Geotechnical Aspects of Underground Construction in Soft Ground, Mair & Taylor (eds)
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Prismatic failure – A new method of calculating stability against boiling of sand within a cofferdam

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ABSTRACT: In an excavation of soil with high ground water level, the soil is subjected to an upward seepage flow. For sandy and/or silty soil within a cofferdam, boiling is a problem. There are several methods of calculating the stability against seepage failure of the soil. Seepage failure sometimes occurs even in cofferdams designed by these methods. In this study, a new method of calculating the stability against boiling of soil — the Prismatic failure concept— is presented and applied to some examples. The prismatic failure concept gives us a good estimation of critical hydraulic head differences for these examples. The concept is useful for calculating the stability against seepage failure of soil in a pit surrounded by sheet pile walling.

1. INTRODUCTION

Let us consider a single wall cofferdam shown in Fig.1(a). When water is pumped out of the space surrounded by the cofferdam, a flow of water occurs through the soil towards the bottom of the space because of the difference in hydraulic heads on both sides of the sheet pile wall. For sandy and/or silty soil within a cofferdam, boiling is a problem. There are several methods of calculating the stability against seepage failure of the soil. Seepage failure sometimes occurs even in cofferdams designed by these methods. There are still questions left in these methods.

Terzaghi(1943) presented a method of calculating the stability against seepage failure of soil behind a sheet pile wall. He assumed that the body of sand which is lifted by water has the shape of a prism with a width of $D/2$; half of the penetration depth D of the wall. In the cases where shapes of critical prisms are different from the shape of Terzaghi's prism: e.g. anisotropic soil, multi-layered soil, soil with a loaded filter etc., Terzaghi's method can not be applied. In this study, a new method of calculating the stability against boiling of soil —the Prismatic failure concept— is presented and applied to some examples. The mechanism of seepage failure of soil within a cofferdam is also discussed using equi-safety-factor lines based on the concept.

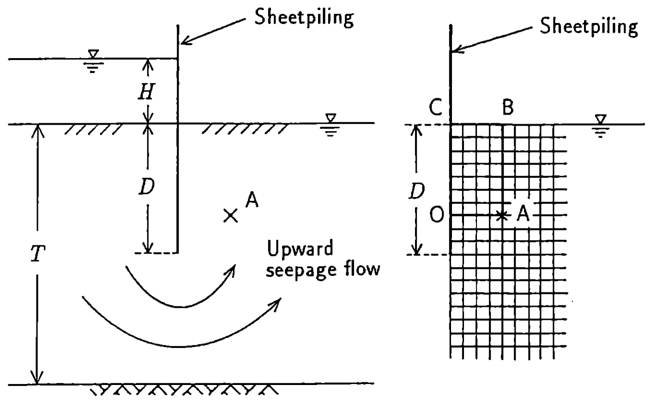


Fig.1. Soil behind sheet piles

2. TERZAGHI'S METHOD (A)

Fig.2 shows a section through a single wall cofferdam. The subsoil with a thickness T is penetrated to a depth D below its surface by sheet piles. The sand adjoining the sheet piles remains stable provided the hydraulic head difference H is smaller than a certain critical value. However, as soon as this critical value is approached, the surface of the sand rises and the subsoil finally collapses.

Terzaghi assumed, from experimental evidence, that the body of sand lifted by water has the shape

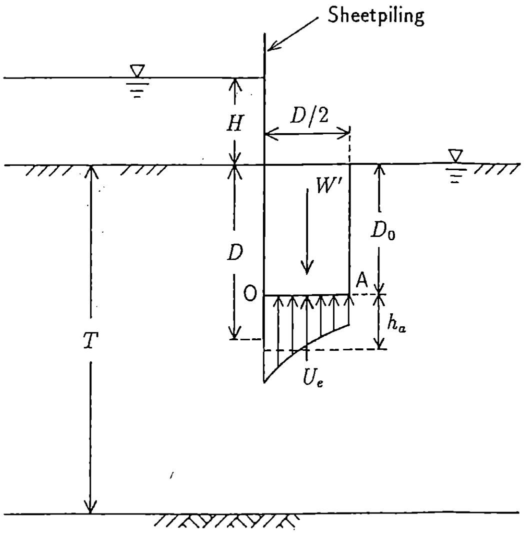


Fig.2. Terzaghi's method (A)

