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## Influence of intermediate principal stress on correction formula of bearing capacity of rock foundation

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**ABSTRACT:** Generally the bearing capacity of rock foundation is high, and practically it's very hard to figure out the exact value of the bearing capacity. As a result the researches related to the bearing capacity of rock foundation are less likely to be taken. However with the increasingly growing load of buildings, the requirements for the bearing capacity of rock foundation are improving in response. The bearing capacity of rock foundation involves the structure of the rock mass and strength of rock, thereby the solution of the bearing capacity of rock foundation is far less perfect compared with the theory of the bearing capacity of soil foundation which is developed from the theory of plasticity. Presently the method of limit equilibrium and the slip line field theory and the limit analysis are available to explore the ultimate bearing capacity of rock foundation, nevertheless the results derived from these theories are most likely to be on the small side. The paper employs the Twin Shear Theory to obtain the theoretical formula of the ultimate bearing capacity of strip-type rock foundation, considering the impacts of intermediate principle stress to the bearing capacity of rock foundation. With the comparison of the project case, it indicates that the results are bigger than the ones applying the Mohr-Coulomb strength theory, and it will bring great economic benefits in the event it is put into the practical projects.

**Keywords:** intermediate principal stress; twin shear theory; bearing capacity; strip foundation

### 1 INTRODUCTION

Resulting from the commonly high bearing capacity of the rock foundation the buildings were seldom built on the rock foundation in the past, accordingly, the researchers who explored the bearing capacity of the rock foundation were found very few. In recent years, the scale and the carrying capacity of modern architecture has been increasingly growing as a result of the rapid development of the urban size, thereby the solution of the bearing capacity of the rock foundation presents further requirements. As a common test to ascertain the bearing capacity of the soil foundation the in-situ loading test now is widely applied to the rock foundation, particularly for the soft rock foundation. Nevertheless the test presents a high demand for the equipment, takes a longer period, and costs a lot as well. Therefore it is generally used to explore the soft rock foundation. As to the highly intense rocks few test results are found, so currently the solution of the bearing capacity of rock foundation is carried out by the uniaxial compressive strength of rock based on the national standard *Code for Design of Building Foundation* (GB50007-2011)

and *Standard for Classification of Engineering Rock Masses* (GB50318-94). At times the bearing capacity of the rock foundation is apparently not in accord with this national standard, particularly for some soft rock foundation, and it still happens that the bearing capacity is lower than that of the common soil, for which the designers don't get a clue to deal with at all. During the process of the practical designing it is an unexpected waste that enlarging the pile diameter and rock foundation diameter for the high bearing capacity of rock foundation. As a result, it is of great meaning for the application of engineering to conduct the theoretical research for the bearing capacity of rock foundation. Nevertheless the bearing capacity of rock foundation involves the structure of the rock mass and strength of rock. The solution of the bearing capacity of rock foundation is far less perfect compared with the theory of the bearing capacity of soil foundation which arose from the theory of plasticity. Presently the Method of Limit Equilibrium and the Slip Line Field Theory and the Limit Analysis are available to explore the ultimate bearing capacity of rock foundation, and these theories mostly related to Mohr-Coulomb strength theory (Terzaghi, 1960) and Hoek-Brown non-linear failure

criterion (Song et al. (1999), Song & Yu (2002)) are employed to get access to the formula of the ultimate bearing capacity of rock foundation. In fact these formulas don't take the impacts of the rock yield and damage of the rock and soil mass from the intermediate principal stress into consideration, thus they are less likely to reflect the real condition of rock foundation. The twin shear theory refers to the intermediate stress  $\sigma_2$ , which is more comprehensive than the Mohr-Coulomb strength theory and Hoek-Brown non-linear failure criterion, and many scholars (Zhou et al. (2002) like, Ajay et al. (2012), and Ghazavi & Eghbali (2013)) prove that the theory is applicable to soil foundation. The paper employs the formula of twin shear theory about the plane strain problem indicated in the literature (Yu et al. (1997)) and modifies the ultimate bearing capacity formula of rock foundation regarding the intermediate principal stress.

