Engineering Decisions Associated with Applying Unsaturated Soil Mechanics

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Abstract: The boundary between the saturated and unsaturated soil occurs at the top of the capillary zone and corresponds to the air-entry value, *aev*. The capillary zone may be a few centimeters in thickness for sand but may be meters in thickness for clay soil. One of the primary pieces of information of value to a geotechnical engineer is the *aev* for the uppermost soil strata. It is difficult to agree upon a single-valued *aev* because of the hysteretic nature between the drying and wetting modes. The unsaturated soil properties have been shown to be most accurately correlated with the degree of saturation of the soil. Generally, gravimetric water content versus suction is measured and then the degree of saturation is calculated through use of the shrinkage curve, SC. This paper highlights the important role of measuring and/or estimating the shrinkage curve for the quantification of unsaturated soil property functions, USPFs.

Introduction

Virtually every engineered structure is founded on or near to the earth's surface; and still the general field of soil mechanics has continued to evolve giving primary consideration solely to the saturated soil zone. Consequently, the long-term maintenance costs associated with light engineered structure arise out of changes in the pore-water stress state initiated by fluctuating climatic conditions. It is generally assumed that the stress state for unsaturated soils starts at the water table and continues to the ground surface. However, the zone immediately above the water table, commonly referred to as the "capillary zone", remains essentially saturated until the air-entry value of the soil is exceeded.

This paper asserts that the air-entry value, *aev*, of the soil constitutes an extremely important piece of information that needs to be assessed by the geotechnical engineer [1]. Stated another way, the unsaturated (nonlinear) soil properties do not need to be taken into consideration unless the stress state in the pore-water exceeds the *aev* of the soil. The *aev* may be in the order of a few kPa for a sand soil and this translates into a capillary zone of a few centimeters of hydraulic head above the water table. On the other hand, the *aev* for an intact clay soil may be in the order of 1000 kPa and this may translate into a theoretical capillary zone that is in the order of 100 meters. Consequently, an understanding of the *aev* of the soil may largely control the extent to which unsaturated soil mechanics properties and principles need to be invoked. The author is aware of the differences associated with drying and wetting modes (i.e., hysteresis), however, based on past research studies it is recommended that attention should first be given to the drying mode.

The objectives of this paper are: i.) to identify and define the primary factors associated with the vadose zone, ii.) separate the independent roles of volume change and degree of saturation change on the determination of the soil-water characteristic curve, SWCC, and iii.) formulate the physical and mathematical aspects of the Shrinkage Curve, SC, on unsaturated soil behavior. The shrinkage curve may need to be either measured in the laboratory or estimated based on Atterberg Limit classification properties [2].

This study is based upon the assumption that it is possible to separate the effects of volume change and degree of saturation change with respect to the suction stress state (i.e., matric suction and total suction), to the exclusion of the secondary effects related to changes in the total stress state components [3, 4].

The scope of the paper is limited to the behavior of soils that are initially in a slurry condition (or are in a near-saturation condition) and then allowed to dry in response to applied soil suction. Consideration is given to the drying branch of the Soil-Water Characteristic Curve; however, it is well-known that there are independent drying and wetting stress paths. The intent is not to address all theoretical aspects of unsaturated soil behavior, but rather to suggest methodologies that best coincide with prudent soil mechanics practices. Also, not all suggested unsaturated soil-water characteristic curve equations proposed in the research literature are not addressed in this paper. The focus is on the use of the Fredlund and Xing (1994) sigmoidal SWCC equation which is defined over the entire range of possible soil suctions.

Some relevant geotechnical engineering questions to consider when dealing with unsaturated soils

There are a series of basic questions that the geotechnical engineer should address when undertaking a study that involves unsaturated soils as part of the soil's profile [5]. Questions may differ somewhat from those addressed for a conventional site investigation when saturated soils are involved. Following are examples of some questions that might be considered:

1.) Where is the water table relative to the ground surface?
2.) What is the approximate air-entry value, *aev*, for the soil types that are in the soils profile at the site?
    a. What is the physical relationship between the *aev* and the thickness of the "capillary zones" associated with each soil strata?
3.) Will the weather conditions influence the long-term performance of the engineered structure?

    a. Will the foundation for the engineered structure be located within a net drying or net wetting environment? In other words, "Will there be changes in the pore-water stress state with respect to time?"
    b. Will the boundary conditions for the site be changed as a result adding a structure onto the site?
4.) What are the primary soil properties that would be of interest in the assessment of the performance of the structure in the future?
    a. What effect will water movement have with respect to time on the near ground surface soils?
    b. Will changes in the shear strength of the soil occur with respect to time and what effect might these changes have on the performance of the structure?
    c. Is slope instability likely to become an issue of concern in the future?
    d. Is it possible that future deformations might occur due to volume increase, (i.e., swelling), volume decrease (i.e., shrinkage or collapse), or lateral deformation (i.e., resistance to volume change)?
    e. Might there be changes in chemicals placed on or near the ground surface?
    f. Might there be long-term thermal conditions that will affect the engineered structure (i.e., effects of freezing and thawing or above-freezing temperature changes)?
5.) What are the likely future risks and costs involved if the structure needs remedial measures?

There are also many other questions that may arise out of consideration of surface modifications such as the design and long-term performance soil cover systems. The 1990s witnessed the expansion of the scope of research into unsaturated soil mechanics to include numerous conditions beyond the swelling actions of soils [1].

It is the unsaturated soil zone between the ground surface and the top of the "capillary zone" that undergoes the most dynamic changes over time; still this zone is generally given the least attention. Minimal attention is often given to the soils in the vadose zone simply because of the perceived complexity of unsaturated soil behavior.

The past several decades of research into the behavior of unsaturated soils has led geotechnical engineers to the place where considerable future risk to engineered structures can be mitigated through use of "Estimation Techniques" for unsaturated soil properties. These "Estimation Techniques" have arisen out of unsaturated soils research studies that have been carried out around the world [6]. The "Estimation Techniques" being referred to are mainly those associated with the determination of unsaturated soil property functions, USPFs, such as the: i.) permeability function, ii.) water storage function, iii.) shear strength function, iv.) as well as other unsaturated soil property functions.

The Family of Soil-Water Characteristic Curves, SWCCs

There is no single relationship between soil suction and the amount of water in a soil. Rather, there are many possible curves that are contained by a bounding drying curve and a bounding wetting curve. Geotechnical engineers have benefited immensely from the research studies undertaken in agronomy-related disciplines such as soil science and soil physics. This is particularly true as it relates to the drying and wetting hysteretic nature of soils.

Klute (1965; 1986) proposed a combination of soil-water characteristic curves that form a limited number of bounding curves that are useful for geotechnical engineering applications [7, 8]. The amount of water in the soil is quantified in terms of degree of saturation of the soil [9, 10]. The physical properties turn out to be particularly sensitive to changes in degree of saturation. It is also convenient (and logical) to use matric suction measurements in the range of suctions up to approximately 1500 kPa and then use total suction measurements in the suction range above 1500 kPa [4].

