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A finite element study of ground movements measured in centrifuge model tests of tunnels

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ABSTRACT: The combination of numerical and physical modelling is a powerful means by which a comprehensive insight into ground movements due to tunnelling can be obtained. Results are presented of plane strain finite element analyses of a tunnel in clay which correspond to a simulation of tunnelling used in centrifuge model tests. In this modelling exercise, the influence of an overlying sand layer and its effect on subsurface movements was studied.

1 INTRODUCTION

Tunnelling is increasingly used as a method of reducing the environmental impact of large scale civil engineering works in urban areas. Consequently recent research has concentrated on improving predictions of both surface and subsurface ground movements that result from tunnel construction. Empirical formulae giving profiles of ground movements (O'Reilly and New, 1982, Mair et al. 1993) are used routinely in engineering practice. However, for the complex geometries and construction methods which are now often associated with tunnelling schemes numerical computations, such as the finite element method, which can provide a comprehensive picture of ground movement throughout the soil, may be preferred.

Finite element analyses that use simple models for soil such as linear or even non-linear elasticity generally do not predict realistic patterns of deformation around tunnels. Better results can be obtained if more sophisticated constitutive models are used such as the 3-surface kinematic hardening model (3-SKH model) (Stallebrass, 1990), which was developed specifically to simulate the main features of the stress-strain behaviour of overconsolidated clays such as non-linearity and the effect of the recent loading history of the soil. The model was used successfully to back analyse field data from a tunnel which was assumed to be constructed in uniform ground (Stallebrass et al. 1994). In practice, it is more common to find layered ground conditions and this paper examines ground movements around a tunnel constructed in a stiff overconsolidated clay with an overlying layer of sand. To investigate this problem a series of preliminary analyses has been conducted in which the clay was represented by the 3-SKH model and the sand by an isotropic linear elastic model. A more sophisticated model for the sand will be developed to progress this work. In order to evaluate the ground movements computed by the finite element analyses, two of the four analyses simulated centrifuge model tests, one of which was a test with no sand layer present.

2 CENTRIFUGE MODEL TESTS

Figure 1 shows a sketch of the model used in the centrifuge tests, which represents a plane strain section through a tunnel excavated in layered ground. The model was tested at 100g and this corresponded to a prototype tunnel of 5m diameter excavated in a block of soil 55m wide and up to 30m deep. To prepare the model the clay was consolidated in the model container

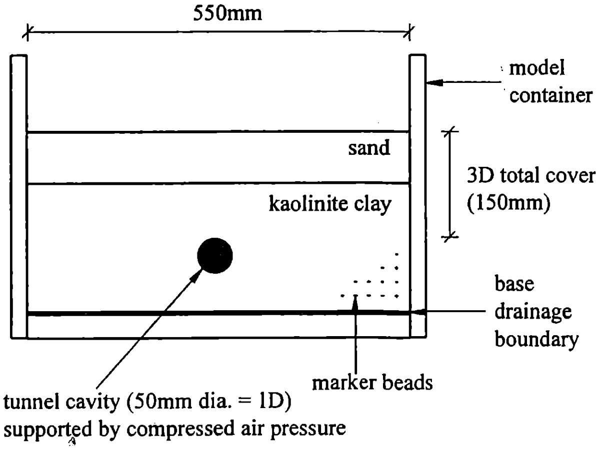

Figure 1. Schematic diagram of the plane strain centrifuge model.

to a maximum vertical effective stress of 500kPa and swelled back under a vertical stress of 250kPa. It was then removed from the consolidation press and the tunnel was excavated, marker beads were placed on the front of the clay and, if required, sand was rained on to the clay surface. The model was then ready for the centrifuge test.

The tunnel was supported by compressed air pressure within a latex rubber membrane. As the centrifuge speed increased to give the test acceleration of 100g, the air pressure was adjusted so that it always balanced the overburden pressure at tunnel axis level. After a period of about 16 hours, which allowed the pore pressures to reach equilibrium, excavation of the tunnel was simulated by reducing the supporting air pressure.

Ground movements were measured in two ways, firstly, by transducers placed both on the sand surface and on the clay/sand interface and secondly, by the movement of the marker beads determined from analysis of digital images captured by CCTV cameras. Further details of the experimental procedure are given by Grant and Taylor (1996).