of a prism with a width $D/2$ and a horizontal base at some depth D_0 below the surface ($0 \leq D_0 \leq D$). The rise of the prism is resisted by the weight and vertical side friction of the prism. It is assumed that at the instant of failure the effective horizontal stress on the sides of the prism and the corresponding frictional resistance are practically zero. Therefore the prism rises as soon as the total excess water pressure U_e on its base OA becomes equal to the submerged weight of the prism W':

$$W' = \frac{D}{2} D_0 \gamma' \tag{1}$$

$$U_e = \frac{1}{2}\gamma_w D h_a = \frac{1}{2}\gamma_w D C_0 H \tag{2}$$

where γ' is the buoyant unit weight of soil, γ_w is the unit weight of water, $h_a (=C_0 H)$ is the average excess hydraulic head, and C_0 is constant and independent of H.

The hydraulic head difference at the instant of rise of the prism H_0 is calculated from Eqs.(1) and (2) as follows:

$$H_0 = \frac{D_0 \gamma'}{C_0 \gamma_w} \tag{3}$$

The calculation is repeated for different horizontal sections, through the sand, located at different depths D_0 below the bottom of the pit. The critical head difference H_c is determined by the condition H_0=minimum:

$$H_c = \min\left\{ H_0 = \frac{D_0\gamma'}{C_0\gamma_w};\ 0 \leq D_0 \leq D \right\} \tag{4}$$

The horizontal section to which this minimum refers is the critical section. For the single wall cofferdam as illustrated in Fig.1(a) an investigation has shown that the critical section passes almost exactly through the lower edge of the wall, or $D_0 = D$ (Terzaghi's method (B), 1948).

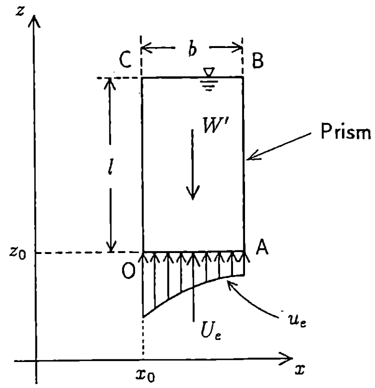


Fig.3. Prismatic failure (No friction)

At a given hydraulic head H the factor of safety against boiling of a prism F_s is

$$F_s = \frac{W'}{U_e} = \frac{D D_0 \gamma'/2}{h_a D \gamma_w/2} = \frac{D_0\gamma'}{C_0\gamma_w}\frac{1}{H} = \frac{H_0}{H} \tag{5}$$

The safety factor against boiling of the soil behind sheet piles $F_{s\min}$ is determined by the condition F_s = minimum:

$$F_{s\min} = \min\left\{ F_s = \frac{W'}{U_e} = \frac{H_0}{H},\ 0 \leq D_0 \leq D \right\}$$

$$= \frac{H_c}{H} \quad (\because \text{From Eqs.(4) and (5)}) \tag{6}$$

3. PRISMATIC FAILURE

We assume that the body of soil, lifted by seepage water, has the shape of a prism with a certain height and width adjoining a sheet pile wall as shown in Fig.1(b).

(1) Prismatic failure (No friction)

Here we assume that the side frictional resistance is zero at the instant of failure. We consider the vertical prism OABC as shown in Fig.3. The prism has the submerged weight W' and is subjected to the total excess hydraulic pressure on its base U_e:

$$W' = \gamma' b\, l \tag{7}$$

$$U_e = \int_{x_0}^{x_0+b} u_e|_{z=z_0}\, dx = C_1 H \tag{8}$$

where b and l are the width and depth of a prism, and u_e is the excess pore water pressure. Therefore the safety factor with respect to the rise of the prism OABC is defined as