Figure 1 identifies three main branches that appear to meet the constitutive needs within geotechnical engineering [7, 8]. The uppermost curve illustrates the drying SWCC where the soil has been initially forced to 100% saturation and then dried throughout the entire suction range. The lowermost SWCC corresponds to the case where the dry soil has been wetted to zero suction. It is also assumed that there may be some entrapped air in the soil after it is wetted. The third (intermediate) SWCC represents a typical drying SWCC for an initially unsaturated soil sample that is allowed to imbibe water and then tested in the drying mode.

The drying mode is easiest to measure in the laboratory; however, it is not easy to directly measure the degree of saturation of a soil specimen during testing. The measurement of the SWCC is the easiest when the amount of water is quantified in terms of gravimetric water content. This paper will later suggest and detail the most convenient procedure(s) to use for converting a gravimetric water content SWCC (i.e., *w*-SWCC) to a degree of saturation SWCC (i.e., *S*-SWCC).

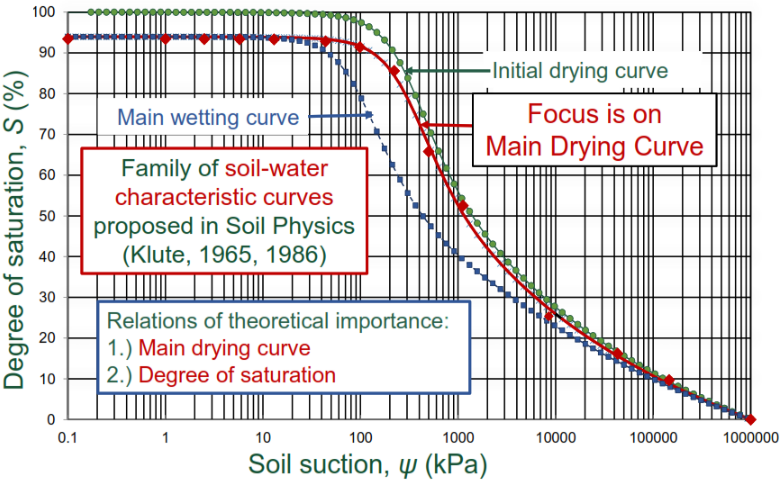


Figure 1: Family of degree of saturation versus soil suction SWCCs [7, 8]

The most important unsaturated soil characteristic that the geotechnical engineer must assess when applying unsaturated soil mechanics is the air-entry value, *aev* [11]. The *aev* must then be related to the field conditions. In other words, "How is the *aev* assessed from the degree of saturation SWCC in the laboratory (i.e., *S*-SWCC), and then applied to the engineering problem at-hand, in the field?".

Significance of the Air-Entry Value, *aev*

The air-entry value, *aev*, of a soil can be empirically defined as the suction at which the soil starts to desaturate when a saturated or "near saturated" soil specimen is subjected to increasing applied matric suctions. If the soil changes volume as matric suction is increased, the *aev* should be defined using the degree of saturation SWCC (i.e., *S*-SWCC). The *aev* is given the symbol, u_{ae}, and can be identified as the point at which the physical soil properties become a function of soil suction. In other words, soils become unsaturated at the point where the pore-water pressure u_w, becomes less than the *aev*, (i.e., u_{ae}).

The above definition provides a standardized definition for geotechnical engineering applications; however, it does not take all questions related to hysteretic effects into consideration. The term, water-entry value, *wev*, can be determined from the wetting branch SWCC as the point where an initially dry soil commences to have the air in the pore spaces replaced by water. The water-entry value can be designated using the symbol, u_{we}. The water-entry value should be defined on the degree of saturation SWCC, (*S*-SWCC).

The concept of the *aev* can be applied to hydrostatic equilibrium field conditions. The top of the capillary zone forms the beginning of the unsaturated soil properties. In other words, the thickness of the capillary zone can be approximated by the magnitude of the *aev* of the soil [4, 5]. This "approximation" does not take into consideration the effects of hysteresis or volume change as suction increases and decreases. The case of a relatively "thick" capillary zone is shown in Figure 2. The case of a relatively "thin" capillary zone is illustrated in Figure 3. It is important to relate the *aev* of a soil to the thickness of the capillary zone because the thickness of the capillary zone controls the point at which unsaturated soil properties need to be taken into consideration.

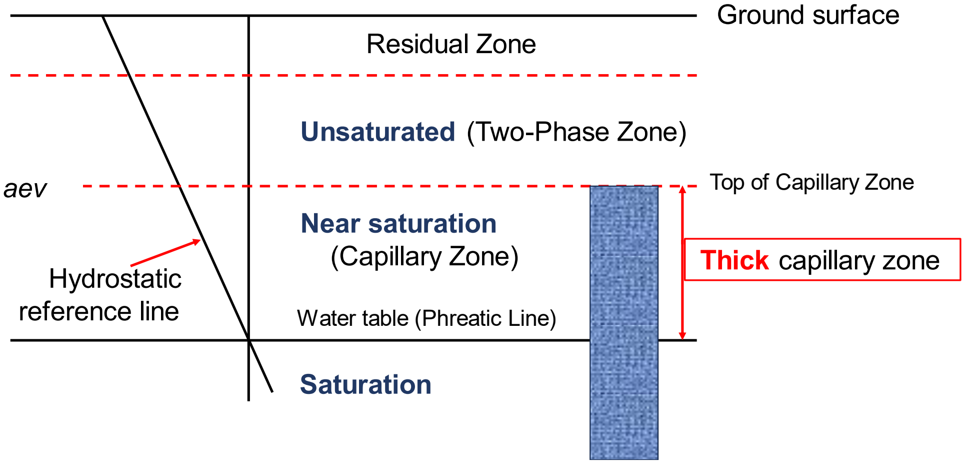


Figure 2: Relationship between the air-entry value, *aev*, for a "thick" capillary zone

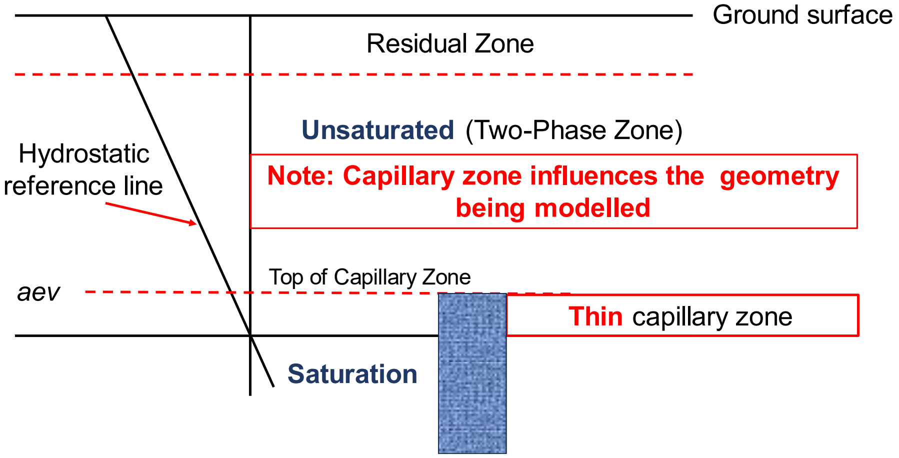


Figure 3: Relationship between the air-entry value, *aev*, for a "thin" capillary zone

Figure 4 illustrates the importance of assessing the thickness of the capillary zone for the case of a homogeneous earth-fill dam. Stated another way, if the capillary zone is relatively "thick", it is possible to model seepage through the dam using constant soil properties for hydraulic conductivity and water storage. Modelers are generally aware that the calculation of hydraulic heads are quite insensitive to the unsaturated soil properties as opposed to the calculation of moisture fluxes.

