Geotechnical Aspects of Underground Construction in Soft Ground, Kusakabe, Fujita & Miyazaki (eds)
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Evaluating plasticity solutions for the response of clay around tunnels

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ABSTRACT: Existing plasticity solutions for predicting the behaviour of clay around tunnels are evaluated using comprehensive high quality data from a plane strain centrifuge model test. A simple linear elastic perfectly plastic formulation is shown to be a useful tool in analysing the distribution of movements around a tunnel in clay. Pore pressure changes in the clay are reasonably predicted using an analysis which also incorporates non-linear elasticity. The detailed comparisons between the plasticity calculations and measured data increase confidence in using the plasticity solutions for interpreting less complete sets of data, for example, from field measurements.

1 INTRODUCTION

Knowledge of the stress changes and ground movements caused by tunnelling has become increasingly important in recent years. Predictions are generally based on empirical techniques since the development of analytical solutions for this problem has proved to be particularly difficult. The main reasons for this are the presence of a free ground surface and the fact that the behaviour of the soil not only varies with stress level, *ie.* depth below ground surface, but is also highly non-linear.

Sagaseta (1987) presented a 2-dimensional solution for deformation due to ground loss in an incompressible, or undrained, elastic material. The solution accounts for the free ground surface by considering a virtual source/sink, symmetric about the free-surface with the source of movements. Although the solution enables prediction of surface and near surface settlement troughs it still requires an assumption of the magnitude of volume loss and was shown to over-predict considerably the width of the settlement troughs when compared to field measurements from the Caracas Metro.

A simpler approach was presented by Mair and Taylor (1993) who developed plasticity solutions for the unloading of both a spherical and a cylindrical cavity. The condition of axisymmetry was imposed which meant that surface or near surface settlement troughs could not be predicted. However, the solutions provided a useful insight into the analysis of physical data, especially the variation of ground movements with vertical and horizontal distance from the tunnel axis. They are given below for the unloading of a long cylindrical cavity.

2 PLASTICITY SOLUTIONS

Mair and Taylor (1993) presented solutions for the unloading of a plane strain cylindrical cavity, as shown in Figure 1, with the following conditions.

(a) The soil behaviour is undrained.
(b) The initial stress state is isotropic.
(c) The medium is infinite and therefore the problem is geometrically axisymmetric.

Assuming that the clay behaves as a linear elastic perfectly plastic continuum the radial ground movements can be shown to be described by:

$$\frac{\delta_r}{a} = \frac{s_u}{2G}\left(\frac{a}{r}\right)\exp(N-1) \qquad (1)$$

where δ_r = radial movement at radius r; a = radius of

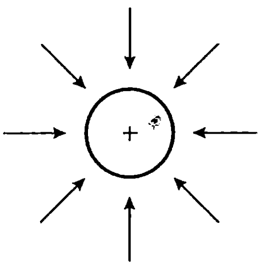

Figure 1. Unloading of a cylindrical cavity.

the tunnel cavity; N = the stability ratio defined as $(\sigma_v - \sigma_T)/s_u$ (σ_v is the vertical total stress (overburden) at tunnel axis level and σ_T = the tunnel support pressure); s_u = the undrained shear strength at tunnel axis level; and G = the elastic shear modulus.

The pore pressure changes assuming a linear elastic perfectly plastic soil model are given by:

$$\frac{\Delta u}{s_u} = 1 - N + 2\ln\left(\frac{r}{a}\right) \tag{2}$$

and the plastic zone extends to a radius $r = R_p$, as:

$$\frac{R_p}{a} = \exp\left(\frac{N-1}{2}\right) \tag{3}$$

As stated by Mair and Taylor (1993), no change in pore pressure is predicted in the elastic region, beyond $r = R_p$, if the linear elastic perfectly plastic soil model is used.