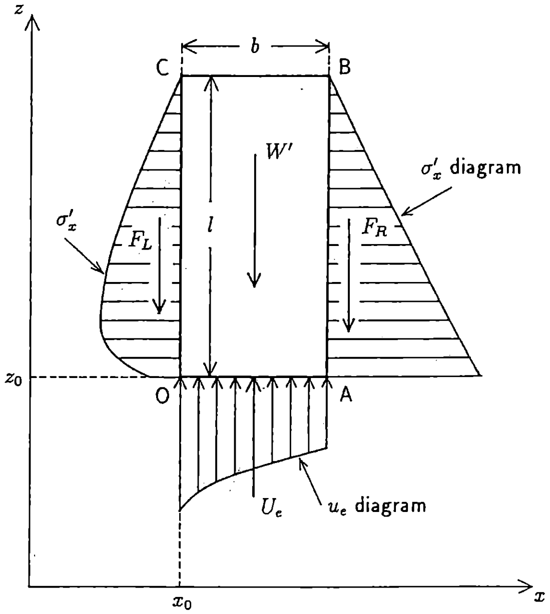


Fig.4. Prismatic failure (Friction)

$$F_s = \frac{W'}{U_e} = \frac{W'}{C_1 H} \tag{9}$$

where C_1 is constant and independent of H. The value of W'/C_1 is inherent in a prism, because W' is also constant for the prism. The Prismatic failure (No friction) is defined as follows:

"Consider the soil behind sheet piles subjected to a hydraulic head difference $H=H_1$. Safety factors for various prisms in the soil are calculated. The critical prism is determined by the condition that the safety factor is minimum among all of the prisms. The critical prism is the same for another hydraulic head difference $H=H_2$, because the value of W'/C_1 is inherent in a prism. The critical prism is, therefore, separated from its bottom and rises when H increases and approaches the critical value H_c."

Let us consider the critical prism. We can get the relation $H_c=W'/C_1$ from the substitution of $F_s=1$ and $H=H_c$ into Eq.(9). Eq.(9) and $H_c=W'/C_1$ give us

$$F_{s\min} = \frac{H_c}{H} \tag{10}$$

where $F_{s\min}$ is the safety factor for the critical prism which is minimum among all of the prisms.

If we define the safety factor for the prism with a bottom right point A as the safety factor at the point A, the distribution of equi-safety-factor lines can be drawn. The distribution gives us useful information for boiling.

(2) Prismatic failure (Friction)

Here we consider the case where the side frictional resistance is not zero at the instant of failure. The frictions F_L and F_R on the left and right hand sides are exerted because of the presence of the horizontal effective stress σ'_x:

$$F_L = \int_{z_0}^{z_0+l} \sigma'_x|_{x=x_0} \tan\delta_1 dz \tag{11}$$

$$F_R = \int_{z_0}^{z_0+l} \sigma'_x|_{x=x_0+b} \tan\delta_2 dz \tag{12}$$

where $\delta_1=\frac{2}{3}\phi'$; an angle of wall friction, and $\delta_2=\phi'$; an angle of internal friction of sand. Sand particles tend to move into the soil because of seepage force, and the passive state is developed within the soil behind sheet piles. So the horizontal effective stress σ'_x is:

$$\sigma'_x = K_p \sigma'_\zeta \tag{13}$$

The coefficient of passive earth pressure K_p is given by Rankine as $K_p=(1+\sin\phi')/(1-\sin\phi')$. The vertical effective stress σ'_ζ is calculated approximately as follows:

$$\sigma'_\zeta = \int_0^\zeta (\gamma' - i_\zeta \gamma_w) d\zeta = \gamma'\zeta - h_\zeta \gamma_w \tag{14}$$

in which h_ζ is the excess hydraulic pressure head at the depth ζ from the surface of a pit.

The rise of the prism OABC is resisted by the submerged weight W' and the side frictions F_L and F_R as shown in Fig.4. The safety factor with respect to the rise of the prism OABC, which is subjected to the total excess water pressure on its base U_e, is

$$F_s = \frac{W' + F_L + F_R}{U_e} \tag{15}$$

The critical prism is determined by the condition that the minimum safety factor among all prisms becomes just equal to 1.0.

