Geotechnical Aspects of Underground Construction in Soft Ground – Yoo, Park, Kim & Ban (Eds)

A new approach for seismic active thrusts in deep excavations

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ABSTRACT: A new theory to take into account the important cut, which constitutes a deep excavation, is here analyzed. Starting from the classical Mononobe–Okabe theory, considerations are drawn to rightly evaluate the resultant thrust as a consequence of a complex dynamic equilibrium polygon, triggered from the ground motion, with also reference to the rotational aspects, summing up two different internal sub-wedges in order to figure out the intensity and the point of application of the actual total seismic thrust (ST). In particular the more internal sub-wedge (swe2), constrained by rotational equilibrium induced by the only outer wedge weight could yield the outer pressure p_2, while the external sub-wedge (swe1) mass is stopped as well alternatively swe1 can move outward yielding the outer pressure p_1, while swe2 is stopped. In the latter case swe1 keeps its point of application, according to rational mechanics knowledge, at 2/3H, but the equivalent height h_G representing the actual point of application of the seismic total thrust could tends to the value 2/3H*STswe1/(STswe1 + STswe2), while in the former case swe2 still keeps at 1/3 H its point of application as it is in a rotational equilibrium.

1 INTRODUCTION

1.1 *Preamble*

Following the important research work resumed in Al Atik & Sitar (2010) for cantilever retaining Structures and bearing in mind the great value of Mononobe Okabe theory, it was natural to find further physical meanings, beside the important explanations given in Al Atik & Sitar (2010), as in rational mechanics it is not possible to apply a force of a continuous body in a point that were not the Centroid, as in some occasion the practice, or better the experimental facts, apparently overwhelm the very important statements of the theory i.e. the rational mechanics, the strength of materials (et cet.) principles in judging physical phenomenon, that are similar.

1.2 *Background*

The important experimental and theoretical work of Al Atik & Sitar (2010) on seismic earth pressures and bending moments on cantilever retaining structures, states the main following conclusions:

1. There seems to be no basis for the currently accepted position of the dynamic earth pressure force in dynamic L.E.A. at 0.6 to 0.67 H and, instead, the point of application should be at 1/3 H, as originally suggested by Mononobe and Matsuo (1932);
2. Maximum dynamic earth pressures and maximum wall inertial forces do not tend to occur simultaneously. As a result, the current design methods based on the Mononobe Okabe theory were found to significantly overestimate dynamic earth pressures and moments.
3. Seismic earth pressures on cantilever retaining walls can be neglected at accelerations below 0.4 g.

It's here, also of uttermost importance to recall the Hessian thrust maximization method introduced in numerous slope stability thrust evaluations: Garini (2010), Garini (2011), Garini (2012), Garini (2013a), Garini (2013b), where it can be seen that the process of determining the maximum thrust indeed depends on various parameters, each one very important in individuating the actual phenomenon behavior. For example Garini (2011) and Garini (2013a) demonstrate that a moment equilibrium may be decisive in judging slope stability, whilst the translational equilibrium is not. Conversely Garini (2010) shows the opposite phenomenon.

In this paper we will focus on a new hypothesis for the actual behavior of a flexible retaining wall typically used in deep excavations, extending the Al Atik & Sitar (2010) findings about the non simultaneity of soil earth pressures and the wall inertial forces to a soil sub wedge immediately behind the wall. In particular the active wedge considered in the Mononobe Okabe theory would not act as a unique body, but as 2 different sub wedges, the outer one pushed away towards the wall yielding the seismic active pressure p_1 while the inner one is acting in a static condition and alternatively the outer sub wedge in a static condition whilst the inner one pushes dynamically against it yielding the seismic active pressure p_2. This hypothesis moves from the experimental evidence claimed in the Al Atik & Sitar (2010) paper with regard to wall inertia forces, and, in particular, would clarify why the application point of the active thrust is not at 2/3H and finally light will further be thrown on the actual behavior of active seismic pressures.

