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# Back analysis for shield tunnel using beam-joint model

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ABSTRACT: In the paper, a beam-joint model will be stated first and then a back analysis, using the beam-joint model, will be carried out to evaluate the external pressure on the lining structure based on the in-situ measured data like the axial forces, the bending moments of the segment pieces. An optimization technique--the simplex method is utilized here. The partial-peripheral ground spring model and the whole-peripheral one have been compared using the back analysis technique.

## 1 INTRODUCTION

It is well known that the joints between segment pieces (called segment joint below) and those between segment rings (called ring joint below) in the lining structure of shield tunnel affect significantly the internal forces and deformations of the lining structure, especially those of the joint parts.

A beam-spring model(Murakami & Koizumi, 1978), composed of beam for segment piece and spring for joint, has been used to simulate the behavior of the segment pieces and the joints and been employed in practice increasingly in recent years in Japan. The model is derived to simulate the linearly elastic model.

However, the above model can not be used to simulate and describe the behaviors of nonlinearity of the segment joint thoroughly and exactly. A lot of laboratory experiments have shown that the internal force-deformation relations behave a serious nonlinearity during a whole loading process;

Furthermore, the lining structure of shield tunnel consists of several segment pieces that are linked with bolts and thus discontinuity is inherent in it. Therefore, the rotation angle between the two adjacent segment pieces with a segment joint is discontinuous;

Moreover, the beam-spring model can not obtain the results of the internal forces of segment bolt and the open distance of the joint directly and precisely. It is only concerned and needed by designers and researchers.

Regarding to the discussed above, a new model--beam-joint model was proposed by authors(1994). With starting from the discontinuity of the structure, the idea of Goodman element (1968) in the numerical analysis of discontinuous medium mechanics was introduced and a point-to-point joint element with tensile action is conceived.

In another side, a great attention has been paid recently to the determinations of the earth pressure pattern on the lining structure, which includes the external earth pressure and the deformation-dependent pressure. It is shown from a lot of the measured data that the design pressure is much greater than the actual earth pressure. The distribution mechanisms of the two kinds of the earth pressure remain to be unsolved and not explained clearly, both theoretically and practically.

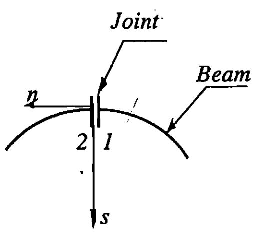
In the paper, the beam-joint model will be stated first and then a back analysis, using the beam-joint model, will be carried out to evaluate the external earth pressure on the lining structure based on the in-situ measured data like the axial forces and bending moments of the segment pieces. The distribution pattern of the external pressure is supposed to vary in parabolic form with the acting position along the horizontal direction and the vertical one, and an optimization technique--the simplex optimization method is utilized here. The partial-peripheral ground spring model and the whole-peripheral one for simulating the reaction of ground to structure will be compared using the back analysis approach. In the meantime, the effect of the cross joint in the longitudinal direction of tunnel will be considered in numerical simulation.

## 2 BEAM-JOINT MODEL

### 2.1 Segment joint

The lining structure of a shield tunnel is assembled by several segment pieces, then the discontinuous plane or the joint is inherent in it. Consequently, if the segment piece is discretized by several beam elements, we consider a joint element with point to point should be inserted into the two segment pieces, shown as Fig.1 to simulate the discontinuity of the linking part. The stiffness of the straight or curved beam element is given by use of the same method as the previous one for the beam-spring model (Murakami & Koizumi, 1978) and the establishment for the stiffness of the joint element is discussed only.