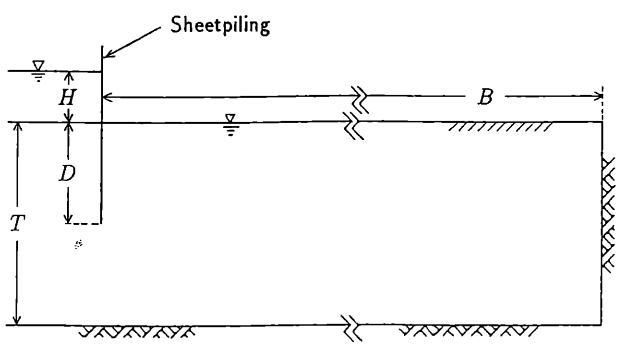
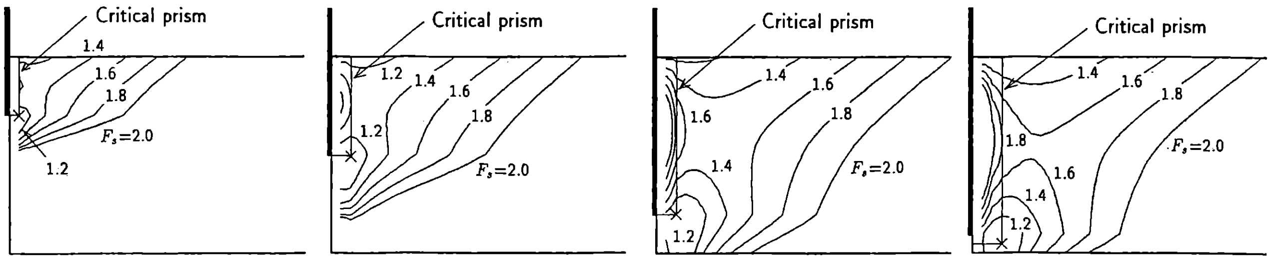


Fig.5. One-layered soil



(a) D/T=0.3 (b) D/T=0.5 (c) D/T=0.8 (d) D/T=0.9

Fig.6. Equi-safety-factor lines by the Prismatic failure (Friction): One-layered isotropic soil

4. ANALYTICAL RESULTS

The three typical examples were analysed using FEM with 4CST elements: One-layered isotropic and anisotropic soils, and two-layered soil. The four methods of calculating stability were used: Terzaghi's method (A), Prismatic failure (No friction and Friction), and Method of 1D assumption (the soil adjoining sheet piles is in one-dimensional state of stress and seepage flow).

4.1 *One-layered isotropic soil*

We consider the homogeneous isotropic soil with a thickness T, a width B=2.5T and a penetration depth D by sheet piles (Fig.5). For different penetration of sheet piles, the stabilities were analysed.

Fig.6 shows the distribution of equi-safety-factor lines by Prismatic failure (Friction) for D/T= 0.3, 0.5, 0.8 and 0.9. For Prismatic failure (No friction) the critical prisms have no width (b=0). On the other hand for the Prismatic failure (Friction) the critical prisms have a width which is smaller than that of the Terzaghi's method. The critical section is the same level as the lower edge of sheet piles when D/T is small, but becomes deeper than the lower edge as D/T is large.

Fig.7 shows the relationship between D/T and $H_c\gamma_w/T\gamma'$. It is observed from Fig.7 that the Prismatic failure (No friction) and 1D assumption give us the same critical head difference H_c, which is the smallest estimation for H_c. The critical head difference by the Prismatic failure (Friction) is nearly equal to that by the Terzaghi's method.

4.2 *One-layered anisotropic soil*

Consider the homogeneous anisotropic soil with a thickness T, a width B=2.5T~ 12.5T and a penetration depth D by sheet piles (Fig.5). For different penetration of sheet piles, the stabilities for the anisotropic soils of k_x/k_z= 1/3, 3, 9 and 25 were analysed, where k_x and k_z are the coefficients of permeability in the x and z directions.

Fig.8 shows the distribution of equi-safety-factor lines by the Prismatic failure (Friction) for D/T= 1/2 in the cases of k_x/k_z=1/3, 3, 9 and 25. The critical prisms by the Prismatic failure (No friction) have zero width (b=0), but on the other hand the critical prism by the prismatic failure (Friction) have a width. The larger the degree of anisotropy becomes, the deeper and wider the critical prism becomes. The critical prism by the Prismatic failure (Friction) becomes larger than that of Terzaghi's method in some cases.

