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# Numerical solution model for instability of underground cavity

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ABSTRACT: Failure of soft soils like sandy media after underground opening, usually involve three dimensional stress conditions in the materials. Furthermore, the behavior and stability of complex ground commonly involve a number of behavioral components such as: elasticity, plasticity, shear hardening, strength deterioration and high potential shear band location etc.. However, to determine the stress conditions which govern fracture of intact materials, a general three dimensional failure criterion is required. In this paper, a multilaminate based model capable of predicting the failure specifications of granular soil on the basis of sliding mechanisms and elastic behavior of particles has been introduced. This model is able to present numerical distribution of plastic strain through even highly anisotropic medium. Active orientations regarding the plastic shear strain, thereafter, is clarified. The orientation takes priority in plastic strain will fail first.Constitutive formulation of multilaminate elasto-plasticity has been implemented in appropriate finite element formulation and a computer program is developed. The generalization of multilaminate framework for all gauss points through the medium leads the solution to clarify rational failure mechanism which may occur first.

## 1.0 INTRODUCTION

Many different analytical solution already proposed for stability of underground cavities in soft ground [1],[2]. Lack of the conformity of predictions and analytical solutions led the solution to physical modeling. However, to obtain a better solution, representing the most probable mode of failure as well as deformations and stress analysis, a mathematical model will do the job.

Multilaminate framework by defining the small continuum structural units as an assemblage of particles and voids which fill infinite spaces between the sampling planes, has appropriately justified the contribution of interconnection forces in overall macro-mechanics.

Plastic deformations are assumed to occur due to sliding, separation/closing of the boundaries and elastic deformations are the overall responses of structural unit bodies. Therefore, the overall deformation of any small part of the medium is composed of total elastic response and an appropriate summation of sliding, separation/closing phenomenon under the current effective normal and shear stresses on sampling planes.

According to these assumptions overall sliding, separation/closing of intergranular points of grains included in one structural unit are summed up and contributed as the result of sliding, separation/closing surrounding boundary planes. This simply implies yielding/failure or even ill-conditioning and bifurcation response to be possible over any of the randomly oriented sampling planes. Consequently, plasticity control such as yielding should be checked at each of the planes and those of the planes which are sliding will contribute to plastic deformation. Therefore, the granular material mass has an infinite number of yield functions usually one for each of the planes in the physical space. Therefore, the essential to look for a practically simple constitutive relations that are capable of incorporating such features.

## 2.0 FORMULATION OF ELASTOPLASTIC STRESS-STRAIN LAW

The classical decomposition of strain increments under the concept of elasto-plasticity in elastic and plastic parts are schematically written as follows:

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p} \qquad (1)$$

The increment of elastic strain ($d\varepsilon^e$) is related to the increments of effective stress ($d\sigma$) by:

$$d\varepsilon^e = C^e \cdot d\sigma \qquad (2)$$

where, $C^e$ is elastic compliance matrix, usually assumed as linear.

Conceptually, it is possible to compute $C^e$ by using multilaminate framework. Also, plastic strain increments ($d\varepsilon^p$), is expressed as:

$$d\varepsilon^p = C^p \cdot d\sigma \qquad (3)$$

where, $C^p$ is plastic compliance matrix.

Clearly, it is expected that all the effects of plastic behavior be included in $C^p$. To find out $C^p$, the constitutive equations for a typical slip plane must be considered in calculations. Consequently, the appropriate summation of all provided compliance matrices corresponding to considered slip planes yields overall $C^p$.

$$C^p = 4\Pi \sum_{i=1}^{n} W_i L^T \hat{C}_i^p L \qquad (4)$$

where, L is a proper transformation matrix to transfer $\hat{C}_i^p$ from ith plane coordinate to global coordinate.

### 2.1 *CONSTITUTIVE EQUATIONS FOR A SAMPLING PLANE*

A sampling plane is defined as a boundary surface which is a contacting surface between two structural units of polyhedral blocks. These structural units are parts of an inheterogeneous continuum, for simplicity defined as a full homogeneous and isotropic material.

Therefore, all inheterogeneities behavior are supposed to appear in inelastic behavior of corresponding slip planes. Figure 1 shows these defined planes (say 13).