The joint element is composed of a double-node shown as Fig.2, so the discontinuity of the two adjacent segment pieces is described by the following relative displacements with three variables between the two nodes 1, 2 in a local coordinate system n, s



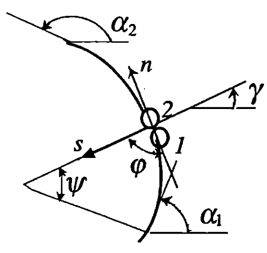




Fig.1 Beam-joint model Fig.2 Double nodes of joint

$$\Delta u = u_1' - u_2', \Delta v = v_1' - v_2', \Delta\vartheta = \vartheta_1' - \vartheta_2' \quad (1)$$

where the local coordinates is defined as: $s$ is the direction of the bisecting angle between the two beams with a positive direction inward to opening and $n$ is the direction perpendicular to $s$ as illustrated in Fig.2. Accordingly, $\Delta u$ is called a relative axial displacement of the joint along $n$, $\Delta v$ is a relative shearing displacement along s and $\Delta\vartheta$ is a relative rotational displacement; $u_i', v_i', \vartheta_i' (i = 1,2)$ are the displacement components of node i along the local coordinate directions.

The point-to-point joint element is analogy with the line-to-line Goodman joint element in the two-dimensional space or the plane-to-plane one in the three-dimensional space with rotational effect. Under a definite external load, the force-deformation relationship of the point-to-point joint element is written as

$$\begin{Bmatrix} N \\ Q \\ M \end{Bmatrix} = \begin{bmatrix} k_n & & \\ & k_s & \\ & & k_\vartheta \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta v \\ \Delta\vartheta \end{Bmatrix} \quad (2)$$

Recording that $\{F\} = \{N \quad Q \quad M\}^T$, $\{\Delta d\} = \{\Delta u \quad \Delta v \quad \Delta\vartheta\}^T$ and $[K] = diag[k_n \quad k_s \quad k_\vartheta]$, we have

$$\{F\} = [K] \bullet \{\Delta d\} \quad (3)$$

where $N$, $Q$, $M$ are the forces along the direction $n$, $s$ and the moment of the joint element, respectively.

Translating $u_i', v_i', \vartheta_i' (i = 1,2)$ into the components of displacement $\{\delta\} = \{u_1, v_1, \vartheta_1, u_2, v_2, \vartheta_2\}^T$ along the local coordinate directions for each related beam, $\{\Delta d\}$ becomes

$$\{\Delta d\} = [E] \bullet \{\delta\} \quad (4)$$

and

$$[E] = \begin{bmatrix} T & -T^T \end{bmatrix}, \; [T] = \begin{bmatrix} sin\varphi & cos\varphi & 0 \\ -cos\varphi & sin\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

where $[E]$, $[T]$ are the translation matrices related with a bisecting angle $\varphi$ between the two tangent lines of the adjacent beams which $\varphi$ is 90° in Fig.2.

The following equation is obtained easily

$$[K_J] = [E]^T [K] \cdot [E] \quad (6)$$

where $[K_J]$ is the stiffness of the joint element in the local coordinates. And the global stiffness of the joint element in the global coordinate system is

$$[K_G] = \begin{bmatrix} A_1 & A_3 & 0 & -A_1 & -A_3 & 0 \\ & A_2 & 0 & -A_3 & -A_2 & 0 \\ & & k_\vartheta & 0 & 0 & -k_\vartheta \\ & & & A_1 & A_3 & 0 \\ & & & & A_2 & 0 \\ sym. & & & & & k_\vartheta \end{bmatrix} \quad (7)$$

where $A_1 = k_n sin^2\gamma + k_s cos^2\gamma$, $A_2 = k_n cos^2\gamma + k_s sin^2\gamma$ and $A_3 = (k_s - k_n) cos\gamma \, sin\gamma$, $\gamma = \alpha_1 + \psi - \varphi$, $\psi$ is the central angle of a curved beam element.

By the way, we known that this model is applicable to elastic bodies that follows a non-linear force-displacement relationship because linearity has not been invoked. A nonlinear relation of the rotational stiffness coefficient of the segment joint which depends upon the relative rotation $\Delta\vartheta$ is given

$$k_\vartheta = (k_{\vartheta_1} - k_{\vartheta_2}) e^{-\beta \cdot \Delta\vartheta} + k_{\vartheta_2} \quad (8)$$

where $k_{\vartheta_1}$, $k_{\vartheta_2}$ and $\beta$ are the constants obtained from the bending test of the segment joint.