Fig.9 shows the relationship between D/T and $H_c\gamma_w/T\gamma'$ with a parameter of k_x/k_z. For the Prismatic failure (Friction) and Terzaghi's methods, the larger the degree of anisotropy grows, the smaller the H_c value becomes. Anisotropy has more effective impact on H_c for the Terzaghi's method than the Prismatic failure (Friction). The H_c values by the two methods are more separated from each other, as the degree of anisotropy grows larger. The D/T vs $H_c\gamma_w/T\gamma'$ curve is the same as the isotropic case (Sec.4.1) for the Prismatic failure (No friction) and 1D assumption.

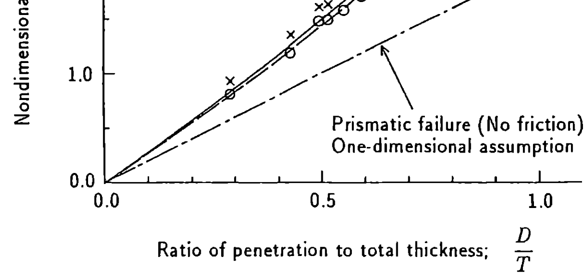
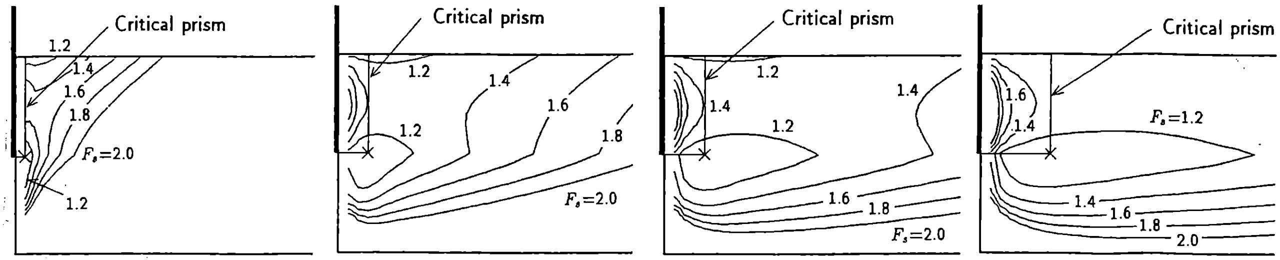


Fig.7. Relationship between D/T and $H_c\gamma_w/T\gamma'$ (One-layered isotropic soil)



(a) $k_x/k_z=1/3$ (b) $k_x/k_z=3$ (c) $k_x/k_z=9$ (d) $k_x/k_z=25$

Fig.8. Equi-safety-factor lines by the Prismatic failure (Friction): One-layered anisotropic soil

4.3 *Two-layered soil*

We consider the two-layered soil with a thickness T, a width B=2.5T and a penetration depth $D=T/2$ by sheet piles (Fig.10). The thickness of the upper layer is $D/2$. For different ratios of permeabilities of the upper and lower layers $k_r(=k_2/k_1)$, the stabilities were analysed.

Fig.11 shows the distribution of equi-safety-factor lines by the Prismatic failure (Friction) for k_r=0.3, 0.635, 0.636 and 2.0. For the Prismatic failure (Friction), the critical section is located at the boundary of the two layers when k_r is smaller than 0.635, and is shifted to the lower edge of sheet piles when k_r is larger than 0.636. The same may be said of the other methods (Table 1). It should be noted that, for a two-layered sand column in one dimension, the critical section is shifted from the boundary of the two layers to the bottom of the sand column when k_r exceeds just 1.0. It means that a sheet pile wall has, in a hydraulic sense, an effect of increase in stability of soil behind the wall.