## 2 THE TWIN SHEAR THEORY

As the Mohr-Coulomb Strength Criterion and other related strength theories applied to the geotechnical engineering fails to reveal the influencing factors of the bearing capacity of rock foundation caused by intermediate principle stress, Professor Yu Maohong indicated that the influence from the intermediate principle stress to the twin shear theory should be taken into account in 1983, and that further developed the general theory of twin shear, which is compatible with the mechanical model for the most engineering materials that are respectively offered with different mathematical expression (Yu (2004)). The commonly used expression of general theory of twin shear in geotechnical engineer is shown in the following:

$$(1) \text{ While } \sigma_2 \leq \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{\sin \phi}{2}(\sigma_1 - \sigma_2)$$

$$F = \sigma_1 - \frac{1 - \sin \phi}{(1+b)(1 + \sin \phi)}(\sigma_1 + \sigma_2) = \frac{2C_0 \cos \phi}{1 + \sin \phi}$$
(1)

$$(2) \text{ While } \sigma_2 \geq \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{\sin \phi}{2}(\sigma_1 - \sigma_2)$$

$$F' = \frac{1}{(1+b)}(\sigma_1 + b\sigma_2) - \frac{1 - \sin \phi}{1 + \sin \phi}\sigma_3 = \frac{2C_0 \cos \phi}{1 + \sin \phi}$$
(2)

where  $C_0$ ,  $\phi$  respectively are cohesive strength and internal friction angle of material;  $\sigma_1, \sigma_2, \sigma_3$  are respectively the first, second and third principle stress;  $b$  is the coefficient reflecting the impact of intermediate principle stress.

As to the plane strain the twin shear theory can be indicated (Yu (2004)).

$$R = P \sin \phi + C_t \cos \phi$$
(3)

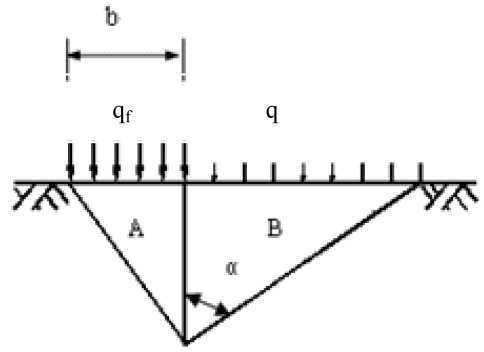


Figure 1. Shear failure model of rock foundation.

where  $\phi_t$  is the internal friction angle of twin shear  $\phi_t = \arcsin \frac{(1-b)+(3+b)\sin \phi_0}{3+\sin \phi_0}$ .  $C_t$  is cohesive strength of twin shear,  $C_t = \frac{4C_0 \cos \phi_0}{3+\sin \phi_0} \frac{1}{\cos \phi_t}$ .  $b$  is coefficient of intermediate principal stress  $m, \sigma_2 = \frac{b}{2}(\sigma_1 + \sigma_3)$ . Substituting equations:  $P = \frac{1}{2}(\sigma_1 + \sigma_3)$ ,  $R = \frac{1}{2}(\sigma_1 - \sigma_3)$  into Equation (1), we get

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin \phi_t + C_t \cos \phi_t$$
(4)

while  $\alpha = 45^\circ + \frac{\phi_t}{2}$ , For Equation (4) making triangular transformation, we get

$$\sigma_1 = \sigma_3 \tan^2(45^\circ + \frac{\phi_t}{2}) + 2C_t \tan(45^\circ + \frac{\phi_t}{2})$$
(5)

## 3 THE FORMULA OF THE ULTIMATE BEARING CAPACITY OF ROCK FOUNDATION

### 3.1 Basic hypothesis

In Figure 1 it presumes that the equispaced strip-type load  $q_f$  with the width of  $b$  works on the semi-infinite body, and  $q$  is the equispaced load working on the surface of the rock foundation near the load  $q_f$ . The hypothesis is made in order to simplify the calculation: 1) The load  $q_f$  is effective in such a long distance that the resistance originating from the sides of both ends can be ignored; 2) The plastic zone of the rock foundation can be simplified as the active zone A of Rankine and passive zone B of Rankine; 3) The failure surface is straight, smooth and perpendicular, and the boundary surface of the active wedge and the passive wedge has no shear stress; 4) The average body force is applied to each failure wedge. The failure model of the rock foundation is illustrated in Figure 1.