Mair and Taylor (1993) presented data which showed that pore pressure changes did occur in the elastic region and suggested a non-linear elastic function to improve predictions. They assumed a simple model of the secant stiffness G increasing linearly with radius as:

$$G = G_0\left(\frac{r}{x}\right) \tag{4}$$

where G_0 is an arbitrary reference stiffness at $r = x$. As the strain level is proportional to $1/r^2$ for a cylindrical cavity, this simple function implies a parabolic stress-strain curve. The use of equation (4) for the variation of stiffness does not change the predicted pore pressures in the plastic zone which are thus given by equation (2). However, when using non-linear elasticity the radius of the plastic zone, R_{pnl}, is given by:

$$\frac{R_{pnl}}{a} = \exp\left(\frac{N}{2} - 1\right) \tag{5}$$

and a change in pore pressure is predicted in the elastic zone which is given by:

$$\frac{\Delta u}{s_u} = -\frac{R_{pnl}}{r} \tag{6}$$

Mair and Taylor (1993) examined displacements around tunnels using field measurements taken along lines projected vertically and horizontally from the tunnel axis. They noted that equation (1) implies: a linear distribution of normalised radial movements, δ_r/a, with the inverse of the radial distance from the tunnel axis, a/r; that the gradient of this distribution increased with stability ratio, N; and that the movements would tend to zero as r became large compared to a. The field data for tunnels in London clay were limited to a single value of stability ratio and showed that the distributions of radial movements along lines projected vertically and horizontally from the tunnel axis were reasonably linear with a/r. However, as may be expected, rather than movements tending to zero as the radius becomes large the data indicated that horizontal movements were practically zero at a finite radius. Although vertical movements must tend to zero as r approaches infinity, the field data, though reasonably linear with a/r, indicated a non-linear distribution as a/r becomes very small.

To compare measurements at different stability ratios Mair and Taylor (1993) used data from tunnels in two other types of clay: soft clay from Ontario and hard Boom clay from Belgium. As with the measurements from tunnels in London clay, both sets of data showed a linear variation of δ_r/a with a/r. However, normalisation of the data for stability ratio, as suggested by equation (1), did not yield a unique line.

By comparison with field data from a deep tunnel in Boom clay Mair and Taylor (1993) also showed that assuming non-linear elasticity in the form of equation (4) produced a significant improvement in the prediction of pore pressure changes in comparison to those predicted assuming linear elastic perfectly plastic soil behaviour.

3 CENTRIFUGE MODEL TESTS

The observations made by Mair and Taylor (1993) and described above have shown that plasticity solutions can be useful in analysing the response of clay around tunnels. However, the field data used for comparison were limited to movements along lines projected vertically and horizontally from the tunnel axis, and to single values for stability ratio in any one type of soil.

Centrifuge model testing can provide more detailed measurement data to evaluate the plasticity solutions. It provides a means of conducting well controlled effective stress path scale model tests using real soil. Recent advances in digital image processing at City University have increased the quantity and quality of measurement data that can be obtained from centrifuge model tests. Considerable insight into ground movements throughout the soil depth can now be achieved in plane strain models.

As part of a large series of tests conducted to examine the distributions of movement around tunnels in two-layer ground (Grant, 1998) a test was carried out on a tunnel in a single layer of clay. The plane strain model is shown schematically in Figure 2.

The soil was a layer of kaolinite clay which had been preconsolidated in a press to a vertical effective

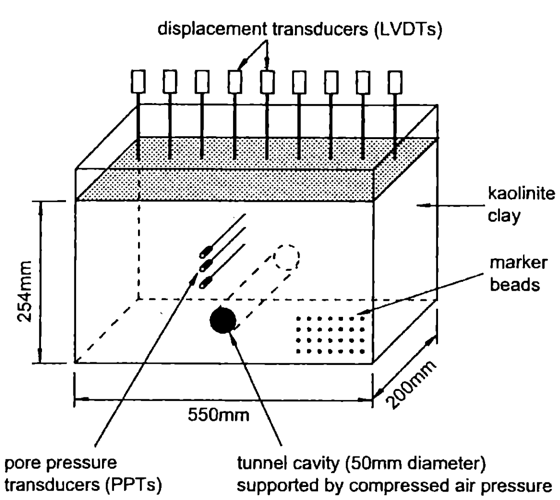

Figure 2. Schematic diagram of centrifuge model.