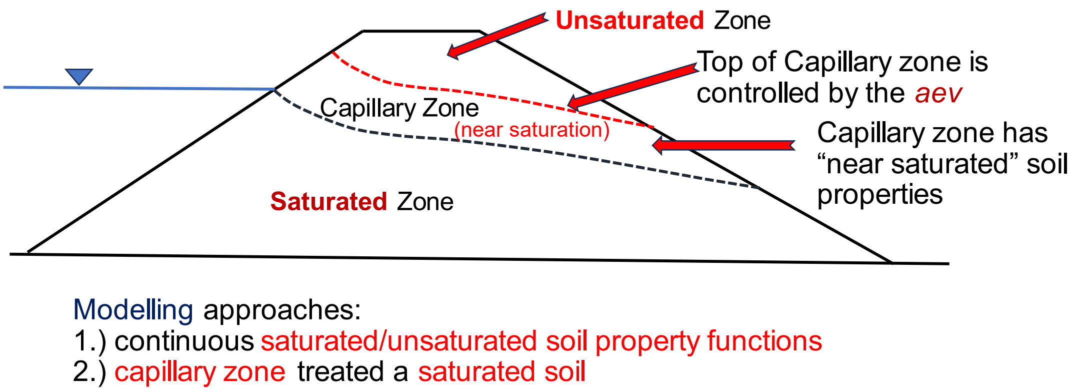


Figure 4: Influence of capillary zone on the numerical modeling of a seepage problem

Table 1 illustrates the influence of the *aev* on the "capillary zone" when modeling seepage through a saturated-unsaturated soil system. The geotechnical engineer needs to be aware that the *aev* can vary over several log cycles which translates into a wide range of possible capillary zone thicknesses. The capillary zone may be in the order of a few centimeters to many meters in thickness. In some cases, the capillary zone may extend near to the ground surface and saturated soil properties (i.e., constant properties) can be used for numerical model simulations.

Table 1: Relationship between air-entry value, *aev*, and the thickness of the capillary zone.

air-entry value, *aev* (kPa)	1	10	100	1000
capillary zone (m)	0.1	1	10	100
capillary zone (cm)	10	100	1000	10000
typical soil	Coarse	Sand	Silt (some clay)	Clay (without fractures)
Note: *aev* designates thickness of the capillary zone with near-saturated soil				

It is noteworthy that in some situations (e.g., calculation of hydraulic heads), the same seepage solution is obtained when using either saturated soil properties or unsaturated soil properties. On the contrary, the calculation of moisture fluxes or quantities of water flow are likely to be highly sensitive to the selected unsaturated soil properties. Consequently, the seepage analysis may rely heavily upon the interpretation of the air-entry values for the soil continua. Figure 5 emphasizes the distinct change in the coefficient of permeability values whenever the *aev* of a soil is exceeded.

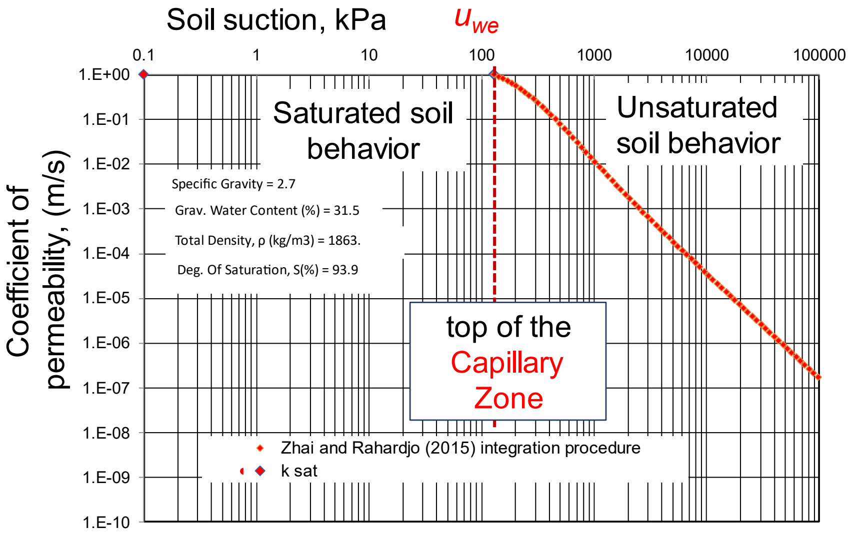


Figure 5: Distinct coefficient of permeability changes at the air-entry value, *aev*

Preference for Laboratory Testing

Unsaturated soil properties have been shown to be most closely related to the situation where the amount of water in the soil is quantified in terms of the degree of saturation [1, 10]. Early researchers in agronomy showed that there is not a single, unique relationship between the amount of water in the soil and the state variable referred to as soil suction. There is an infinite array of possible *S*-SWCC relationships, however, the *S*-SWCC curves can be reduced to three main relationships as shown in Figure 6.

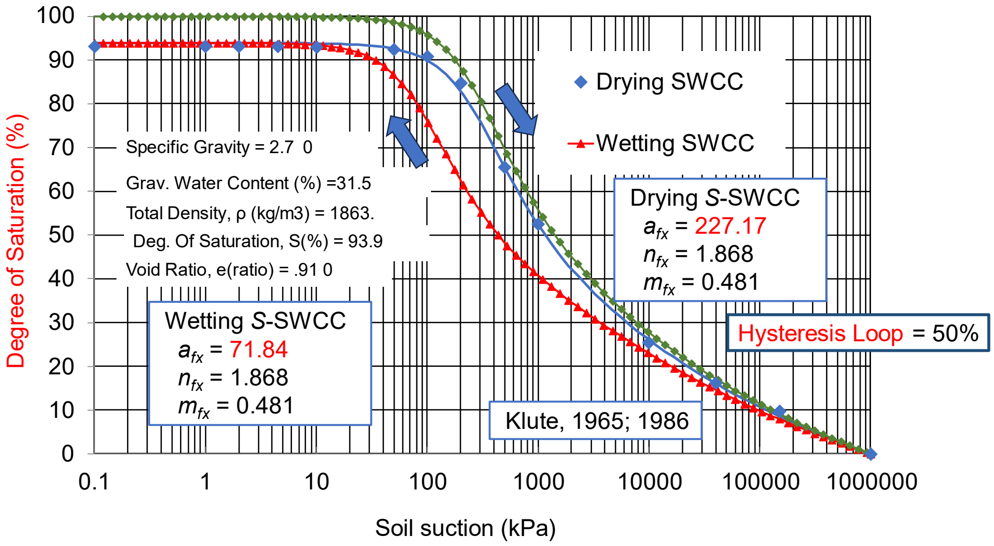


Figure 6: Family of soil-water characteristic curves illustrating the interpretation of the air-entry value [7, 8].