### 3.2 Considering intermediate principle stress

For the passive zone B impacted by the wedge A with the  $q_f$  it causes horizontal normal stress  $\sigma_{1B}$  acting on the wedge B which suffers the biggest principle

stress. However the resultant force of the self-weight stress and the equispaced load  $q$  of rock surface is the smallest principle stress  $\sigma_{3B}$  working on the wedge B, as shown in the following:

$$\sigma_{3B} = \frac{1}{2} \gamma h^2 \tan \alpha + q \quad (6)$$

From the equation (6) it gets:

$$\sigma_{1B} = \sigma_{3B} \tan^2 \alpha + 2C_i \tan \alpha \quad (7)$$

where  $\gamma$  is the rock bulk density,  $\alpha = 45^\circ + \frac{\varphi_t}{2}$ .

For the wedge A,  $q_f + \frac{1}{2} \gamma h$  is the biggest principle stress working on the top, while  $\sigma_{3A} = \sigma_{1B}$  is the smallest principle stress working on the wedge A, as shown in the following:

$$q_f + \frac{1}{2} b h \gamma = \sigma_{3A} \tan^2 \alpha + 2C_i \tan \alpha \quad (8)$$

Substituting Equations (6), (7) in (8) we get:

$$\begin{aligned} q_f &= \frac{1}{2} \gamma b \tan^5 \alpha + 2C_i \tan \alpha [1 + \tan^2 \alpha] \\ &+ q \tan^4 \alpha - \frac{1}{2} \gamma b \tan \alpha \end{aligned} \quad (9)$$

For the equation (9) the last part is far smaller than other part thus it can be omitted, where

$$\begin{aligned} N_\gamma &= \tan^5 \left( 45^\circ + \frac{1}{2} \arcsin \frac{(1-m) + (3+m) \sin \varphi_0}{3 + \sin \varphi_0} \right) \\ &= \tan^5 \left( 45^\circ + \frac{\varphi_t}{2} \right) \end{aligned}$$

$$N_c = 2 \tan \left( 45^\circ + \frac{1}{2} \arcsin \frac{(1-m) + (3+m) \sin \varphi_0}{3 + \sin \varphi_0} \right)$$

$$\times \left[ 1 + \tan^2 \left( 45^\circ + \frac{1}{2} \arcsin \frac{(1-m) + (3+m) \sin \varphi_0}{3 + \sin \varphi_0} \right) \right]$$

$$= 2 \tan \left( 45^\circ + \frac{\varphi_t}{2} \right) \left[ 1 + \tan^2 \left( 45^\circ + \frac{\varphi_t}{2} \right) \right]$$

$$N_q = \tan^4 \left( 45^\circ + \frac{1}{2} \arcsin \frac{(1-m) + (3+m) \sin \varphi_0}{3 + \sin \varphi_0} \right)$$

$$= \tan^4 \left( 45^\circ + \frac{\varphi_t}{2} \right)$$

The equation (9) can be presented as:

$$q_f = \frac{1}{2} \gamma b N_\gamma + C_i N_c + q N_q \quad (10)$$

Consequently the modified formula of the bearing capacity of rock foundation is drawn involving the intermediate principle stress.

#### 4 PROJECT CASE

The data for some rock foundation:  $\gamma = 25 \text{ kN/m}^3$ ,  $c = 30 \text{ kPa}$ ,  $\varphi_0 = 30^\circ$ , to give a strip-type load and the width  $b = 1 \text{ m}$ , (Sheng, 2006) the following respectively take the linear shear failure and the formula drawn in the paper to calculate the ultimate bearing capacity of foundation and compare the results of them.