stress, σ_v', of 500kPa and then swelled back to a σ_v' of 250kPa throughout the soil mass. The tunnel was cut through the clay and the cavity lined with a latex rubber membrane. The model was accelerated to 100 times gravity (100g) while supplying compressed air pressure to the tunnel membrane which balanced the vertical total stress at tunnel axis level (175mm below the ground surface). Water was supplied to the model to maintain a water-table approximately 25mm below ground level and equilibrium with the new stress regime was achieved by leaving the model at 100g overnight (reconsolidation). The acceleration of 100g produced stress similitude with a prototype of dimensions 100 times that of the model, *ie.* a 5m diameter tunnel at 17.5m depth, with the soil in an overconsolidated state. After effective stress equilibrium had been achieved the tunnelling event was simulated by reducing the compressed air pressure within the tunnel membrane to zero over a period of about three and a half minutes; during this event the clay response would be essentially undrained, *ie.* zero volumetric strain. Measurements were made of pore pressures changes around the tunnel using miniature pore pressure transducers (Druck PDCR81) and of displacements both at the ground surface using conventional displacement transducers (linearly variable differential transformers, LVDTs), and throughout the full depth of soil using digital image processing techniques.

4 DIGITAL IMAGE PROCESSING

At City University, London, advances in digital image processing have provided a measurement system which enables movements to be measured throughout the soil depth in plane strain models. A typical image from the centrifuge test described is given in Figure 3.

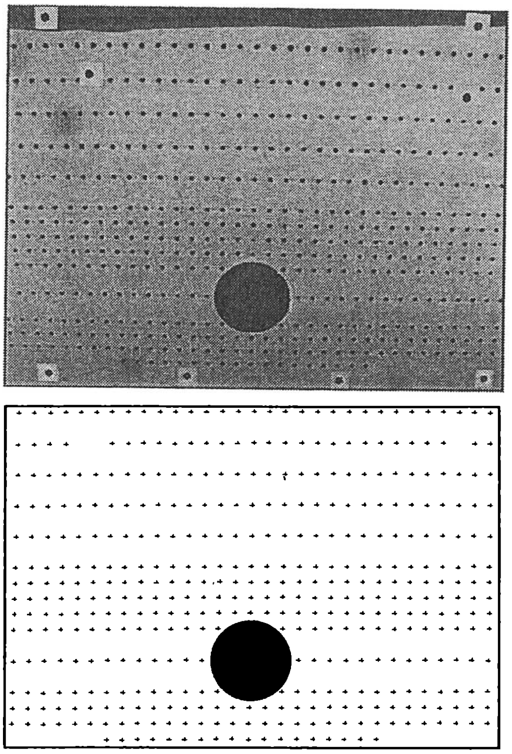

Figure 3. Digitised image from the CCD camera in-flight and calculated positions of the targets in real (object) space after reconsolidation and before tunnel pressure reduction.

A CCD camera mounted on the centrifuge is able to view the front face of the soil model through a thick perspex window forming one side of the model container. The images are transferred to the centrifuge control room where they are stored on computer at a rate of approximately one frame every second. Using techniques of close-range photogrammetry the positions of black marker beads which have been pressed into the front face of the soil can be determined in real (object) space to a precision of approximately ±60μm (Taylor *et al.* 1998). Thus for the tunnel test described comprehensive displacements throughout the soil depth and for a complete range of tunnel support pressures can be achieved. This would not be possible for a tunnel in the field.

5 MOVEMENTS AROUND THE TUNNEL

Figure 4 shows the displacement vectors for the marker beads at a discrete time in the test. Originally the targets were spaced either 10mm or 20mm apart on a right-rectangular grid and it can be seen that only a few of them were lost during the automated tracking process. Such comprehensive measurement data can be achieved only by image processing and scale model tests at present. The centrifuge model test provides data from a real, undrained, plane strain event with which to evaluate the plasticity solutions

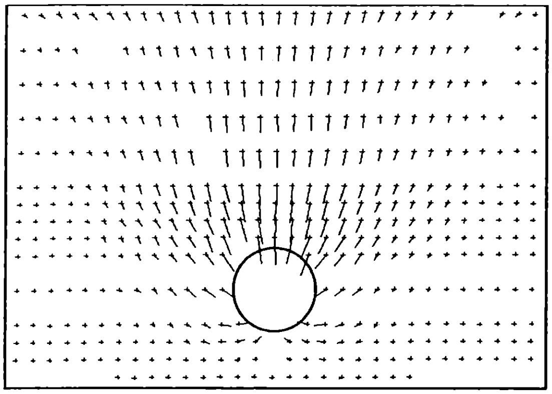

Figure 4. Vectors of ground movement (x10) when the movement near the tunnel crown is ~1.7mm.