Theoretical considerations favor relating unsaturated soil properties solely to the degree of saturation SWCC since it isolates the effects of volume change as soil suction is increased. Most laboratory testing apparatuses make use of the axis-translation concept and relate suction to gravimetric water content, *w*. Gravimetric water content combines the effects of volume change and desaturation within its measurement. As a result, the interpretation of the *w*-SWCC does not yield the "true" air-entry value of the soil. Gravimetric water content measurement produces a slope to the *w*-SWCC function in low suction range making it somewhat more difficult to best fit a single mode, sigmodal function to the laboratory measurements.

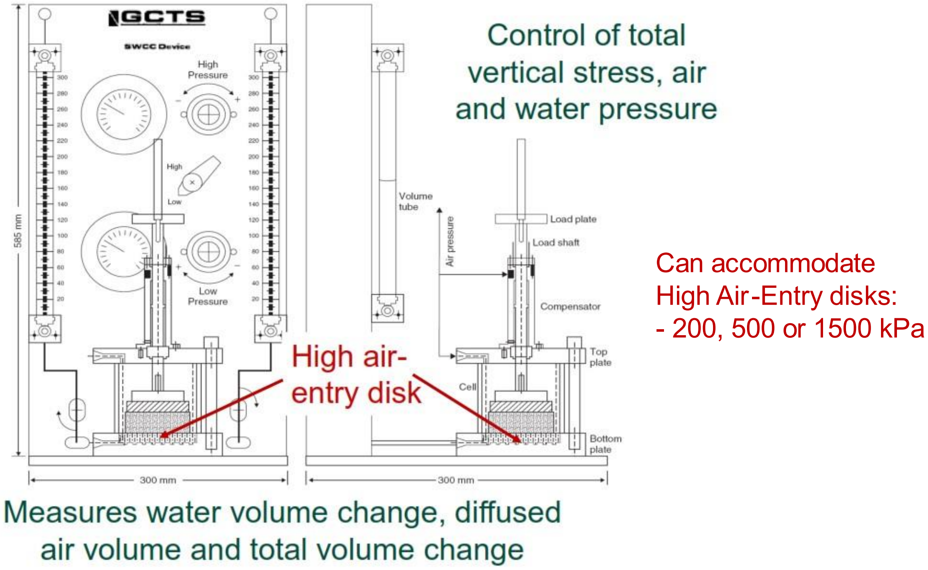
It is noted that most laboratory testing apparatuses are limited to measuring the drying branch of the family of SWCC curves. In consideration of the above-mentioned factors, it is suggested the drying branch of the SWCC be used as the "reference" branch for laboratory data interpretation and that other means be sought for the conversion of the *w*-SWCC to the *S*-SWCC. The conversion between the two functions can readily be accomplished either through the measurement or estimation of the shrinkage curve, SC, of the soil. It is out of convenience and expediency rather than theoretical considerations that the *w*-SWCC is combined with the shrinkage curve, SC, to determine the air-entry value, u_{ae}, of the soil. Much of the remainder of this paper deals with the conversion of *w*-SWCC to the *S*-SWCC through consideration of the shrinkage curve, SC.

Laboratory Measurement of SWCCs For Geotechnical Engineering Applications

Soil suction can range from a fraction of one kPa (~ zero) to 1,000,000 kPa. It is advantageous to have a single continuous mathematical function that embraces the entire range of suctions. The range of soil suction can be arbitrarily subdivided into a low suction range that is less than 1500 kPa and a high suction range that is greater than 1500 kPa. The low suction range uses matric suction equilibration along with the axis-translation technique applied to a high air-entry ceramic disks for the measurement of the *w*-SWCC. The high suction range uses the vapor pressure equalization on an apparatus such as the chilled-mirror PotentiaMeter. The amount of water in the soil is measured in terms of gravimetric water content.

The low and high suction range branches are assumed to essentially overlap at suction values of 1500 kPa. This assumption does not appear to create serious difficulties in the interpretation of laboratory data even though it is matric (or capillary) suction that is measured in the low suction range and total suction that is measured in the high suction range [12].

Figure 7 shows a "low soil suction range" pressure plate device specially designed to measure the *w*-SWCC relationship [13]. The pressure plate device has interchangeable high air-entry disks pressure plates ranging from 200 to 1500 kPa. The device allows for the independent control of total vertical stress state in addition to the control of the air and water pressures. The water volume change is measured independent of the diffused air volume and overall total volume change. Both the drying and wetting branches of the w-SWCC have been measured on several soils using this device [14]; however, this paper emphasis the measurement of the drying SWCC. The design features on this device have taken into consideration the main performance requirements associated with the measurement of the low suction matric suction branch of soils.



Control of total vertical stress, air and water pressure

Can accommodate High Air-Entry disks:
- 200, 500 or 1500 kPa

High air-entry disk

Measures water volume change, diffused air volume and total volume change

Figure 7: Low soil suction range pressure plate apparatus for measuring the *w*-SWCC relationship [13].

Figure 8 shows the use of a Chilled-Mirror PotentiaMeter for measuring the *w*-SWCC relationship in the high soil suction range [15].

Chilled-mirror water PotentiaMeter

- Measurement in a few minutes
- Measure total suction at 3 or 4 elapsed times of drying
- Total suction is 1,000,000 kPa at zero water content

Figure 8: High soil suction range Chilled-Mirror PotentiaMeter for measuring the w-SWCC relationship [15].

Data Interpretation for the Gravimetric Water Content SWCC, (*w*-SWCC)

Figure 9 shows a typical *w*-SWCC for silty clay soil. The *w*-SWCC function shows features that are of similar shape to those observed on the theoretical *S*-SWCC; however, there are some important distinctions between the *S*-SWCC and the *w*-SWCC. For example, the downward bend in the SWCC occurs at a different point on the two functions except for the case where there is no volume change in the soil specimen as soil suction is increased.

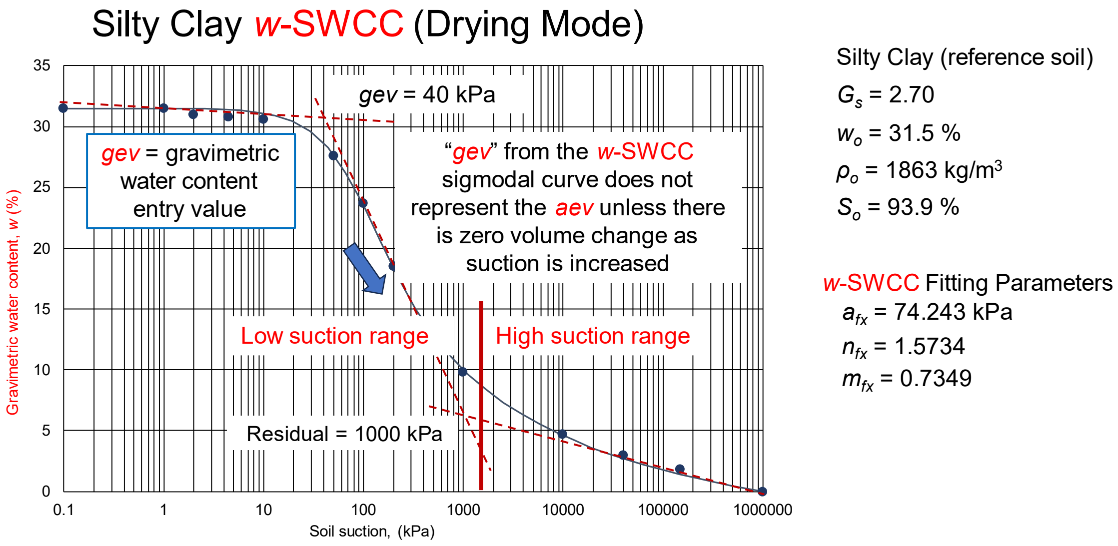


Figure 9: Typical data for the drying mode *w*-SWCC for a silty clay [16].