Figure 2 shows the vertical effective stress in the clay at two stages during the model preparation process and also after pore pressures had equalised at 100g. The levels at which the clay/sand interfaces occurred are also indicated. It is assumed that the effective stress in the soil remained the same from the time that the model was removed from the consolidation press until drainage was allowed on the centrifuge, i.e. no free water entered the model and evaporation during the preparation of the model was minimised. Hence all the soil continued to swell during the pore pressure equalisation phase of the test (on the centrifuge), with the largest change in stress at the ground surface. This assumption was confirmed by pore pressure measurements made during the test. The final stress distribution shown in Figure 2 is for the far field. It will be altered slightly around the tunnel to accommodate the constant tunnel support pressure.

Two model tests Cent15 and Cent16 are simulated by the finite element analyses reported in this paper. Cent15 was a test with no sand layer and 3D of cover above the tunnel crown. In Cent16 there was 1.5D of clay overlain by 1.5D of sand to give the same overall cover to diameter ratio as Cent16. The clay used was Speswhite kaolin and the sand was well graded silica sand in a medium/dense state after it had been rained onto the clay surface.

3 METHOD OF ANALYSIS

3.1 *Constitutive models for soils*

The clay was represented by the 3-SKH model which is described in detail by Stallebrass (1990). The novel features of the model are two nested kinematic surfaces that exist inside the conventional Modified Cam-clay state boundary surface (Roscoe and Burland, 1968). These surfaces are defined in stress space as shown in Figure 3. The yield surface defines the onset of elastoplastic deformation which ensures that the stress-strain response of the model is non-linear and that shear and volumetric deformation is coupled except at very small stress or strain increments following a significant change in stress path direction. The history surface defines the stress change required for the previous loading paths to have negligible effect on the stress-strain response of the soil. The configuration of both surfaces relative to the current stress state, which they translate to follow, defines the soil stiffness predicted by the model. In this way the model can simulate all the major characteristics of the behaviour of overconsolidated soils. The model is described by three new soil parameters in addition to the five parameters conventionally used with Modified Cam-clay. The definition of each parameter and the value of the parameter in these analyses is given in Table 1. The values were all obtained from triaxial tests by Stallebrass (1990) except for the coefficients in the equation for G'_{ec} which are taken from Viggiani (1992).

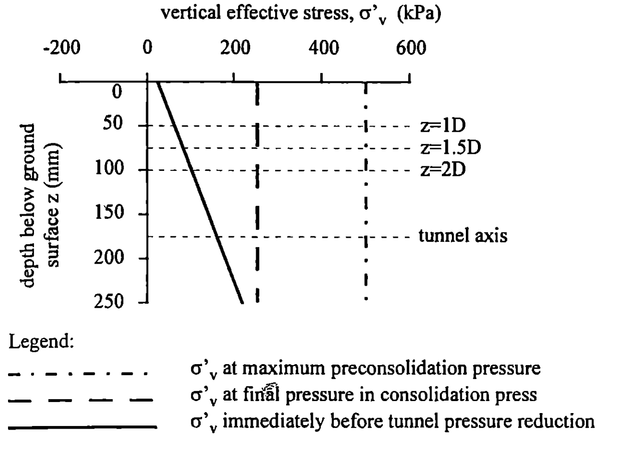

Figure 2. Stress history of clay.

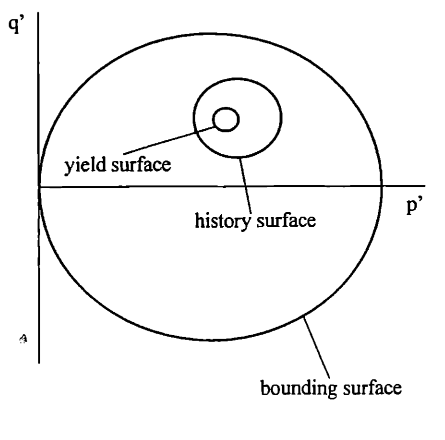

Figure 3. Basic features of the 3-SKH model.

For simplicity and due to constraints in the finite element program the sand has been represented as isotropic linear elastic with a Young's Modulus and Poisson's ratio as given in Table 1. The Young's Modulus was derived from preliminary triaxial tests on the sand used in the centrifuge model. The value is taken from a test carried out on sand at a representative voids ratio and the mean stress level in the 1.5D sand layer.

