 2.

Essler, R.D. & Droof E.R., Falk E., 2000. Compensation Grouting Concept, Theory and Practice. *Advances in Grouting and Ground Modification—Geotechnical Special Publication No 104,* Denver, Colorado: ASCE.

Garini, A.D. 2008. An analytical calculation of multianchored walls and some consequences. *Proceedings of the Second BGA International Conference on Foundations, ICOF 2008*. Dundee: IHS BRE Press.

Garini, A.D. 2009. A dynamic load calculation for multianchored flexible walls. *Performance Based Design in Earthquake Geotechnical Engineering—from Case Histories to Practice*. London: Taylor & Francis Group

Oreste, P. 2002. The convergence-confinement method: roles and limits in modern geotechnical tunnel design. *Gallerie:* Bologna. P atron Editore, 24 (66): 44–50.

Russo, G. & Repetto L., Piraud J., Laviguerie R. 2009. Back—analysis of the extreme squeezing conditions in the ecxploratory adit to the Lyon-Turin base tunnel. *Proceedings of the 3rd CANUS Rock Mechanics Symposium* Toronto: Ed. M. Diederichs and G. Grasselli.