## .2 Ring joint

n the present designing method only the shearing ction is used to simulate the longitudinal linking oint. The shearing action includes the radial and ircumferential shearings along the segment body emonstrated in Fig.3 and their shearing force versus hearing displacement relation is expressed as

$$\begin{Bmatrix} f_{nq} \\ f_{sq} \end{Bmatrix} = \begin{bmatrix} k_{nq} & \\ & k_{sq} \end{bmatrix} \begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} \qquad (9)$$

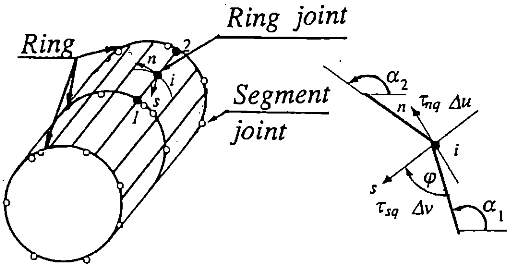



Fig.3 Shearing model for the ring joint

where $f_{nq}, f_{sq}$ are the circumferential and radial hearing forces, respectively, $\Delta u, \Delta v$ are the relative eformations between the two segment rings corresonding to $f_{nq}, f_{sq}$ and $k_{nq}, k_{sq}$ are their corresonding shearing stiffness coefficients.

$\Delta u, \Delta v$ are given by the first two equations in q.(1), but here $u_i', v_i'(i = 1,2)$ are the nodal displacements at its two neighbouring segment rings in a ocal coordinate system shown in Fig.3. Similar to he demonstrated previously, we have

$$\{\Delta d_q\} = [E_q]\{\delta_q\} \qquad (10)$$

here $\{\Delta d_q\} = \{\Delta u \quad \Delta v\}^T, \{\delta_q\} = \{u_1 \quad v_1 \quad u_2 \quad v_2\}^T$

$[E_q] = \begin{bmatrix} T_q & -T_q^T \end{bmatrix}, [T_q] = \begin{bmatrix} \sin\varphi & \cos\varphi \\ -\cos\varphi & \sin\varphi \end{bmatrix}$, $\varphi$ is a isecting angle between the two tangent lines of the djacent beams as shown in Fig.3.

We also have the stiffness of the ring joint element n a local coordinate system

$$[K_q] = [E_q]diag[k_{nq} \quad k_{sq}] \cdot [E_q]^T \qquad (11)$$

nd its global element stiffness is similar to that of q.(7) without the rotational term.

## PATTERN OF EXTERNAL EARTH PRESSURES

he external earth pressure components on the segent lining structure are assumed as a shape of a arabolic function as shown in Fig.4. In the figure, we take the upper left point of the lining layout as the original point $O$ of the $x$-$y$ coordinate system and suppose $p_i, q_i (i = 1,3)$ as unknowns to be found. Then we can write the vertical and horizontal pressure components $p(x,y)$, $q(x,y)$ in the following forms:

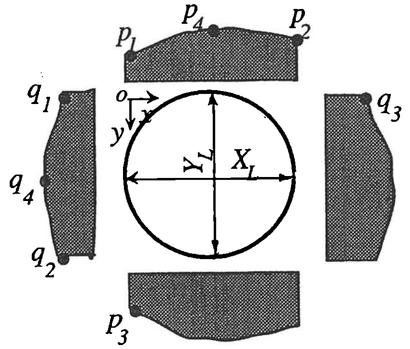



Fig.4 External earth pressure pattern without ground reaction

$$p(x,y) = a_0 + a_1 x + a_2 x^2 + a_3 y$$
$$q(x,y) = b_0 + b_1 y + b_2 y^2 + b_3 x \qquad (12)$$

where $a_i, b_i (i = 0,3)$ are dependent of $p_i, q_i$. By substituting $a_i, b_i$ into $p_i, q_i$, we can rewrite eqs(10),(11) into

$$p(x,y) = p_1 + (2d_x - 1)d_x p_{21} + 4(1 - d_x)d_x p_{41} + d_y p_{31}$$
$$q(x,y) = q_1 + (2d_y - 1)d_y q_{21} + 4(1 - d_y)d_y q_{41} + d_x q_{31} \qquad (13)$$

in which $d_x = x / X_L, d_y = y / Y_L, r_{i1} = r_i - r_1$ ($i$=2, 4; $r$=$p$,$q$).