Table 1. Soil properties.

Parameter	Value
STALLEBRASS MODEL - SPESWHITE KAOLIN	
G'_{ec} - elastic shear modulus $1964(p'/p_r')^{0.65}R_o^{0.2}$	(kPa)
κ - gradient of a swelling line in lnv:lnp' space	0.005
λ - gradient of compression line in lnv:lnp' space	0.073
M - critical state friction coefficient	0.89
Γ - specific volume at critical state when p'=1kPa	2.994
k_v - vertical permeability (m/sec)	0.7E-9
k_h - horizontal permeability (m/sec)	1.8E-9
T - size ratio for history and bounding surfaces	0.25
S - size ratio for yield and history surfaces	0.08
ψ - exponent in the hardening modulus	2.5
γ_w - unit weight of water (kN/m^3)	9.81
γ - saturated unit weight of soil (kN/m^3)	17.44
ISOTROPIC LINEAR ELASTIC MODEL - SAND	
E - Young's modulus (kPa)	46800
ν - Poisson's ratio	0.3
γ - saturated unit weight of soil (kN/m^3)	19.00
γ' - buoyant unit weight of soil (kN/m^3)	9.19

Note:- p_r' is a reference pressure equal to 1kPa.
R_o is the overconsolidation ratio defined in terms of the preconsolidation pressure.

3.2 *Finite element analyses*

The analyses were carried out using the CRISP (CRItical State Program) finite element program (Britto and Gunn, 1987), in which the 3-SKH model has been implemented.

Four analyses were carried out, two of which were designed to replicate centrifuge tests Cent15 and Cent16 as closely as possible. The other two analyses investigated different thicknesses of sand but maintained the same overall cover to diameter ratio for the tunnel. The analyses are listed in Table 2

Figure 4 shows the mesh that was used for all the analyses, which contains approximately 600 linear strain triangles all with three pore pressure degrees of freedom (except those representing the sand).

The mesh is restrained horizontally at the axis of symmetry and at its right hand boundary representing the interface between the model container and the clay, which was greased to minimise friction. The base of the mesh is restrained both horizontally and vertically to account for the friction which occurs at this interface. In the initial stages of the analyses there are elements

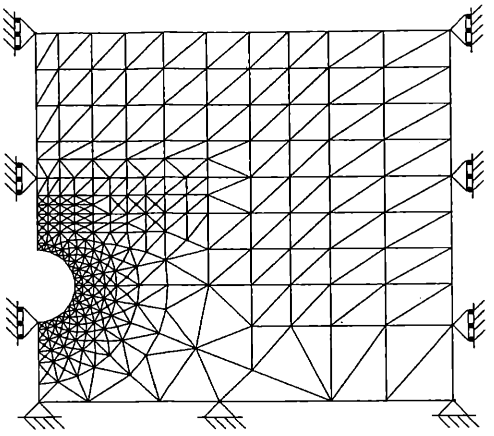

Figure 4. The finite element mesh.

within the tunnel cavity, which are later replaced by a pressure as described below.

The CRISP finite element program can model coupled consolidation events and this facility was used to model the clay in all the analyses. The horizontal and vertical permeability of the clay, given in Table 1, were calculated from formulae in Al Tabbaa (1987) and the rate at which the pressure in the tunnel was reduced, 1kPa/s, was the rate used in the model tests. The sand was considered to be drained.

Table 2. Summary of finite element analyses.

Analysis	Soil cover above tunnel crown
C1	3D clay only
S1	1.5D sand over 1.5D clay
S2	1D sand over 2D clay
S3	2D sand over 1D clay

3.3 *Modelling stress changes*

The stress changes applied to the model can be divided into three main stages:

1. One dimensional compression and swelling in the consolidation press.
2. Pore pressure equalisation on the centrifuge with the model at 100g, the tunnel supported by a constant pressure and the sand layer placed (if present).
3. Reduction of the support pressure inside the tunnel at a rate of 1kPa/s.

The soil model uses the changing configuration of the kinematic surfaces during loading to represent the important effect of the recent stress history of the soil on its stress-strain response. Hence, it is important to recreate the recent stress history of the centrifuge model in the analyses so that the surfaces are orientated correctly at the start of excavation. In these analyses the changes in stress are reproduced from the start of one-dimensional swelling in stage 1, i.e. from point B in Figure 5.