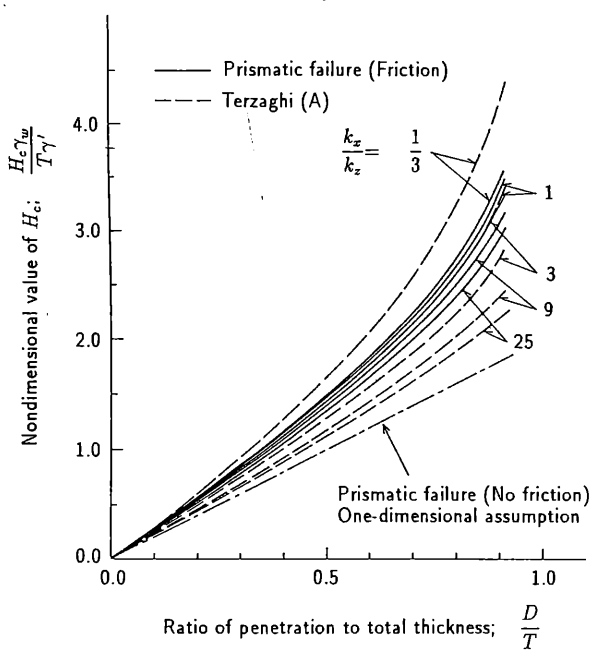


Fig.9. Relationship between D/T and $H_c\gamma_w/T\gamma'$ (One-layered anisotropic soil)

Table 1 Changes in critical section

Method	Critical section is located at the boundary of the two layers	Critical section is located at the lower edge of sheet piles
Terzaghi	$k_r\leqq0.752$	$k_r\geqq0.753$
P.f. (No friction)	$k_r\leqq0.435$	$k_r\geqq0.436$
P.f. (Friction)	$k_r\leqq0.635$	$k_r\geqq0.636$
1D assumption	$k_r\leqq1.0$	$k_r\geqq1.0$

Fig.12 shows the relationship between k_r and $H_c\gamma_w/T\gamma'$. It is found from Fig.12 that the 1D assumption gives us the smallest estimation for the critical hydraulic head difference H_c. When k_r is less than 0.40, the results obtained by the Prismatic failure (No friction and Friction) and Terzaghi's method (A) approach the same line. It is thought that, for $k_r<0.40$, the soil adjoining sheet piles in the upper layer is nearly in one-dimensional state of stress and seepage flow. The theoretical line of the above three methods for $k_r<0.40$ is, therefore, predicted to fit in well with experimental data, because the theory coincides with the experiment for quicksand of one-layered soil in one dimension (Tanaka et al., 1991). As k_r goes beyond 0.40, the H_c values by the above three methods are separated from one another, and the magnitudes of H_c by the Prismatic failure (Friction), Terzaghi's method (A) and Prismatic failure (No friction) are larger in this order.

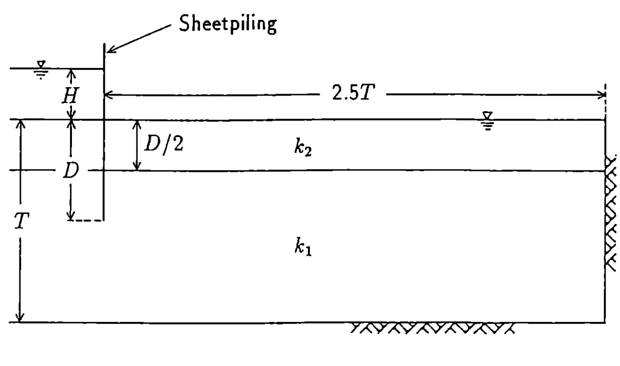
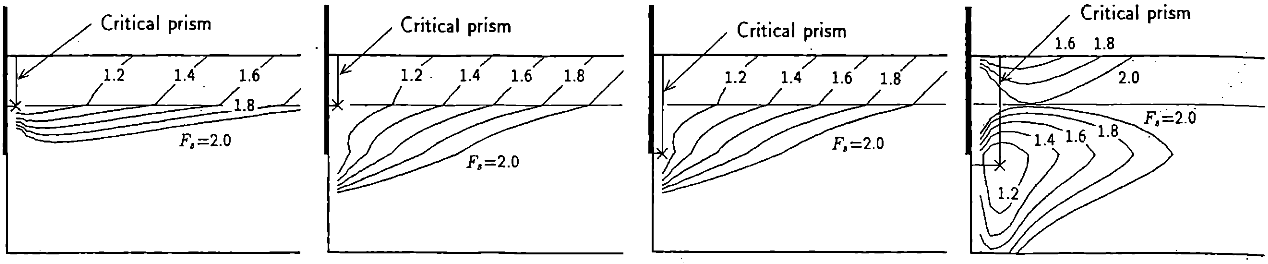