The starting or initial volume-mass soil properties for the drying branch of the shrinkage curve are shown in Figure 9. Also shown are the fitting parameters for the Fredlund and Xing (1994) SWCC equation [17]. The soil properties are essentially the same as those published for Regina clay [16]. Subsequent analyses referred to in this manuscript are assumed to be for the same soil.

The *w*-SWCC bend in the curvature will occur at a lower value than the "true" *aev* observed on the *S*-SWCC. However, the same Fredlund and Xing (1994) sigmodal equations can be used to mathematically fit either the *w*-SWCC or the *S*-SWCC datasets. The difference between the two datasets occurs because the gravimetric water content function responds to both volume change and degree of saturation change as soil suction is increased.

The Fredlund and Xing (1994) equation has four fitting parameters including residual suction. Residual suction need only be approximated when using the empirical construction procedure [18]. The fitting variables associated with the SWCC equation can be obtained using a nonlinear regression analysis. The fitting parameters for *w*-SWCC will have a slightly different physical meaning from those assigned to the degree of saturation SWCC, (*S*-SWCC) if the soil undergoes volume change as soil suction is increased. The Fredlund and Xing (1994) SWCC equation is shown for the *w*-SWCC in mathematical form in Equation (1) [17].

$$w(\psi) = \frac{w_s C(\psi)}{\left\{\ln\left[\exp(1) + (\psi/a_f)^{n_f}\right]\right\}^{m_f}} \qquad (1)$$

where $w(\psi)$ = water content at any soil suction, ψ, a_{fx} = fitting parameter related to the suction near the (arithmetic) inflection point of the *w*-SWCC, n_{fx} = fitting parameter related to the maximum rate of gravimetric water content change, m_{fx} = fitting parameter related to the curvature near residual gravimetric water content conditions, ψ_r = suction near residual conditions of the soil, and $C(\psi)$ = correction factor directing the *w*-SWCC towards a suction of 106 kPa at zero water content, written as:

$$C(\psi) = 1 - \frac{\ln\left(1+\psi/\psi_r\right)}{\ln\left(1+10^6/\psi_r\right)} \tag{2}$$

The fitting parameters along with the initial volume-mass soil properties are shown for the silty clay soil on Figure 9. A close fit can generally be achieved with laboratory data when using the Fredlund and Xing (1994) equation; however, this is not always the case. It has been observed in some cases that the dataset may show a gradual downward trend even under relatively low applied suction values.

Importance of the Shrinkage Curve

The shrinkage curve for a soil has historically been viewed as a means of evaluating the shrinkage limit, *SL*, of the soil [19]. The drying curve, starting from a slurry state or from a near-saturated state, is used to calculate the SL of the soil. However, the SC can also play an important role in quantifying the continuous relationship between the volume change and degree of saturation change of a soil during the drying process. Being able to relate these two behavioral mechanisms allows for the conversion of the *w*-SWCC into the *S*-SWCC.

The shrinkage curve can readily be measured in the laboratory; however, it can also be estimated based on the Atterberg Limits of the soil. Both the measurement procedure and the estimation procedure are described in this paper. The shrinkage curve and the *w*-SWCC must be “blended” together through use of the common gravimetric water content for the starting point for the calculation of the *S*-SWCC (Figure 10).

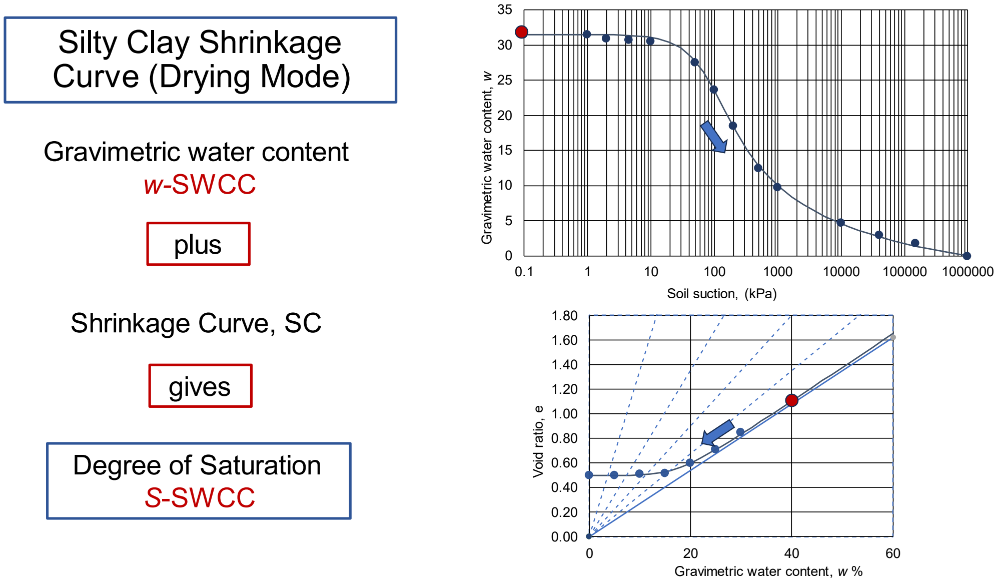
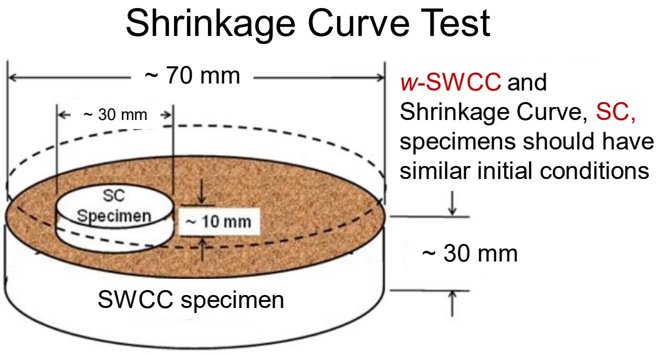


Figure 10: Use of the shrinkage curve, SC, and the gravimetric water content SWCC, (*w*-SWCC), to calculate the S-SWCC.

Measurement of the Shrinkage Curve SC

The shrinkage curve is generally measured on soil specimens that are about 3 centimeters in diameter and one centimeter in thickness. The relatively small sized specimen generally allows the soil to undergo complete drying without cracking into small pieces. A micrometer caliper can be used to periodically measure the diameter and thickness of the soil specimen as the specimen is allowed to slowly go towards its completely dry state (Figure 11).



Possible Test Procedures:
1.) Periodic measurements of volume and mass as the soil dries, or
2.) Measurement of initial and completely dried volume and mass

Figure 11: Measurement of the Shrinkage Curve, SC.

Figure 12 shows the drying branch for a typical shrinkage curve for a silty clay soil. The soil commences drying from a near saturated state. The volume-mass pathway is asymptotic to a high degree of saturation line. As the soil approaches a completely dry state, it becomes asymptotic to a constant but minimum void ratio designated by the symbol, a_{sh}.