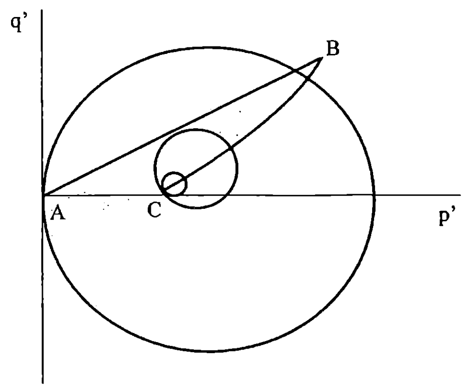

Figure 5. Sketched stress path for an element at tunnel axis level.

In all the finite element analyses the elements representing the clay were initially unloaded under essentially drained conditions from σ'_{vmax} = 500kPa to σ'_v = 250kPa. Loading stage 2 was separated into 3 phases: the change from constant stresses in the consolidation press to a linearly varying stress at 100g; the replacing of the tunnel elements with a uniform air pressure; the construction of the sand layer which hitherto had been represented by a surcharge on the clay layer (this last phase was not incorporated in analysis C1). The first two phases involved applying or removing appropriate loads or elements and then allowing consolidation of excess pore pressures. In the last stage there was negligible change in stress in the clay layer and the sand elements were defined as drained so no consolidation phase was necessary. The transfer between a constant stress throughout the model to a linearly varying stress at 100g was achieved by manipulating the gravitational force and surcharges using a method devised by Labiouse (1995).

The water table is at 25mm below the surface of the model for all the analyses. In analyses S1, S2 and S3 in order to represent the effective stresses in both the sand and underlying clay correctly the buoyant unit weight of the sand was used below the water table and a surcharge equal to the difference between the saturated and buoyant unit weight applied at the clay surface. Horizontal stresses in both the sand and clay were generated by following the stress paths described above and were not specified separately during the analysis. The model is able to make relatively accurate predictions of K_0 during one-dimensional swelling and reloading as shown in Stallebrass and Taylor (1995).

4 MOVEMENTS IN THE CLAY

Numerical analyses have been carried out which have sought to examine the effect of an overlying sand layer on the deformations occurring in overconsolidated clay in which a tunnel is excavated. The sand was modelled as an isotropic linear elastic material with a constant stiffness chosen from test data. There are clearly some approximations inherent in this representation, but it does enable the analysis to model a layer of material deforming under drained conditions and with an appropriate stress regime, $K_0 < 1$ and σ_v' reducing to zero at the ground surface. However, stress-strain characteristics of the material will be inaccurate and so the results of the numerical computations will be compared by examining ground movements in the clay layer only.

Figures 6 and 7 compare computed settlements from analysis C1 with measured settlements from centrifuge test Cent15, providing a datum from which the results of the other analyses can be evaluated. Figure 6 shows that the overall response of the centrifuge model, as represented by the surface settlement at the centreline of the tunnel, is initially much stiffer than the computed response but after a stress change of about 100kPa becomes less stiff.

In addition the computed settlement troughs are wider than the mean Gaussian distribution fitted to the model test data, as shown in Figure 7. The centrifuge data can be represented by a single normalised curve because the width of the distribution does not change significantly

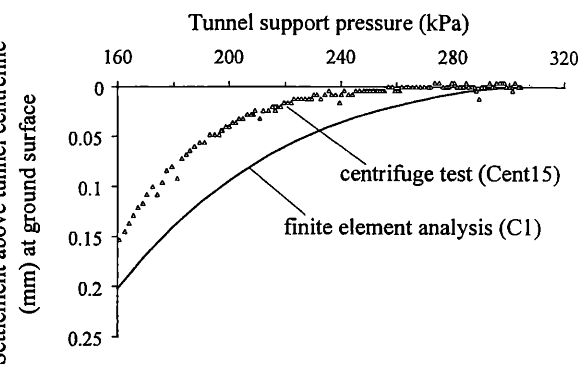

Figure 6. Settlements above tunnel centreline at ground surface for a cover of 3D clay only.