2 THE METHOD

2.1 *General hypothesis*

To face the active pressure behavior we will focus on a simple case depicted in Figure 1 with the following important hypothesis:

1. We consider a frictional soil behind a vertical wall;
2. The rupture wedge is divided in two sub wedges as visible in Figure 2 in a way that the outer wedge constrains only by its own weight the inner one because the seismic force in the outer sub wedge is not simultaneous to the seismic pressures in the inner sub wedge. In particular, the two sub wedges are individuated by a rotational equilibrium equation.
3. The inner sub wedge remains in a static condition and the soil reaction long the failure surface apply at 1/3 of the rupture surface length, as usual in static behavior of frictional soils;
4. Alternatively the outer sub wedge acts with all his seismic active pressure whilst the inner one has no seismic thrust;
5. For the sake of simplicity and also to well compare our results with Al Atik & Sitar (2010), in the Mononobe Okabe formula, it is assumed $K_V = 0$.

2.2 *Sub wedges individuation*

With the above said hypotheses we will have in general the case depicted in Figure 2, so that the seismic rotational equilibrium of the inner sub wedge about the point at 1/3 of the slipping surface of sub wedge 1 over sub wedge 2 is:

$$R\cos(\alpha-\phi)\frac{(L-L_1)}{3}+\frac{1}{2}\gamma\frac{H^2}{3}(L-L_1)-\frac{1}{2}\gamma H(L^2-L_1{}^2)=0 \tag{1}$$

where R is given from the Forces polygon as shown in Figure 3

$$R=\frac{P_{AE}\cos(\phi-\beta)+K_H\frac{\gamma}{2}H(L-L_1)}{\sin(\phi-\alpha)} \tag{2}$$

From equations (1) and (2), we get:

$$L_1=\frac{B+C+D\pm\sqrt{(B+C+D)^2-4A_1[-A_2L^2+L(B+D)]}}{2A_1} \tag{3}$$

where:

$$A_1=1+\frac{K_H}{\tan(\phi-\alpha)} \tag{4}$$

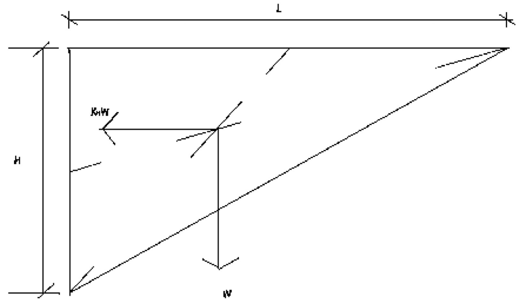

Figure 1. Seismic active Thrust on a wedge for a vertical wall.

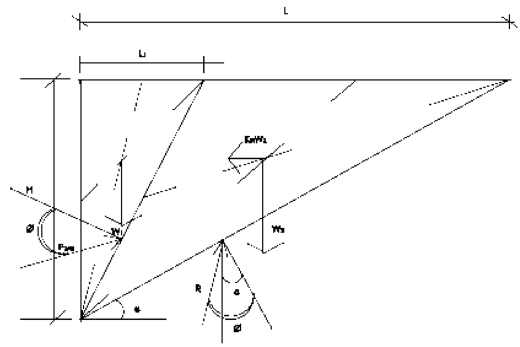

Figure 2. The original wedge is the divided in two sub wedges individuated by the length L1 and the rotational equilibrium is imposed about the third of the separation surfaces.

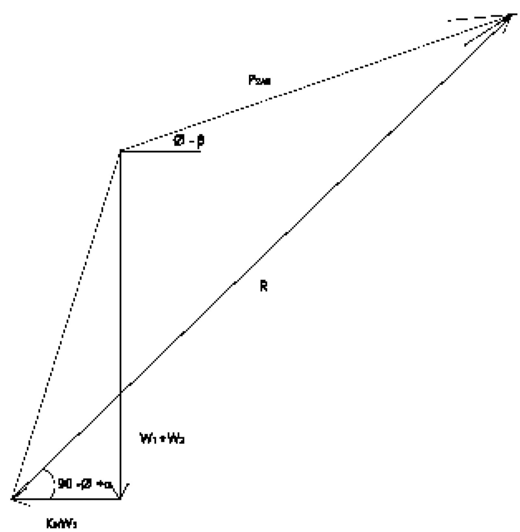

Figure 3. Forces Polygon to figure out the Reaction R.