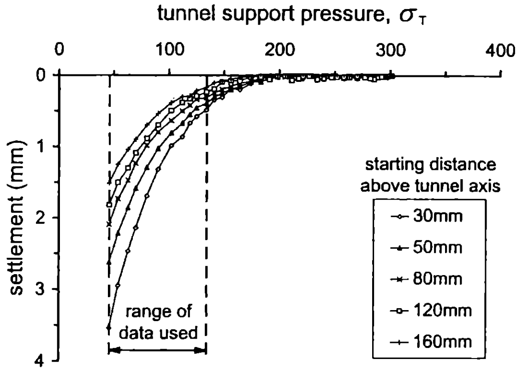

Figure 5. Settlement of some targets vertically above the tunnel axis.

described in Section 2. Firstly, it is clear from the displacement vectors given in Figure 4 that the movements were not truly radial to the tunnel. There is a shift in the level of the focal point of the displacement vectors in the vicinity of the tunnel but, in general, for the regions above the crown the vectors are directed below the tunnel invert. The conditions assumed to formulate the plasticity solutions were listed earlier and imply that movements must be radial to the tunnel. However, this is a purely qualitative comparison and with the data available a more detailed quantitative evaluation can be made.

Figure 5 shows settlements throughout the test for marker beads along a line projected vertically from the tunnel axis. The movements along this line should be truly radial as it was an axis of symmetry in the test and reference to the vectors in Figure 4 confirms this, within practical limits. On the abscissa of Figure 5 are values of tunnel support pressure, σ_T, and so the test proceeded from right to left on the graph. As expected the movements decreased with radial distance from the tunnel axis. Figure 5 indicates the range of tunnel pressures over which displacement data were used to evaluate the plasticity solutions, as shown in Figures 6 to 9. Clearly, this covers a wide range of movements throughout the test.

The measured settlement of all the targets along the line projected vertically above the tunnel axis are plotted against the inverse of the radial distance from the tunnel axis, a/r, in Figure 6 for a range of tunnel support pressures, σ_T. For each σ_T the normalised distribution of settlement, or movement radial to the tunnel, δ_r/a, with a/r is approximately linear. Both the magnitude of δ_r/a and the gradient of the distribution of movement with a/r increased as σ_T reduced. The distributions of δ_r/a with a/r are approximately linear for a wide range of σ_T and therefore stability ratio, N, which is consistent with equation (1). However, as stated previously, equation (1) logically predicts zero radial movement as r becomes large relative to a and therefore the variations of δ_r/a with a/r along the line projected vertically above the tunnel axis cannot continue to be linear at very large radii. In this respect these data are consistent with the field data presented by Mair and Taylor (1993).

Figure 7 shows a similar plot to Figure 6 for the radial components of movement, δ_r/a, along a line projected horizontally from the tunnel axis. The magnitudes of movement are less than those along the line projected vertically above the tunnel axis but the distributions with a/r are also approximately linear and again both the magnitudes of movement and gradient of the distribution of δ_r/a with a/r increased with the reduction of σ_T. Along this line projected horizontally from the tunnel axis the movements tend to zero at a finite distance from the tunnel axis. Again, these data are consistent with the field data presented by Mair and Taylor (1993), and it is logical that movements tend to zero at a finite distance from the tunnel along this horizontal line.

In contrast to the field data, the comprehensive measurements obtained from the centrifuge model test also allows examination of the components of movement along a line projected at 45° towards the ground surface from the tunnel axis. The data are shown in Figure 8. The distributions of δ_r/a with a/r could be described as approximately linear but close to the tunnel, as a/r approaches unity, the tendency is for smaller components of radial movement. This can be observed in the non-radial movements illustrated by the vector plot given in Figure 4.