As already defined, the vector of plastic strain is calculated from the study of the glide motion over an individual sampling plane. To start explaining the plasticity constitutive law for a sampling plane, the main features of plasticity law (i.e. yield criterion, plastic potential function, flow rule and hardening rule) must also be considered.

### 2.2 *YIELD CRITERION*

In this constitutive formulation, two yield criteria are defined by two ratios of the shear stress components ($\tau_{xi}, \tau_{yi}$) to the normal effective stress ($\sigma_{ni}$) on ith sampling plane. The simplest form of yield function i.e. a straight line on $\tau$ versus $\sigma_n$ space is adopted.

As the ratios $\tau_{xi}/\sigma_n$ and $\tau_{yi}/\sigma_n$ increase the yield boundaries represented by the straight lines rotate anti-clock-wise due to hardening and approaches Mohr-Coulomb's failure surface and finally failure on corresponding plane takes place.

The equation of yield functions for two perpendicular orientations (xi,yi) on ith plane are formulated as follows:

$$F_{xi}(\tau_{xi}, \sigma_{ni}, \eta_{xi}) = \tau_{xi} - \eta_{xi}\ \sigma_{ni} \qquad (5)$$

$$F_{yi}(\tau_{yi}, \sigma_{ni}, \eta_{yi}) = \tau_{yi} - \eta_{yi}\ \sigma_{ni} \qquad (6)$$

where, $\eta_{xi} = \tan(\alpha_i)$ and $\eta_{yi} = \tan(\beta_i)$are hardening parameters for two plasticity rules of orientations $x_i$ and $y_i$. However, they are assumed as a hyperbolic function of plastic shear strain components on the ith plane.$\alpha_i$ and $\beta_i$ are the slope of yield lines.

To provide elastic behavior of cohesionless material at the start of stress increment or whenever the direction of stress path changes, a small elastic domain (defined by angle $\phi_e$) is considered. This domain as shown in Figure 2 is small and negligible. Therefore, the value of $\phi_e$ for all sands assumed to be the same. However, the minimum value of $\eta_{xi}$ and $\eta_{yi}$ are $\tan(\phi_e)$ at the first loading process.

### 2.3 *PLASTIC POTENTIAL FUNCTION*

The plastic potential function is stated in terms of $\tau_{xi}$, $\tau_{yi}$ and $\sigma_{ni}$ for ith plane [3] and [5] as follows:

$$\psi_{xi}(\tau_{xi}, \sigma_{ni}) = \tau_{xi} + \eta_{cxi} \cdot \sigma_{ni} \cdot \log(\sigma_{ni} / \sigma_{ni0}) \qquad (7)$$

$$\psi_{yi}(\tau_{yi}, \sigma_{ni}) = \tau_{yi} + \eta_{cyi} \cdot \sigma_{ni} \cdot \log(\sigma_{ni} / \sigma_{ni0}) \qquad (8)$$

where, $\eta_{cxi}$ and $\eta_{cyi}$ are the slope of critical state lines for the plasticity in xi and yi directions and $\sigma_{nio}$ is the initial value of effective normal stress on ith plane. Typical presentations of this function are shown in Figure 2.

The gradient of this function in both directions represents contractant and dilatant behavior in the ranges as:

$0 \leq (\tau_{xi}, \text{or } \tau_{yi} \text{ or })\ \leq \sigma ni\ (\eta_{cxi} \text{ or } \eta c_{yi})$ (contractant behavior) (9)

$(\tau_{xi}, \text{or } \tau_{yi} \text{ or })\ \geq \sigma ni\ (\eta_{cxi} \text{ or } \eta c_{yi})$ (dilatant behavior) (10)

Derivative of this function is found as:

$$d\psi_{xi}/d\sigma_i = \{1\ ,\ \eta_{cxi}-\eta_{xi}\}^T \qquad (11)$$

$$d\psi_{yi}/d\sigma_i = \{1\ ,\ \eta_{cyi}-\eta_{yi}\}^T \qquad (12)$$

Obviously, dilatancy is positive if $\eta_{xi} > \eta_{cxi}$ or $\eta_{yi} > \eta_{cyi}$ and vice versa. However, on critical state line of each direction, $\eta_{xi}$ or $\eta_{yi}$ will be equal to either ücxi or ücyi and there is no volumetric plastic strain.