$$A_2=1-\frac{K_H}{\tan(\phi-\alpha)} \tag{5}$$

$$B=\frac{K_{AE}H\cos(\phi-\beta)}{\tan(\phi-\alpha)} \tag{6}$$

Table 1. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for H = 6 m; Φ = 35o; $\gamma = 17\,\text{kNm}^3$; PGA = 0.66 g; L_1 = 1.41 m (equation 3 sign −); L = 2.33 m.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	6.31	4.68			
1.0	12.62	9.35			
1.5	18.93	14.03			
2.0	25.24	18.71			
2.5	31.55	23.38			
3.0	37.86	28.06			
3.5	44.17	32.74			
4.0	50.47	37.41			
4.5	56.78	42.09			
5.0	63.09	46.77			
5.5	69.40	51.44			
6.0	75.71	56.12			
T.S.T.			227.14	168.36	
Centroid					0.38

*Total Seismic Thrust

$$C = \frac{2K_H L}{\tan(\phi - \alpha)} \tag{7}$$

$$D = K_H H \tag{8}$$

In this way L_1 individuates the dimensions of the two sub wedges so that the inner sub wedge 2 be in a rotational equilibrium during the seismic motion whilst the sub wedge 1 has no acceleration at all.

2.3 *Sub wedges Mononobe–Okabe analytical evaluations*

For calculating a thrust behind a wall with an opposite inclination to the normal gravity wall inclination, it's necessary to reformulate the Mononobe–Okabe seismic active thrust as follow:

$$P_{AE} = \frac{1}{2}\gamma H^2 \frac{[\cot(\alpha) - \tan(\beta)][\tan(\alpha-\phi) + K_H]}{\cos(\beta)[\sin(\delta)\tan(\alpha-\phi) + \cos(\delta)] - \sin(\beta)[\cos(\delta)\tan(\alpha-\phi) - \sin(\delta)]} \tag{9}$$

In the same time keeping in mind the static maximum Coulomb active thrust we have the following:

$$P_A = \frac{1}{2}\gamma H^2 \frac{[\cos(\beta-\phi)]^2}{[\cos(\beta)]^2\cos(\beta+\delta)[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-i)}{\sin(\beta+\delta)\sin(\beta-i)}}]^2} \tag{10}$$

So the two sub wedges can be individuated by the length L_1: the outward one has $\beta = 0$ while the inner one has $\beta = \tan^{-1}(L_1/H)$, and α varies according to the possible values of β, the first derivative of P_{AE}, in particular with respect to α, which entails a minimum P_{AE} and first of all, the extremes of P_{AE}.

2.4 *Example cases for comparison with Al Atik & Sitar (2010)*

We now consider the case described in Al Atik & Sitar (2010) where H = 6 m while the P.G.A. is 0.66 g and Φ = 35°, while δ, this time, was supposed to be 0.7Φ.

We find the pressure p_2 either of sub wedge 2 (inner) when sub wedge 1 (outer) has no seismic force and vice versa as indicated in Table 1. The pressures are comparable with those reported in Al Atik & Sitar (2010) and the total wedge seismic pressures centroid results from the following formula:

$$h_G = \frac{ST_{we1}}{(ST_{we1} + ST_{we2})} \tag{11}$$

Table 2. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for H = 6 m; Φ = 35°; $\gamma = 17\,\text{kNm}^3$; PGA = 0.5 g; L_1 = 1.29 m (equation 3 sign −); L = 4.02 m.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	−0.54	5.51			
1.0	−1.07	11.02			
1.5	−1.61	16.53			
2.0	−2.14	22.04			
2.5	−2.68	27.55			
3.0	−3.21	33.06			
3.5	−3.75	38.57			
4.0	−4.28	44.08			
4.5	−4.82	49.59			
5.0	−5.35	55.11			
5.5	−5.89	60.62			
6.0	−6.42	66.13			
T.S.T.*			−19.27	198.38	
Centroid					n.a.**