- (1) To employ the linear shear failure to work on the ultimate bearing capacity of foundation  
With,  $\varphi_0 = 30^\circ$  then it gets  $N_\gamma = 15.6$ ,  $N_c = 13.9$ ,  $N_q = 9$ , thus the ultimate bearing capacity of foundation with the linear shear failure is shown as:

$$q_f = \frac{1}{2} \gamma b N_\gamma + c N_c + q N_q = 663 \text{ kPa}$$

- (2) To assume the coefficient of the intermediate principle stress  $m = 1$  to calculate the ultimate bearing capacity of foundation with the formula drawn in the paper

With the formula in the paper  $\varphi_t = 34.8^\circ$ ,  $C_t = 36.2$ , to substitute in the equation (10) we get,  $N_\gamma = 25.6$ ,  $N_c = 17.8$ ,  $N_q = 13.3$ , thus the following stated is the ultimate bearing capacity of foundation with the formula in the paper.

$$q_f = \frac{1}{2} \gamma b N_\gamma + C_i N_c + q N_q = 964.36 \text{ kPa}$$

- (3) To assume the coefficient of the intermediate principle stress  $m = 0.9$  to calculate the ultimate bearing capacity of foundation with the formula drawn in the paper

With the formula the paper,  $\varphi_t = 35.9^\circ$ ,  $C_t = 36.7$ , to substitute in the equation,  $N_\gamma = 28.8$ ,  $N_c = 18.9$ ,  $N_q = 14.7$ , thus the following stated is the ultimate bearing capacity of foundation with the formula in the paper.

$$q_f = \frac{1}{2} \gamma b N_\gamma + C_i N_c + q N_q = 1053.63 \text{ kPa}$$

#### 5 CONCLUSION

- (1) For the calculation of the bearing capacity of foundation, the ultimate bearing capacity of the linear shear failure is smaller than the one when the impact of intermediate principle stress is taken into consideration, and it can bring forth the huge economic benefits once it is put into the practical projects. However due to the lack of application background for projects it is still essential

- to conduct the in-situ loading test to verify its practicability
- (2) With the decreasing of the coefficient of the intermediate principle stress the ultimate bearing capacity of foundation is increasing instead.

## REFERENCES

- Ajay Bindlish; Mahendra Singh; N. K. Samadhiya. 2012. Ultimate Bearing Capacity of Shallow Foundations on Jointed Rock Mass. *Indian Geotechnical Journal* 42(3): 169-178.
- Code for design of building foundation (GB50007-2011). Beijing: China Architecture & Building Press (in Chinese).
- Ghazavi, M. and Eghbali, A. 2013. New Geometric Average Method for Calculation of Ultimate Bearing Capacity of Shallow Foundations on Stratified Sands. *Int. J. Geomech.* 13(2): 101-108.
- Standard for classification of engineering rock masses (GB50318-94). Beijing: China planning press (in Chinese).
- Song Jianbo, Yu Yuanzhong, Liu Hanchao. 1999. Method determining ultimate bearing capacity of rock foundation with Hock-Brown strength criterion. *Journal of Geological Hazards and Environment Preservation* 0(4): 67-70.
- Song Jianbo, Yu Yuanzhong. 2002. Bell solution determining ultimate bearing capacity of homogeneous rock foundation under shear failure model. *Chinese Journal of Rock Mechanics and Engineering* 21(3): 410-412.
- Sheng Minrong, Chen Jianfeng. 2006. *Rock mass Mechanics*. Shanghai: Tongji press (in Chinese).
- Terzaghi K. 1960 *Theory Mechanics*. Translated by Xu Zhiying. Beijing: Geological Publishing House (in Chinese).
- Yu Maohong, Yang Songyan, Liu Chunyang. 1997. Unified Plane-Strain Slip Line Field Theory System [J]. *China Civil Engineering Journal* 30(2):14-26 (in Chinese).
- Yu Maohong. 1994. geotechnical materials Unified Strength Theory and its Applications. *Chinese Journal of Geotechnical Engineering* (2):1-10
- Yu Maohong. 2004. Unified strength theory and it's applications. The Press of Springer-verlag Berlin, Germany.
- Zhou Xiaoping, Wang Jianhua, Zhang Yongxing. 2002. Calculation of ultimate bearing capacity of foundation under triaxial compressive loading. *Journal of Chongqing Jianzhu University* 24(3): 28-32. (in Chinese).