Accordingly, the derivatives of the adopted plastic potential function which is based on the conception of energy equation [3], can only be expressed in terms of variable üxi or üyi, identify the components of plastic strain increment ratio as well as the position of no dilatancy/contractancy. This aspect seems to be the most suitable form which conforms to the constitutive formulation of sampling plane in the case of having a double hardening plasticity rule for granular media.

These relations can also be expressed in another form as:

$$d\varepsilon^p_{xi} = \{1/H_{pxi}\}.\{\partial F_{xi}/\partial\sigma i\}.\{\partial\psi_{xi}/\partial\sigma i\}^T.d\sigma i = \bar{C}^p_{x}d\sigma i \qquad (13)$$

$$d\varepsilon^p_{yi} = \{1/H_{pyi}\}.\{\partial F_{yi}/\partial\sigma i\}.\{\partial\psi_{yi}/\partial\sigma i\}^T.d\sigma i = \bar{C}^p_{y}d\sigma i \qquad (14)$$

where $H_{pxi}$ and $H_{pyi}$ are defined as hardening modules of ith plane corresponding to xi and yi directions and are obtained as follows:

$$H_{pxi} = -\{\partial F_{xi}/\partial\varepsilon^{pt}_{xi}\}\{\partial\psi_{xi}/\partial\tau_{xi}\} \qquad (15)$$

$$H_{pyi} = -\{\partial F_{yi}/\partial\varepsilon^{pt}_{yi}\}\{\partial\psi_{yi}/\partial\tau_{yi}\} \qquad (16)$$

$$H_{pxi} = \sigma_{ni}\frac{A_{xi}\ \mathrm{Tan}(\phi_{xi})}{(A_{xi}+\varepsilon^{pt}_{xi})^2} \qquad (17)$$

$$H_{pyi} = \sigma_{ni}\frac{A_{yi}\ \mathrm{Tan}(\phi_{yi})}{(A_{yi}+\varepsilon^{pt}_{yi})^2} \qquad (18)$$

$$C^p_{xi} = \frac{(A_{xi}+\varepsilon^{pt}_{xi})^2}{A_{xi}\ \mathrm{Tan}(\phi_{xi})\sigma_{ni}}\begin{bmatrix} 1 & -\eta_{xi} \\ \eta_{cxi} & \eta_{xi}(\eta_{xi}-\eta_{cxi}) \end{bmatrix} \qquad (19)$$

$$C^p_{yi} = \frac{(A_{yi}+\varepsilon^{pt}_{yi})^2}{A_{yi}\ \mathrm{Tan}(\phi_{yi})\sigma_{ni}}\begin{bmatrix} 1 & -\eta_{yi} \\ \eta_{cyi} & \eta_{yi}(\eta_{yi}-\eta_{cyi}) \end{bmatrix} \qquad (20)$$

## 3.0 DEFINITION OF PLANES IN THREE DIMENSIONAL MEDIA

Figure 1 shows the orientation of all 13 planes in similar cubes.

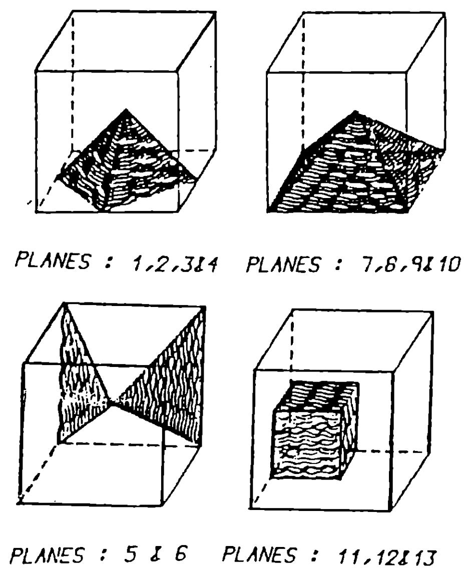


Figure 1. The orientation of 13 planes.