*Total Seismic Thrust
**not applicable

Table 3. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for H = 6 m; Φ = 35°; $\gamma = 17\,\text{kNm}^3$; PGA = 0.4 g; L_1 = 1.74 m (equation 3 sign +); L = 4.78 m.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	−3.02	6.29			
1.0	−6.05	12.57			
1.5	−9.07	18.86			
2.0	−12.09	25.15			
2.5	−15.12	31.43			
3.0	−18.14	37.72			
3.5	−21.17	44.00			
4.0	−24.19	50.29			
4.5	−27.21	56.58			
5.0	−30.24	62.86			
5.5	−33.26	69.15			
6.0	−36.28	75.44			
T.S.T.*			−108.85	226.31	
Centroid					n.a.**

*Total Seismic Thrust
**not applicable

Table 4. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for $H = 6\,m$; $\Phi = 35°$; $\gamma = 17\,kNm^3$; $PGA = 0.3\,g$; $L_1 = 3.22\,m$ (equation 3 sign −); $L = 5.27\,m$.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	−7.00	9.08			
1.0	−14.00	18.16			
1.5	−21.00	27.23			
2.0	−28.00	36.31			
2.5	−35.00	45.39			
3.0	−42.00	54.47			
3.5	−49.00	63.54			
4.0	−56.01	72.62			
4.5	−63.01	81.70			
5.0	−70.01	90.78			
5.5	−77.01	99.86			
6.0	−84.01	108.93			
T.S.T.*			−252.02	326.80	
Centroid					n.a.**

*Total Seismic Thrust
**not applicable

Table 5. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for $H = 6\,m$; $\Phi = 35°$; $\gamma = 17\,kNm^3$; $PGA = 0.7\,g$; $L_1 = 1.03\,m$ (equation 3 sign −); $L = 2.61\,m$.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	13.35	4.39			
1.0	26.71	8.79			
1.5	40.06	13.18			
2.0	53.42	17.57			
2.5	66.77	21.96			
3.0	80.12	26.36			
3.5	93.48	30.75			
4.0	106.83	35.14			
4.5	120.18	39.53			
5.0	133.54	43.93			
5.5	146.89	48.32			
6.0	160.25	52.71			
T.S.T.*			480.74	158.13	
Centroid					0.50

*Total Seismic Thrust

It's clear that the point of application of the total seismic pressures is indeed at about H/3 (i.e. 0.38H).

So far this would explain how it is possible that the rational mechanics statements be anyway respected.

Moreover if we now consider a P.G.A. of 0.5 g we get that p_1 (see Table 2) is indeed about zero as stated by Al Atik & Sitar (2010), where is stated that under a P.G.A. of 0.4 g there is not seismic force at all.

In the same way we find out that for a P.G.A. of 0.4 g (see Table 3) we get a greater negative p_1, and so on for a P.G.A. of 0.3 g (Table 4), while for a P.G.A. of 0.7 g we get a greater p_1 positive value(see Table 5).

Table 6. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for $H = 6\,m$; $\Phi = 40°$; $\gamma = 17\,kNm^3$; $PGA = 0.8\,g$; $L_1 = 1.68\,m$ (equation 3 sign −); $L = 3.05\,m$.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	8.30	8.03			
1.0	16.60	16.07			
1.5	24.90	24.10			
2.0	33.20	32.13			
2.5	41.50	40.16			
3.0	49.79	48.20			
3.5	58.09	56.23			
4.0	66.39	64.26			
4.5	74.69	72.29			
5.0	82.99	80.33			
5.5	91.29	88.36			
6.0	99.59	96.39			
T.S.T.*			298.77	289.18	
Centroid					0.34

*Total Seismic Thrust

Table 7. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for $H = 6\,m$; $\Phi = 45°$; $\gamma = 17\,kNm^3$; $PGA = 0.9\,g$; $L_1 = 0.71\,m$ (equation 3 sign +); $L = 1.03\,m$; $R = 1.15$.