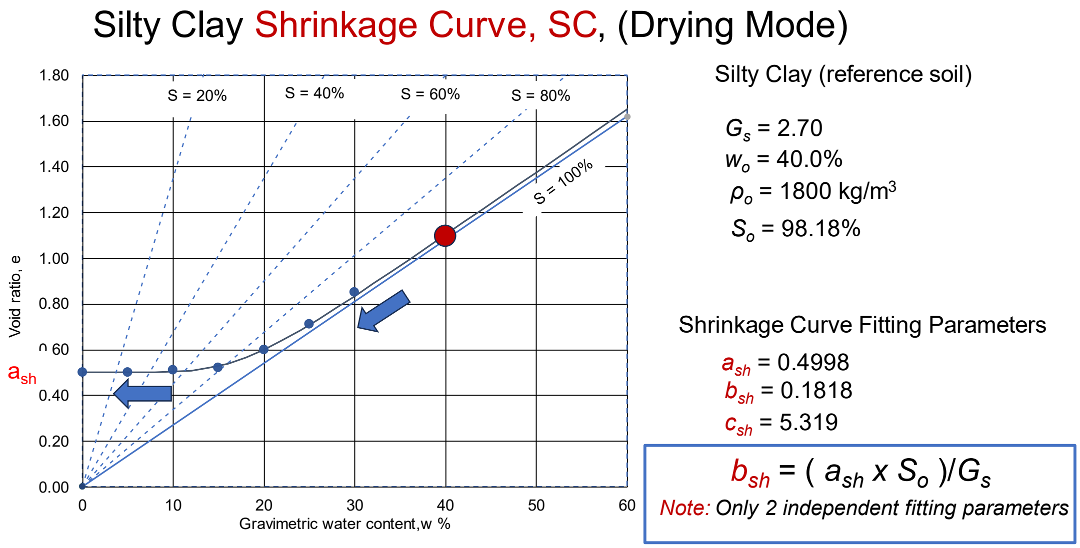


Figure 12: Typical Shrinkage Curve data measured on the drying branch for silty clay.

Constitutive Behavior of the Shrinkage Curve, SC

Wijaya and Leong (2016) summarized several equations that have been proposed for the fitting of complex multi-modal shrinkage curves [20]; the types sometimes encountered when drying from a variety of initial conditions. However, when matching the stress path of the *w*-SWCC test, the M. Fredlund (2000) hyperbole equation can best fit measured shrinkage curve data [21].

$$e(w) = a_{sh}\left[\left(\frac{w}{b_{sh}}\right)^{c_{sh}} + 1\right]^{1/c_{sh}} \qquad (3)$$

where: a_{sh} = minimum void ratio upon complete drying, b_{sh} = variable related to the slope of the drying curve calculated as: $b_{sh} = (a_{sh} \times S_o)/Gs$, and c_{sh} = variable related to the sharpness of curvature as the soil desaturates, and So = conditioned degree of saturation. The emphasis is on characterizing the drying *SC* along a similar stress path to that used for the measurement of the *w*-SWCC. The initial degree of saturation should be near 100% (e.g., S_o = 94%).

The a_{sh} variable is relatively easy to determine since it is the minimum void ratio that a soil can attain upon complete drying. The a_{sh} variable can be determined while measuring the entire shrinkage curve or by simply measuring the minimum void ratio corresponding to an oven-dried soil specimen.

Estimation of the Shrinkage Curve, SC

The a_{sh} variable can also be estimated from the shrinkage limit, *SL*, of the soil. Casagrande (1932) showed that the shrinkage limit, *SL*, of a slurry or remolded soil could be estimated from the liquid limit and plasticity index of a soil. The shrinkage limit of a soil has been shown to lie either above or below the A-Line on the Casagrande plasticity chart. The A-Line on the plasticity chart corresponds to a straight line that is mathematically defined as follows [22].

$$PI_A = 0.75LL_S - 15 \qquad (4)$$

where PI_A = plasticity index on the A-Line, and LL_s = measured liquid limit of the selected soil. The shrinkage limit, SL_S, of the soil can then be written in terms of the plasticity index of the soil, PI_S.

$$SL_S = 20 + \left(PI_S - PI_A\right) \qquad (5)$$

Holtz and Kovacs (1981) suggest that the estimation of the shrinkage limit of a slurry or remolded soil appears to be of similar accuracy to those measured in a laboratory [19]. The ability to accurately estimate the shrinkage limit of a soil from the other plasticity classification values means that one of the primary variables associated with the characterization of the volume-mass behavior of a soil can be determined independent of the measurement of the entire shrinkage curve. The shrinkage limit of the soil, SL_S, is equal to the amount of water required to completely fill the voids when the soil is dried to its minimum void ratio.

$$a_{sh} = SL_S G_s \qquad (6)$$

where the initial degree of saturation of the slurry soil is assumed to be 100%. Consequently, the ash parameter for the shrinkage curve can be calculated from the soil plasticity values.

The second variable required for the hyperbolic characterization of the shrinkage curve equation is b_{sh} [21]. The b_{sh} variable is calculated from the a_{sh} variable, the specific gravity, *Gs*, and the initial degree of saturation, *So*.

$$b_{sh} = a_{sh} S_0 / G_s \qquad (7)$$

The remaining fitting parameter for the shrinkage curve is the c_{sh} variable when using the M. Fredlund (2000) equation. The c_{sh} variable defines the curvature of the shrinkage curve when moving between the near saturated state and the completely dried state. The common range of c_{sh} values for soils lies between 3 and 15. Soils with a c_{sh} value of 15 are of low compressibility soils while soils with a c_{sh} value of 1.5 would have much higher compressibility (or undergo considerable volume change) as soil suction is increased. Several volume-mass measurements on a soil specimen can be used to quantify the c_{sh} variable. A typical value of c_{sh} for a moderately compressible silty soil would be about 5 to 7.

A research study of experimental shrinkage curve test results was undertaken by Wong et al., (2019) [23]. The c_{sh} fitting parameter was found to vary with the plasticity of the soil and a close correlation was found with the plasticity ratio, PL/LL. A total of 27 soils were analyzed and a best fit equation for c_{sh} was found to be as follows.

$$c_{sh} = 1.7 + e^{3.1^{PL/LL}} \qquad (8)$$

where e = is the base of the natural logarithm.

Separation of the Effects of Volume Change and Degree of Saturation Changes

The following steps are suggested for blending the results of a measured gravimetric water content SWCC and a shrinkage curve, SC.

Step #1 involves rotating the usual shrinkage curve plot through 90 degrees. The rotation of the volume-mass axes allows the water content axis to be shared with the gravimetric water content axis on the w-SWCC. The plot on the right side of Figure 13 shows the usual manner for plotting the shrinkage curve. The rotated shrinkage curve plot is shown in the left side of Figure 13. It is the plot on the left side of Figure 13 that is preferred when analyzing the w-SWCC data. Also shown are the initial volume-mass soil properties and the fitting parameters when using the M. Fredlund (2000) shrinkage curve equation.