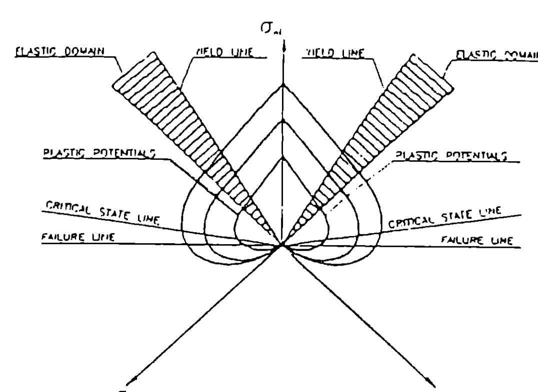


Figure 2. Yield and Failure loci for plane i ($\tau \sim\sigma_n$).

## 4.0 FINITE ELEMENT SOLUTION

The problem analyzed is presented by the finite element mesh shown in Figure 3-a. A rectangular opening of 2 meters high and 2 meters wide is excavated with an overburden cover of 6 meters. A 100 millimeters thick timber temporarily liner is installed in place. No gap between the lining and the surrounding soil is assumed.

The mesh consists of 80 elements and 277 nodes, 8 nodal parabolic isoparametric quadrilateral elements are used.

Four different material properties have been assumed for liners

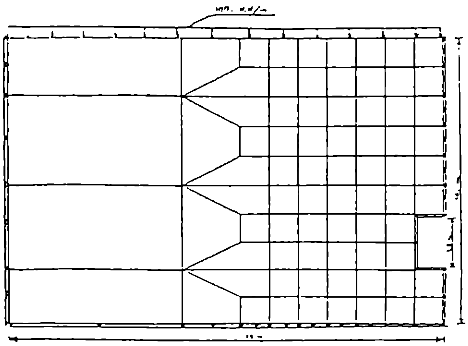

Figure 3-a Finite element mesh

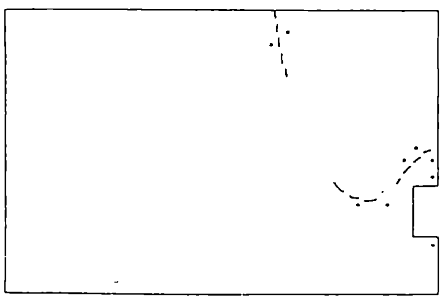

Figure 3-b. Failure mechanism (E=220 MPa.).

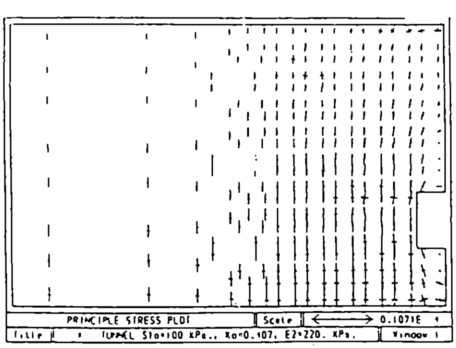

Figure 3-c. Changes of principal stress (E=220 MPa.).

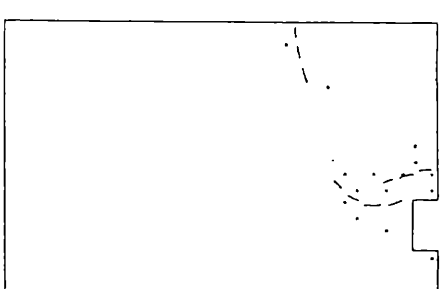

Figure 3-d. Failure mechanism (E=100 MPa.).

(E'=220, 100, 50 and 10 MPa.).

The type of soil around the opening is assumed to be Hostun sand and the level of water table is below the mesh area. Unit weight of dry sand is assumed as 16.9 $KN/m^3$.

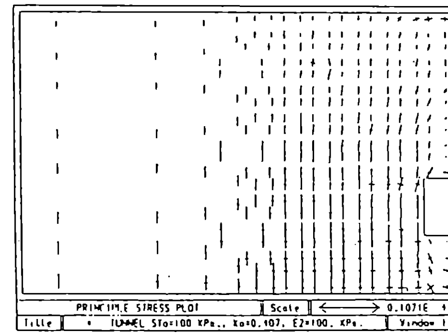

Figure 3-e. Changes of principal stress (E=100 MPa.).