## 4 CONSTRAINT CONDITIONS AND INPUT DATA

Selection for fixed points in calculating will greatly affects the reliability of the back-analyzed results. Generally speaking, it is difficult to choose some suitable positions as the fixed points so as to remove the rigid displacements of the structure due to the unsymmetries of the structure and the loading pattern in calculating. However, if both the normal and tangent stiffnesses of ground spring are introduced simultaneously, it is available to avoid effectively the rigid displacements produced.

In the meantime, it is well known that the cross-joint type in the longitudinal direction of tunnel has a significant influence on the internal forces of segment. Therefore, we need to simulate the strengthening effect of the cross-joint type on the whole structure.

Here, The beam-joint model as discussed previously is used and the mechanical parameters of the model required in back-calculating are determined from the laboratory model tests of the segment joint bending and shearing as well as the segment ring loading. We will account for these with an example below.

Suppose that the axial force $N$, the bending moment $M$ and the convergence $D$ of the segment piece are measured in the field. $N$, $M$ at any point of a beam element can be obtained easily.

## 5 BACK ANALYSIS TECHNIQUE

The optimization method is commonly used for making a back-analysis calculation to find the unknowns because it is available for almost all the problems, in particular the nonlinear ones. Here, the unknowns are supposed to be the components $P_i, Q_i$ of the external earth pressure and the objective function $J$ of optimization is defined as

$$J = w_1 J_N + w_2 J_M + w_3 J_D \quad (14)$$

where

$$J_N = \sum_{i=1}^{L_1}(N_i - N_i^*)^2 \Big/ \sum_{i=1}^{L_1} N_i^2$$

$$J_M = \sum_{i=1}^{L_2}(M_i - M_i^*)^2 \Big/ \sum_{i=1}^{L_2} M_i^2$$

$$J_D = \sum_{i=1}^{L_3}(D_i - D_i^*)^2 \Big/ \sum_{i=1}^{L_3} D_i^2$$

and $L_1$, $L_2$, $L_3$ are the measuring numbers of the axial force, the bending moment and the convergence of the lining structure, respectively, $N_i$, $M_i$, $D_i$ are the calculating values and $N_i^*$, $M_i^*$, $D_i^*$ are their measured data, and $w_1$, $w_2$, $w_3$ are their corresponding weighted coefficients, usually taking $w_1 = w_2 = w_3 = 1$.

The simplex optimization technique which is not concerned with the finding of any derivative is utilized here.

## 6 CASE STUDY

The extension project of Osaka Municipal Subway Line No.7 is under construction now. In the project, 1.2m reinforced concrete(RC) segment is used instead of 1.0m segment because of economical and workable reasons. The normal earth pressure to segment, the axial force and bending moment of the segment pieces and their deformations as well as the ground deformation due to tunneling have been monitored in order to verify the present design method. An observing section was scheduled for the monitoring points in a dense sand with the standard penetration test value greater than 60 at a distance of 30m from the launching shaft, and the overburden is 16.8m and the ground water level at the excavation depth is G.L.-10m.

The measured data for the lining structure are used to back-calculate the components of the external earth pressure. The sketch layout of the measuring points for the axial force and bending moment of the segment pieces and the normal earth pressure to the segments are illustrated in Fig.5 and their measured data are listed in tables 1,2.