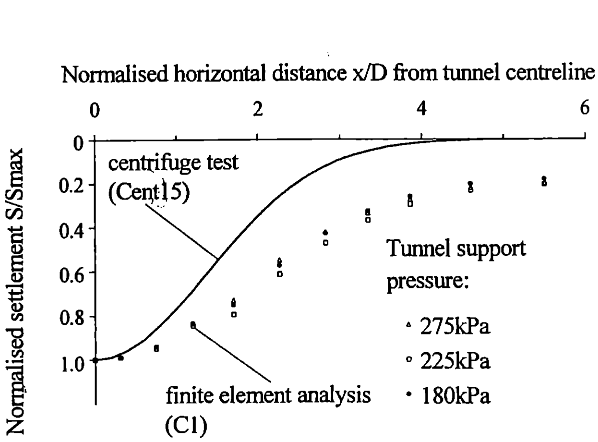

Figure 7. Normalised settlement troughs at depth, z=1.5D for a cover of 3D clay only.

during the reduction of support pressure, (Grant and Taylor, 1996). Figure 7 shows that three normalised curves obtained during analysis C1 are also similar over this range of support pressures.

The data in Figure 7 are for settlement troughs at a depth of 1.5D in models with a clay cover only. They can also be compared directly with settlement troughs at the clay/sand interface obtained from model test Cent16 and analysis S1, shown in Figure 8.

Comparison of the centrifuge test data presented in Figures 7 and 8 shows that the sand layer has acted to widen the trough, such that half the maximum settlement occurs at a horizontal distance, x, of 1.5D from the tunnel centreline in Cent15 and at x=2D in Cent16. Consideration of the computed settlement troughs indicates a similar increase in trough width. In the all clay analysis C1 half the maximum settlement occurs at x=2.5D whereas in analysis S1 this point is reached at x=3D.

As the results of the analyses C1 and S1 are consistent with the centrifuge test data, the additional analyses, S2 and S3 can be used to establish whether the behaviour observed extends to other depths of sand cover. Subsurface settlement troughs at a depth of 2D computed from all the analyses are shown in Figure 9,

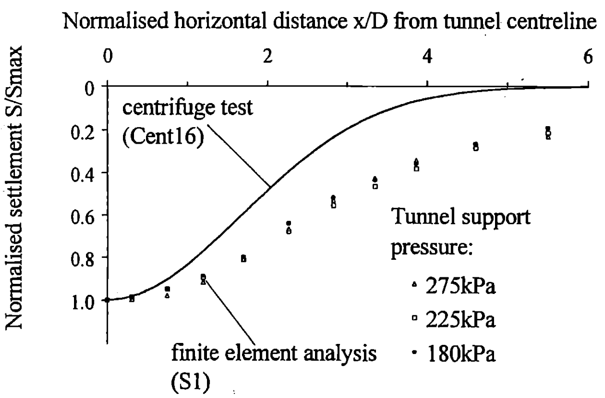

Figure 8. Normalised settlement troughs at depth, z=1.5D (the clay/sand interface) for a cover of 1.5D sand over 1.5D clay.

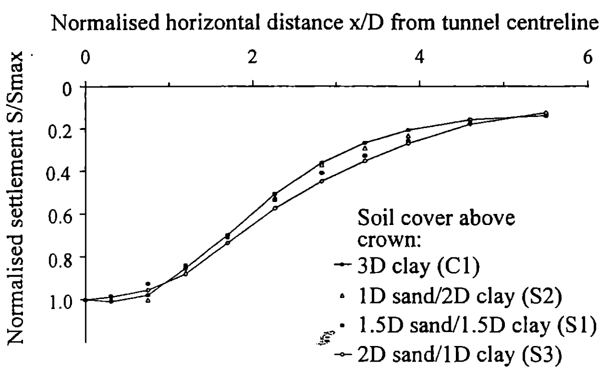

Figure 9. Normalised settlements for the four finite element analyses at a depth z=2D.

and provide evidence of a steady increase in trough width with depth of sand layer. The range of the data is indicated by the solid lines.

Figures 10 and 11 present respectively subsurface movements measured from CCTV camera images, and analysed as described in Grant and Taylor (1996), and movements computed in analysis S1. The sets of vectors are normalised to give a displacement at the centreline of the tunnel crown which is the same in both cases; there are no vectors shown in the sand in Figure 10.