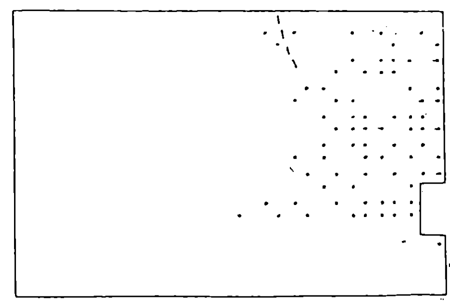

Figure 3-f. Failure mechanism (E=50 MPa.).

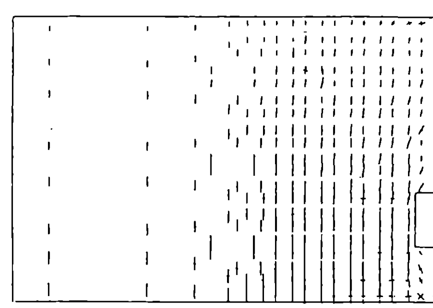

Figure 3-g. Changes of principal stress (E= 50 MPa.).

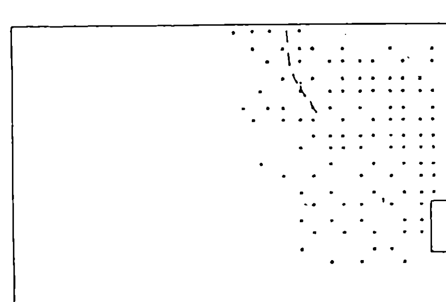

Figure 3-h. Failure mechanism (E=220 MPa.).

Obviously, the weaker the liner material, the more deformations in the liner and settlements of ground surface. Due to the rectangular shape of cavity cross section, the reduction in diameters are quite smaller than reductions in width and height of cavity. The

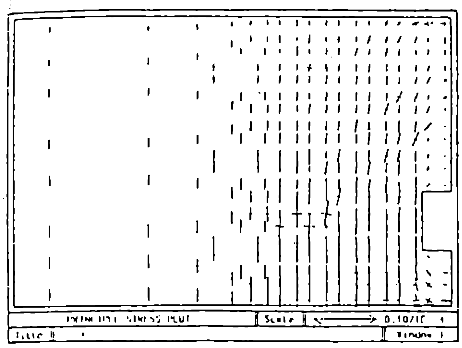

Figure 3-i. Changes of principal stress (E= 10 MPa.).

displacement vector plots represent the rate of departure from the half space plane of the pattern which is affected by cavity creation. Obviously, the weaker the liner material, the wider the affected ground surface is. These Figures also clarify the displaced zone around the cavity.

Figures corresponding to failed gauss points show how progressively the locations failure zone grows up. The gauss points including at least one plane with 5% shear strain or negative normal stress are assumed to be failed. Normally, the failed points show the creation of shear zone close to what is expected according to Terzaghi's hypothesis.

## 5.0 CONCLUSIONS

The comparison of results from integration of constitutive relation, model and finite element solution provided good agreement.

The mechanisms of mobilization of affected zone and the consequent development of failure zones indicate clearly a significant arching process around the cavity. This explains the very low pressure spectrum observed through the surrounding medium in this and similar geological environments.

## 6.0 REFERENCES

Kvapil, R., 1982,"The Mechanics and Design of Sublevel Caving Systems." Sec. 4.1-2 in *Underground Mining Methods Handbook*, W.A.Hustrulid, (ed.,) Soc. Mng. Eng.-AIME, New York, pp. 880-897

Janelid, I., 1975, "Sublevel Caving," Preprint, No. 75-AU-15, *Winter Meeting, Soc. Mng. Eng.-AIME*, New York, 23 pp.

Sadrnezhad S.A. & Pande G.N., A Multilaminate Model For Sand, *Proceeding of 3rd International symposium on Numerical Models in Geomechanics*, NUMOG-III, 8-11 May 1989, Niagara Falls, CANADA.

Terzaghi, K. 1936. Stress distribution in dry and saturated sand above a yielding trap-door, *Proceeding 1st International Conference Soil Mechanics & Foundations Engineering, Cambridge*, Mass., 1, pp. 337-311.

Sadrnezhd S.A., Multilaminate elastoplastic model for granular media, *Journal of Engineering*, Islamic Republic of Iran, vol.5, Nos.1&2, May 1992-11