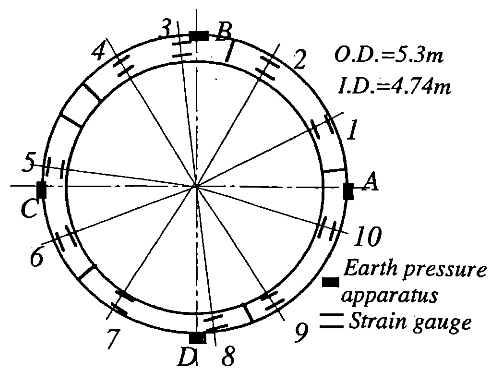



Fig.5 Sketch layout of measuring points

Table 1 Measured data and calculated results

| No. | Axial force (KN/Ring) | | |
|---|---|---|---|
| | Measured | P-P | W-P |
| 1 | 692 | 697(653) | 693(650) |
| 2 | 499 | 558(520) | 569(543) |
| 3 | 458 | 501(442) | 520(479) |
| 4 | 571 | 554(517) | 555(535) |
| 5 | 644 | 647(657) | 665(656) |
| 6 | 746 | 637(522) | 666(513) |
| 7 | 315 | 637(743) | 650(716) |
| 8 | 535 | 545(697) | 513(681) |
| 9 | 399 | 562(645) | 556(618) |
| 10 | 823 | 630(574) | 665(563) |
| No. | Moment (KN.m/Ring) | | |
| | Measured | P-P | W-P |
| 1 | -18.0 | -69.4(-63.6) | -53.0(-50.5) |
| 2 | 41.0 | 13.8(55.8) | 20.3(43.7) |
| 3 | 35.0 | 93.7(102.2) | 84.6(83.1) |
| 4 | 18.0 | 10.4(34.6) | -19.9(29.1) |
| 5 | -9.0 | -71.5(-86.2) | -71.2(-73.3) |
| 6 | 16.0 | -46.7(-103.0) | -65.1(-82.2) |
| 7 | -12.0 | 26.2(3.3) | 16.2(85.7) |
| 8 | 47.0 | 58.9(139.0) | 79.1(106.8) |
| 9 | 12.0 | 28.3(82.3) | 34.7(60.8) |
| 10 | -21.0 | -39.1(-109.5) | -60.8(-84.2) |

() denotes the calculated value without segment joint; P-P represents the partial-peripheral model and W-P is the whole-peripheral model.

The nonlinear relation of $M-\Delta\vartheta$, the linear relations of $N\text{-}\Delta u$ and $Q\text{-}\Delta v$ for the segment joint and the ring joint are employed. Also, both the partial-peripheral model and the whole-peripheral model for the ground spring are used. Their parameters describing the curves from a laboratory model test are given as follows:

(1)segment joint: $k^+_{\vartheta_1}$, $k^-_{\vartheta_1}$ =34050, 37270KN.m/rad/ring, $k^+_{\vartheta_2}$, $k^-_{\vartheta_2}$=6980,4020KN.m/rad/ring, $\beta^+$, $\beta^-$=923, 1534rad$^{-1}$; $k_n$=3×10$^4$KN/m/ring, $k_s$=78600KN/m/ring.

(2)ring joint: $k_{nq}$=$k_{sq}$=39300KN/m;

(3)ground spring:$K_s$=$K_n$/3, $K^+_n$, $K^-_n$=2.4×10$^4$, 1.2×10$^4$KN/m$^3$.

The parameters of RC segment ring(1.2m) are: $E$=3.5×10$^7$KPa, $I=2.195\times10^{-3}$m$^4$, $A$=0.336m$^2$ and $p_g$=6.96KN/m. The external values of $p_i,q_i$ $(i=1,4)$ for optimizing are given from the design pressures listed in table 3. The structural layout with three rings in the cross-joint type is discretized, and the case without the segment joint is also considered for the sake of comparison with that using beam-joint model.