The three main points to note are:

1. The pattern of the computed vectors in the clay above the tunnel axis indicates soil moving towards points below the tunnel and is consistent with the measured vectors.
2. In the centrifuge test the movement at the invert of the tunnel is less than a third of displacements at the crown. In contrast the computed invert movement is about half the computed crown movements, i.e. the finite element analyses are predicting more displacement into the tunnel, below the midpoint of the tunnel, than was measured in the tests.

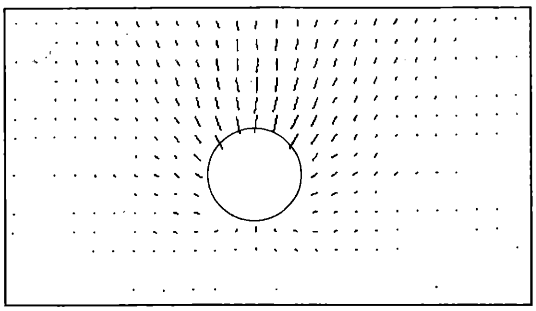

Figure 10. Vectors of movement in the clay layer only at a discrete stage in centrifuge test Cent16 (from digital image analysis), normalised by the displacement at the tunnel crown.

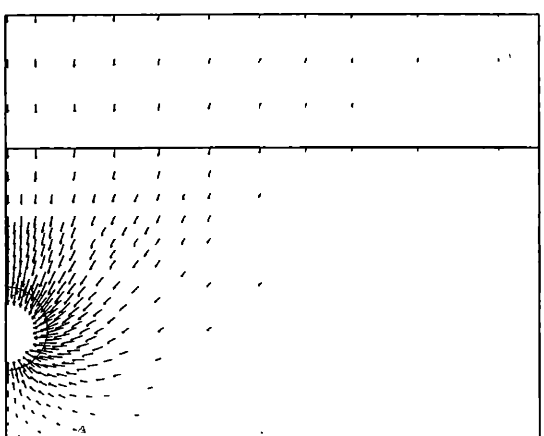

Figure 11. Vectors of movement for the sand and clay layers at a discrete stage in finite element analysis S1, normalised by the displacement at the tunnel crown.

3. Despite the approximations inherent in modelling the sand by a linear elastic material, the pattern of displacements computed in the sand layer as illustrated in Figure 11 is reasonable, indicating that the stress changes that occurred during reduction of the tunnel support pressure were correct.

5 DISCUSSION

In general, for an equivalent reduction in tunnel support pressure, the deformations computed by the finite element analyses are greater than those measured in the centrifuge tests. Reasons for this are that there may be quantitative errors in the representation of the soil by the constitutive model, particularly in the choice of stiffness parameters defining the hardening modulus. Also, the mechanism of deformation predicted by the finite element analyses may not be an accurate representation of the model tests. Wider settlement troughs are computed in the analyses than are measured in the model and this appears to be linked to greater movements at the tunnel invert. The movement at the far field vertical boundary may be exaggerated because the boundary is fixed in the horizontal direction only. This implies an axis of symmetry, i.e. that there is an identical tunnel as a mirror image of the one modelled and the settlements at this boundary are therefore due to both tunnels. Fixing this boundary vertically and horizontally, in order to investigate the effect of friction between the clay and the model container produces an unrealistic v-shaped settlement trough.

Despite the problems outlined above, the finite element computations have predicted a trend of increasing trough width with depth of sand layer, which corresponds to observations from centrifuge model tests. The trend observed in the analyses is probably a consequence of correctly imposing continuity of displacement between the linear elastic sand and the non-linear elastoplastic clay. A tendency for localised deformation in the clay is counteracted by the more uniform spread of deformation in the sand, an effect which increases with the depth of the sand layer. In the centrifuge tests the stiffness in the sand may also have been relatively constant compared to that in the clay. This is partially because the soil is less plastic and therefore less non-linear (Jardine et al., 1984) and also because above the tunnel, elements of soil are being sheared in extension from a low K_0 condition in the sand giving a stress state further from failure than the high K_0 clay so that the stiffness will not decrease so sharply. Further analyses with more complex models for the sand layer are needed to confirm this theory.

6 SUMMARY

A set of preliminary analyses have been carried out which demonstrate that using a linear elastic model to represent the sand, it is possible to predict the basic characteristics of the effect of an overlying layer of sand on deformations around a tunnel driven in clay.

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