z (m)	p_1 kN/m	p_2 kN/m	ST_{we1} kN	ST_{we2} kN	$r = h_G/H$ –
0.0	0.00	0.00			
0.5	16.11	2.29			
1.0	32.22	4.58			
1.5	48.33	6.88			
2.0	64.44	9.17			
2.5	80.55	11.46			
3.0	96.66	13.75			
3.5	112.77	16.04			
4.0	128.88	18.34			
4.5	144.99	20.63			
5.0	161.10	22.92			
5.5	177.21	25.21			
6.0	193.32	27.50			
T.S.T.*			579.95	82.51	
Centroid					0.58

*Total Seismic Thrust

Hence notice (Table 6) that for a P.G.A. of 0.8 g the soil with $\Phi = 35°$ cannot withstand the seismic motion and a $\Phi = 40°$ has been considered yielding a point of application at about H/3.

Finally Table 7 and 8 show that, for a given P.G.A., the higher is the ratio $R = PGA/\Phi$ the higher is the seismic thrust point of application and this is obvious because $ST_{we2} = P_{AE2} - P_{A2}$ is minor as visible in the forces polygon of Figure 3.

3 CONCLUSIONS

An important interpretation of the recent research findings reported in Al Atik & Sitar (2010) regarding

Table 8. Seismic active Thrust pressures distribution in sub wedge 1 and 2 for H = 6 m; Φ = 50°; γ = 17 kNm^3; PGA = 0.9 g; L_1 = 1.03 m (equation 3 sign −); L = 3.05 m; R = 1.03.

z(m)	p_1 kN/m	p_2 kN/m	STwe1 kN	STwe2 kN	$r = h_G/H$ -
0.0	0.00	0.00			
0.5	5.93	8.59			
1.0	11.86	17.17			
1.5	17.78	25.76			
2.0	23.71	34.35			
2.5	29.64	42.93			
3.0	35.57	51.52			
3.5	41.49	60.11			
4.0	47.42	68.69			
4.5	53.35	77.28			
5.0	59.28	85.87			
5.5	65.20	94.45			
6.0	71.13	103.04			
T.S.T.*			213.40	309.12	
Centroid					0.27

*Total Seismic Thrust

the seismic active thrust on cantilever flexible walls has been given resorting to correct rational mechanics statements i.e. the application of the seismic force in the centroid at 2/3H.

These considerations clarify how it is possible, by splitting in two sub wedges the seismic active thrust wedge, that the major findings in Al Atik & Sitar (2010) had indeed its provoking cause.

Moreover it is individuated and shown which actually is the soil mechanics to be accounted for to rightly evaluate the seismic thrust point of application.

REFERENCES

Al Atik, L. & Sitar N. 2010. Seismic Earth Pressures on Cantilever Retaining Structures *ASCE, Journal of Geotechnical Engineering* 111(10): 1161–1176.

Garini, A.D., 2010. Rupture Surfaces of granular Soils linear shaped Slope Stability” *17th SEAGC Conference,* Taipei, TAIWAN.

Garini, A.D., 2011. Circular shaped Rupture Surfaces for undrained Soils Slope Stability” *Conference on Advances in Geotechnical Engineering,* Perth, AUSTRALIA.

Garini, A.D., 2012. A seismic active Thrust calculation for circular Tunnels” *2nd International Conference on performance based design in Earthquake geotechnical Engineeering,* Taormina, ITALY.

Garini, A.D., 2013a. Spheroidal shaped rupture Surfaces for undrained Soils planar Slope Stability” *Geo Congress,* San Diego, CA USA.

Garini, A.D., 2013b. Critical failure Surfaces for Tie-back and Compound Slope Analysis” *Design and Practice of Geosyntheti-Reinforced Soil Structures,* DEStech Publications, Inc., PA USA.

Mononobe, N. & Matsuo, H., 1932. Experimental Investigation of Lateral Earth Pressure during Earthquakes” *Earthquake Research Institute and Research Institute of Public Works, Department of Home Affairs.* JAPAN.