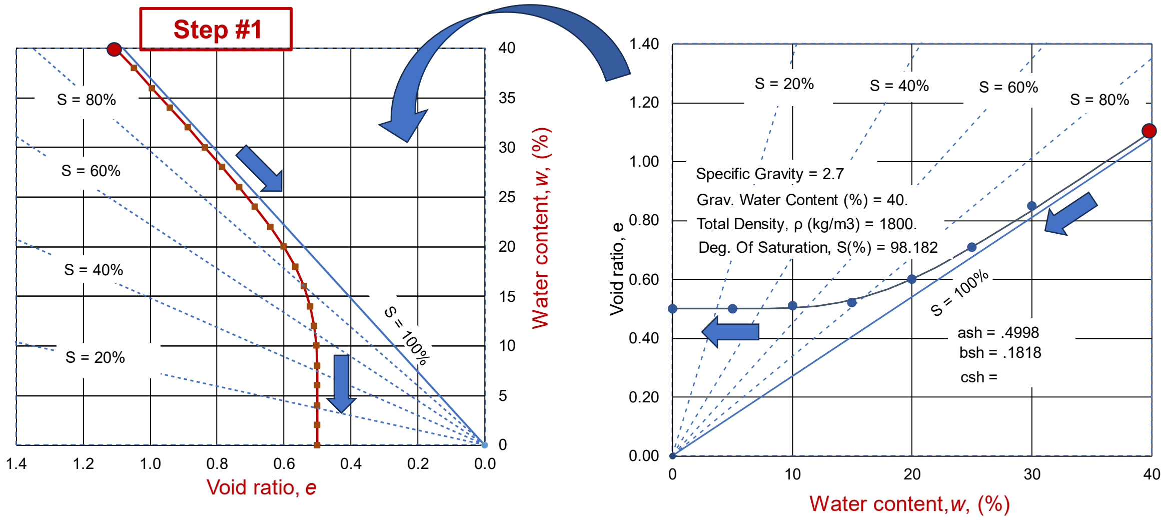


Figure 13: Ninety-degree rotation of the shrinkage curve axes associated with the drying curve.

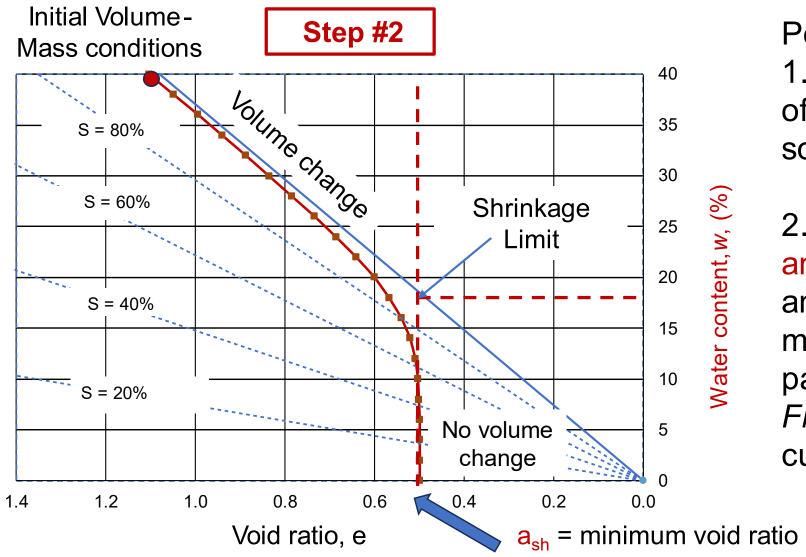
Step #2 illustrates the relationship between the shrinkage limit and the primary fitting parameter, a_{sh} for the shrinkage curve [19]. The shrinkage limit, SL, can either be used to estimate a shrinkage curve for the calculation of the a_{sh} fitting parameter as shown in equation (9). The shrinkage limit, SL, can also be calculated in a vice versa manner as shown in equation (10).

$$a_{sh} = G_s \times SL \qquad (9)$$

or

$$SL = a_{sh}/G_s \tag{10}$$

The data points along the shrinkage curve can be generated through experimental measurements or through use of the empirical hyperbolic equation [21].



Possible Test Procedures:
1.) Periodic measurements of volume and mass as the soil dries.

2.) Measurement of initial and completely dried volume and mass to defines the minimum void ratio parameter, a_{sh}, for the *M. Fredlund (2000)* shrinkage curve equation

Figure 14: Relationship between the shrinkage limit, *SL*, and the a_{sh} fitting parameter used in the M. Fredlund (2000) shrinkage curve equation.

The a_{sh} variable can be viewed as the minimum void ratio that the soil attains when it is completely dry. It is assumed the total stress applied on the soil has a minor and secondary effect on the minimum void ratio, a_{sh}.

Step #3 brings together the shrinkage curve, SC, and the *w*-SWCC using a common gravimetric water content scale as the ordinate axes on both plots. The shrinkage curve is shown on the left side in Figure 15 while the *w*-SWCC is shown on the right side.

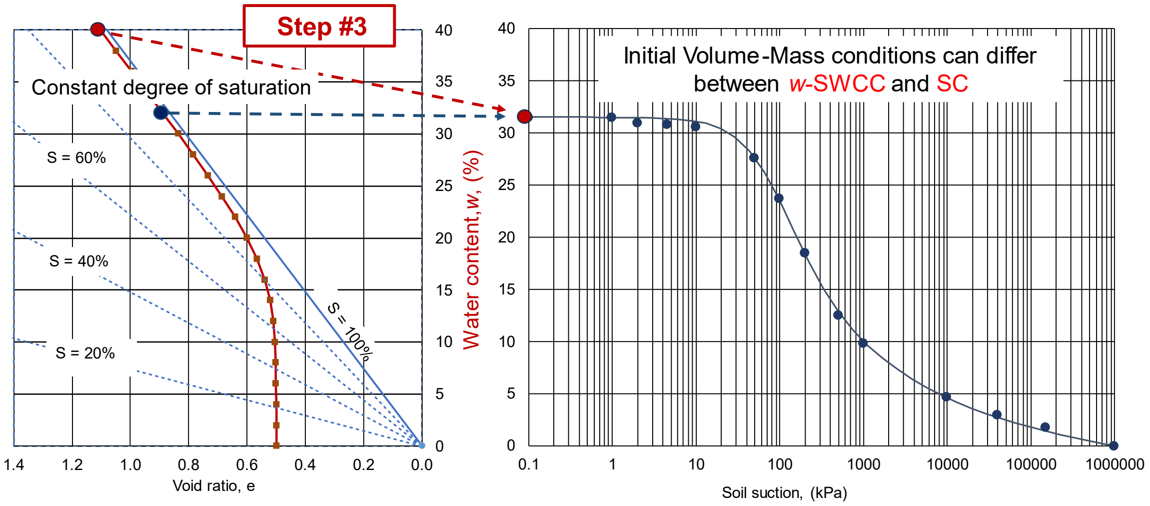
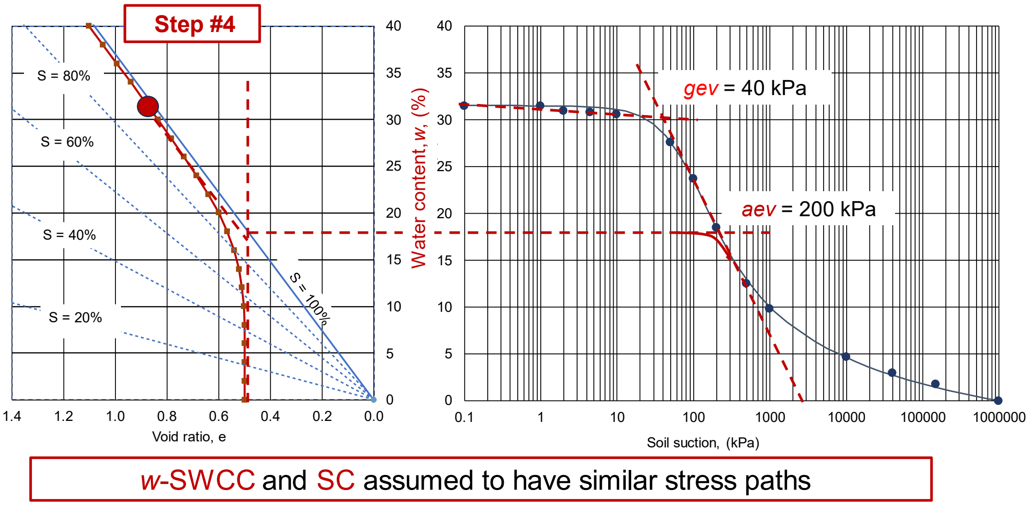