Fig.10. Two-layered soil



(a) k_r=0.3 (b) k_r=0.635 (c) k_r=0.636 (d) k_r=2.0

Fig.11. Equi-safety-factor lines by the Prismatic failure (Friction): Two-layered soil

5. TEST RESULTS AND CONCLUSIONS

Experimental results on seepage failure of the one-layered sand of $D_r \doteqdot 50\%$ behind a sheet pile wall revealed that the soil masses start to deform in the vicinity of the wall at $H=H_y$, or the settlement of the upstream surface and the rise of the downstream surface occur. The deformation enlarges with increase in H and the subsoil finelly collapses at $H=H_f$. H_y is nearly equal to $H_f \times 0.9$.

(1) One-layered isotropic soil: Fig.7 shows the experimental results of H_y at starting to deform and H_f at failure as well as the theoretical results. Theoretical critical head differences H_c by the Prismatic failure (Friction) and Terzaghi's method are nearly equal to each other and coincide with H_y.

(2) One-layered anisotropic soil: For the Prismatic failure (Friction), the larger the degree of anisotropy becomes, the deeper and wider the critical prism becomes. The critical prism is larger than that of Terzaghi's method in some cases. For the Prismatic failure (Friction) and Terzaghi's method, the larger the degree of anisotropy grows, the smaller the H_c value becomes. Anisotropy has more effective impact on H_c by the Terzaghi's method than the Prismatic failure (Friction). The H_c values by the two methods are more separated from each other, as the degree of anisotropy grows larger.

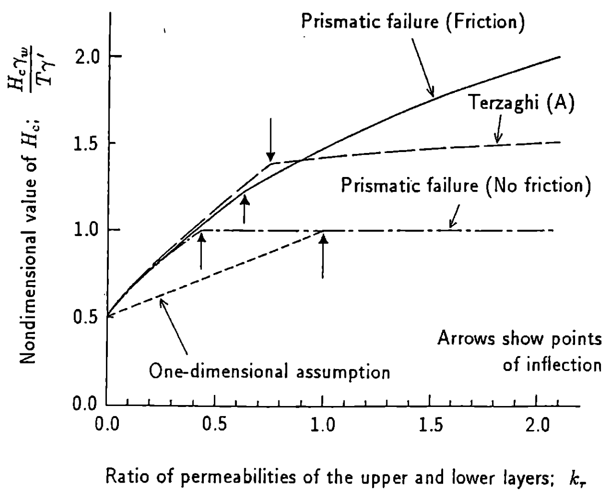


Fig.12. Relationship between k_r and $H_c\gamma_w/T\gamma'$ (Two-layered soil)

(3) Two-layered soil: For the Prismatic failure (Friction), the critical section is located at the boundary of the two layers when k_r is smaller than 0.635, and is shifted to the lower edge of sheet piles when k_r is larger than 0.636 (Table 1). It means that a sheet pile wall has, in a hydraulic sense, an effect of increase in stability of soil behind the wall. When k_r is less than 0.40, the results obtained by the Prismatic failure (No friction and Friction) and Terzaghi's method (A) approach the same line, which is predicted to fit in well with experimental data. As k_r goes beyond 0.40, the H_c values by the above three methods are more separated from one another.

The prismatic failure (No friction) always gives us a value of H_c on the safety side, and the prismatic failure (Friction) gives us a practical value of H_c not only for the isotropic soil but for the anisotropic soil and two-layered soil behind sheet piles. Thus the prismatic failure concept is useful for considering the stability against seepage failure of soil behind a sheet pile wall. The distribution of equi-safety-factor lines in soil is also useful for predicting a mode of failure.

Engineering practice of the methods described in this paper is still now left open.

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