Table 2 Measured and calculated normal earth pressures

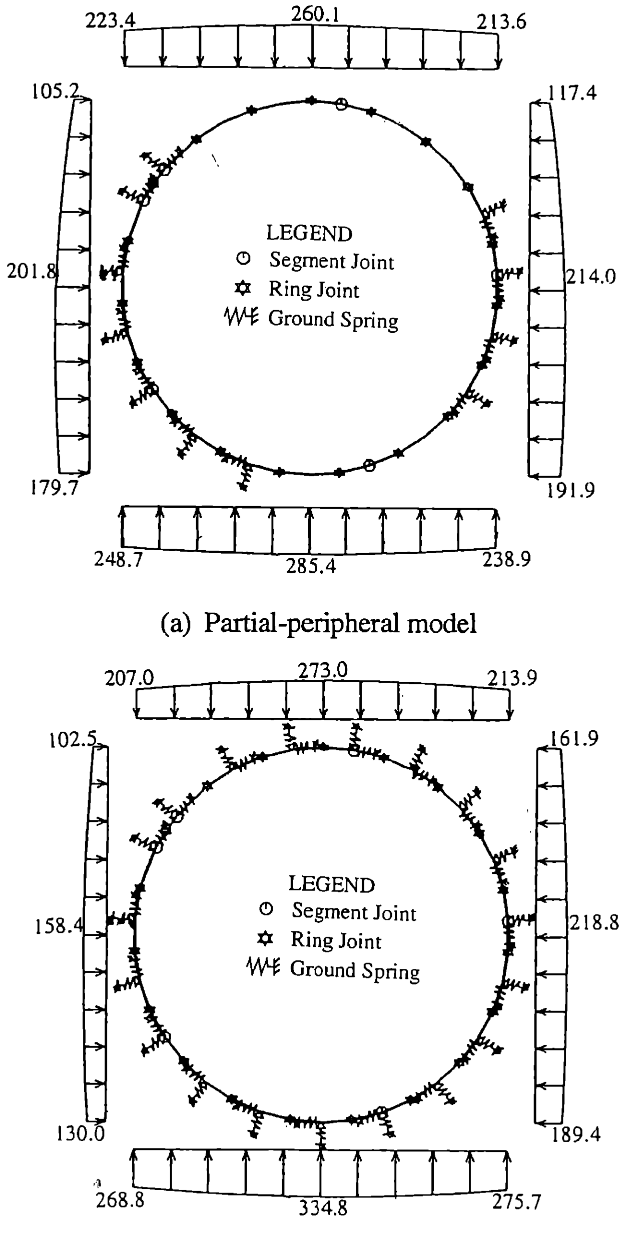
| Point | Measured | Calculated(KN/m$^2$) | |
|---|---|---|---|
| | (KN/m$^2$) | P-P | W-P |
| A | 286.7 | 204.8(194.7) | 205.3(198.3) |
| B | 280.0 | 216.8(203.6) | 208.3(203.3) |
| C | 360.0 | 225.5(193.7) | 198.6(191.7) |
| D | 240.0 | 237.8(277.8) | 244.4(251.7) |

Table 3 Initial and calculated values of external earth Pressures

| Variable | Initial | Calculated(KN/m$^2$) | |
|---|---|---|---|
| | (KN/m$^2$) | P-P | W-P |
| $p_1$ | 178.1 | 223.4(147.3) | 207.0(164.2) |
| $p_2$ | 178.1 | 213.6(212.1) | 213.9(224.7) |
| $p_3$ | 178.1 | 248.7(236.3) | 268.8(271.7) |
| $p_4$ | 178.1 | 260.1(244.3) | 273.0(251.7) |
| $q_1$ | 98.0 | 105.2(77.1) | 102.5(102.4) |
| $q_2$ | 147.7 | 179.7(158.7) | 130.0(151.4) |
| $q_3$ | 98.0 | 117.4(101.6) | 161.9(154.1) |
| $q_4$ | 120.0 | 201.8(150.4) | 158.4(152.6) |

By inputting the above parameters and the axial forces measured in the field, a back-analysis is carried out. The back-calculated values of the external earth pressures, the axial forces, the bending moments and the normal earth pressures including the ground reaction as well as their corresponding measured data are listed by tables 1-3 and illustrated by Figs.6-9 for the partial-peripheral ground spring model and/or the whole-peripheral one. By comparison it is known that the calculated moments have almost the same varying trend as the measured, but most of the calculated ones are greater than the measured. In another side, except that the calculated vertical earth pressures on the segment in the invert part are nearly equal to the measured, the others arrive only at about 65 percent of the measured.