As noted by Mair and Taylor (1993), equation (1) implies that normalising the radial movements by $\exp(N-1)$ should result in a unique distribution of δ_r/a with a/r. This has been applied to the data presented in Figures 6 to 8 and is given in Figure 9, assuming that the undrained shear strength, s_u, is 50kPa (evaluated from Critical State soil mechanics theory and empirical correlations). The result is a series of approximately linear and parallel lines, one for each radially projected line. Thus, the data are consistent with equation (1), except that there is an offset between the sets of data which appears to depend on the direction of the projected radial line. This offset is

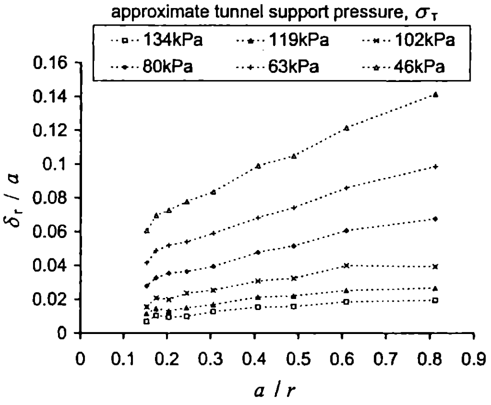

Figure 6. Radial movements along the line projected vertically above the tunnel axis.

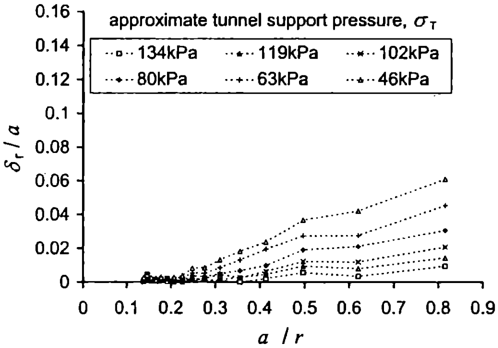

Figure 7. Radial components of movement along a line projected horizontally from the tunnel axis.

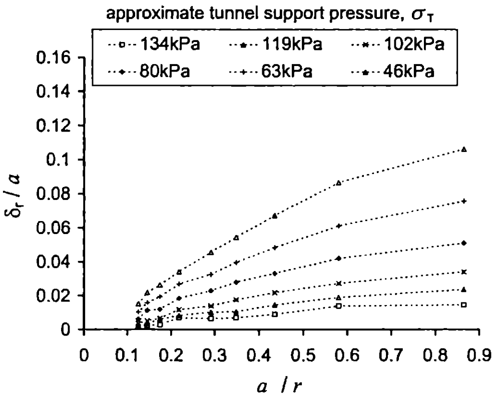

Figure 8. Radial components of movement along a line projected at 45° above the tunnel axis.

probably due to the initial stress state in the centrifuge model test not being isotropic and the test geometry not being axisymmetric.

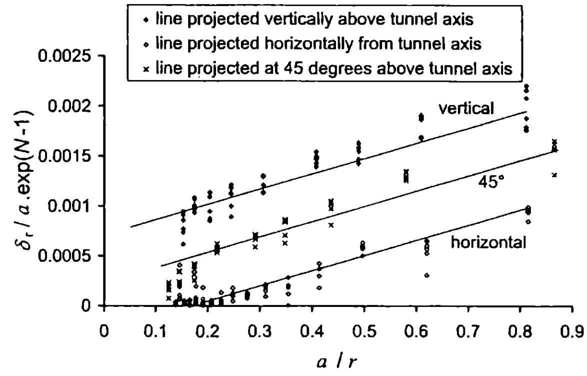

Figure 9. Normalised radial components of movement.

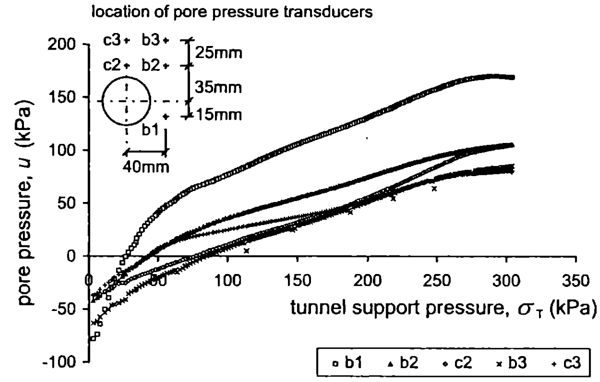

Figure 10. Pore pressure changes during centrifuge model test.

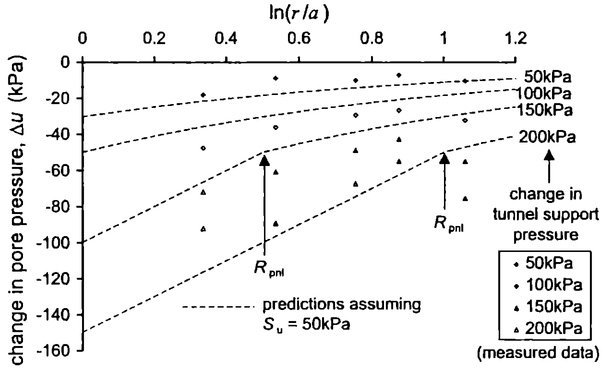

Figure 11. Measured and predicted changes in pore pressure.

6 PORE PRESSURE CHANGES

The pore pressure changes measured around the tunnel cavity during the centrifuge model test are shown in Figure 10. The test proceeded from right to left on the graph, *ie.* with reducing tunnel support pressure, σ_T. As expected for undrained behaviour of overconsolidated clay, a significant reduction in the pore pressures occurred in the vicinity of the contracting tunnel cavity.

As stated in Section 2, no pore pressure change is predicted beyond the plastically deforming zone if equations (2) and (3) are used which assume a linear elastic perfectly plastic soil model. Mair and Taylor (1993) showed that this was not consistent with physical data and that better predictions could be made by assuming a non-linear elastic perfectly plastic soil model, in the form of equation (4). Predictions of pore pressure changes in the plastic zone are still given by equation (2) but the extent of the plastic zone is then given by equation (5) and the pore pressure changes in the elastic zone by equation (6).

Predictions for the change in pore pressure around the tunnel have been made using the proposed non-linear elastic perfectly plastic formulation and are compared with the measured data from the centrifuge model test in Figure 11.

Comparisons are made for a wide range of change in tunnel support pressure from 50kPa to 200kPa. Although there are differences between the predictions and the measured data, it is clear that the general pattern of the distribution of pore pressure change with radius from the tunnel axis is reasonably well predicted by equations (2), (5) and (6).

7 SUMMARY

Existing plasticity solutions for the displacements and pore pressure changes around a tunnel in clay have been evaluated using comprehensive high quality measurement data from a plane strain centrifuge model test.

Predictions of radial displacements using a formulation incorporating a linear elastic perfectly plastic soil model have been compared to the measured radial component of movement along the following lines projected radially from the tunnel axis: vertically above; horizontally; and at 45° towards the ground surface. The predictions have been shown to be consistent with measurements from the real (centrifuge) event in the following ways.

(a) The distribution of δ_r/a with a/r is approximately linear for any pre-collapse value of stability ratio, N.

(b) The gradient of δ_r/a with a/r increases with the value of N.

(c) Normalising δ_r/a by $\exp(N-1)$ produces a single line with a/r for each radial direction examined.

In contrast to the plasticity solutions, normalising δ_r/a by $\exp(N-1)$ does not produce a unique distribution of δ_r/a with a/r for all lines projected radially from the tunnel. For the radial projections examined in the centrifuge test the normalisation yielded a series of approximately linear and parallel lines. It is likely that the difference between the observed and predicted movements is mainly due to the condition of geometric axisymmetry assumed in the formulation of the plasticity solutions.

A formulation incorporating a non-linear elastic perfectly plastic soil model has been shown to predict reasonably the changes in pore pressure with radius from the tunnel axis if the stress changes at the tunnel boundary are known.

Notwithstanding the differences observed, the plasticity solutions described have been shown to be a useful tool for analysing the response of clay around tunnels.

ACKNOWLEDGEMENTS

The centrifuge model test was conducted as part of a series of tests funded by the Engineering and Physical Sciences Research Council (EPSRC) under grant award GR/J53010.

The digital image processing system was developed for the centrifuge largely by members of the Engineering Surveying Research Centre at City University. This was also funded by the EPSRC under grant award GR/J74022. Particular thanks are due to Dr Stuart Robson, Dr Jin Chen and Professor Mike Cooper.

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