In order to evaluate whether the results of the back-analyses are good or not, the two relative errors of the absolute sum between the measured and the calculated are defined as:




(a) Partial-peripheral model

(b) Whole-peripheral model

Fig.6 Loading patterns(KN/m$^2$)

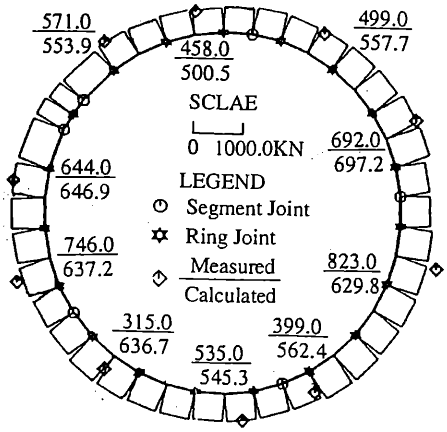



Fig.7 Axial forces( P-P model, KN/ring)

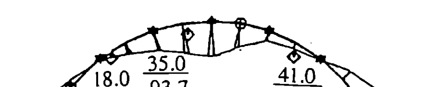



Fig.8 Bending moments(P-P model, KN.m/ring)

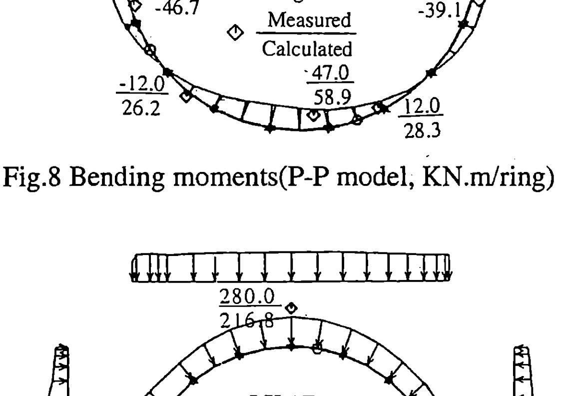



Fig.9 Normal earth pressures(P-P model, KN/m$^2$)

$$(RE)_N = \left| 1 - \sum_i |N_i| \Big/ \sum_i |N_i^*| \right|$$

$$(RE)_M = \left| 1 - \sum_i |M_i| \Big/ \sum_i |M_i^*| \right|$$

They are associated with the axial force and the bending moment separately. It is known from the table 1 that the absolute sums of the calculated are greater than those of the measured, and the values of $(RE)_N$ varying from 5.0% to 6.5% have no obvious differences among the different calculation models. However, the values of $(RE)_M$ change largely from 100.0% to 240.0%. The $(RE)_M$ values without the segment joints are two times of those with segment joints. Under the same conditions with the segment joints, the $(RE)_M$ value of the whole-peripheral ground spring model is less than that of the partial-peripheral one.

For the two ground reaction models, another difference is that the patterns of the boundary displacement in the crown and invert parts due to the action of tension force is not the same between them and it appears that the pattern of the whole-peripheral model is closer to the FEM calculation results and the actual situation.

## 7 CONCLUSIONS

We have proposed a back-analysis method for finding the external earth pressure patterns on the shield lining based on the axial forces measured in the field. The following conclusions are obtained:

1. A new structural model--the beam-joint model is proposed to simulate the mechanical behaviors of segment pieces and joints, including the segment joint and the ring joint. Different from the beam-spring model, the beam-joint model can be used to simulate the above nonlinearities mentioned and the discontinuity of the deformations effectively.

2. A back-analysis method for finding the external earth pressures on the lining with two interaction models between ground and structure is executed. A parabolic function of the external pressure patterns is supposed, and through the back-analysis procedure it is found that the parabolic function is more suitable than the linear function.

3. The calculated results with and without the segment joints are compared. It seems that the structural model with joints is more reasonable than the model without joints.

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