Figure 15: Bringing together the shrinkage curve, SC, and the *w*-SWCC to a common gravimetric water content scale.

The initial gravimetric water content (i.e., $w = 31.5\%$), controls the starting phase properties at the start of the *w*-SWCC function. As the applied soil suction increases, the degree of saturation of the soil remains essentially constant at a value near to 100%. Consequently, it is not necessary that the initial gravimetric water content for the shrinkage curve soil specimen be precisely the same for the shrinkage curve test.

Step #4 explains how it is possible to separate two distinct mechanisms that are embedded within the gravimetric water content SWCC. These are the asymptotic line where the degree of saturation remains near-constant and the asymptotic line where the volume change remains essentially constant. The shrinkage curve function (or dataset) can be used to interpret the two datasets (i.e., *w*-SWCC and SC), in a more precise manner. The assumption is made that the shrinkage limit of the soil can be used to represent the dividing point between water content change and degree of saturation change.

Figure 16 shows that the original bend on the *w*-SWCC occurred around 40 kPa. However, when it is assumed that shrinkage limit constitutes the limiting point of volume change, the air-entry value, *aev*, is shown to be approximately 200 kPa. In other words, an understanding of the shrinkage limit, *SL*, of the soil along with the *w*-SWCC allows for an approximation of the degree of saturation SWCC, (*S*-SWCC), and the air-entry value, *aev*, for the soil.



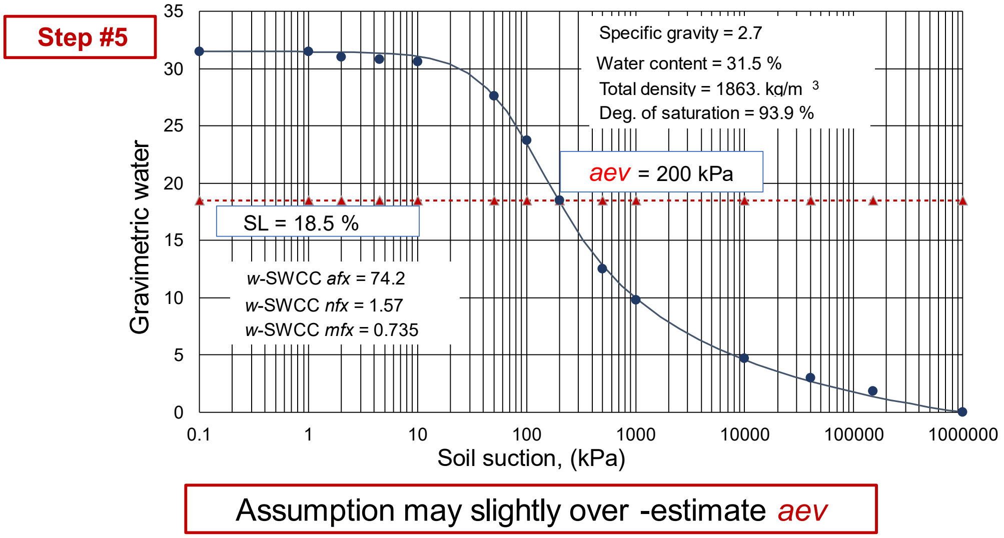
w-SWCC and SC assumed to have similar stress paths

Figure 16: Separation of overall volume change and desaturation components.

It is possible to estimate the shrinkage limit, *SL*, of the soil from the liquid limit and plastic limit of the soil. Consequently, the geotechnical engineer has a valuable estimation tool for characterizing the degree of saturation SWCC and for subsequently estimating the unsaturated soil property functions (e.g., permeability function or shear strength function). Further studies would be warranted to further understand the determination of the unsaturated soil property functions for sands and other noncohesive soils.

Approximate calculation of the air-entry value, *aev*

An approximate value for the *aev* can be calculated from the *w*-SWCC and an understanding of the shrinkage limit of the soil (i.e., Step #5). The analysis involves assuming all volume changes in the soil occur while the water content of the soil is greater than its shrinkage limit. In contrast, all desaturation of the soil commences at the shrinkage limit. In other words, the gradual bending of the shrinkage curve is ignored as the shrinkage limit of the soil is approached.


Step #5

Assumption may slightly over -estimate *aev*

Figure 17: Interpretation of the *w*-SWCC for an approximate value for the air-entry value, *aev*.

The effect of the separation of volume change and desaturation can be illustrated as shown on Figure 17 where a horizontal line is drawn through the shrinkage limit of the soil. The intersection of the *w*-SWCC and the shrinkage limit line occurs at approximately 200 kPa. This *aev* would be expected to be slightly higher than the "true" air-entry value, *aev* since the gradual curvature of the shrinkage curve is ignored in the analysis. The variable *aev* can be obtained as a graphical solution or by using a solver in EXCEL.

More precise calculation of the air-entry value, *aev*

A more precise solution for the air-entry value, *aev*, of the soil can be obtained through use of a continuous function that represents the volume change as soil suction is increased (i.e., void ratio versus suction) (Step #6). The basic volume-mass phase relationship is shown in equation (11). This equation can be used to calculate a more complete and continuous mathematical function for the degree of saturation SWCC.

$$S\ e = w\ \ G_s \tag{11}$$

where: S = degree of saturation, e = void ratio, w = gravimetric water content and G_s = specific gravity of the soil solids.

The degree of saturation can be written as a function of soil suction, ($S(\psi)$) as shown in equation (12). The gravimetric water content can also be written as a function of soil suction, ($w(\psi)$, *w*-SWCC). The void ratio can be written as a function of gravimetric water content, ($e(w)$, the shrinkage curve). In other words, there are continuous mathematical functions for all components of the basic volume-mass relationships.

$$S(w) = \frac{w(\psi)G_S}{e(w)} \tag{12}$$

where: $w(\psi)$ = gravimetric water content is written as a function of soil suction (i.e., the *w*-SWCC), and $e(w)$ = void ratio is written as a function of gravimetric water content (i.e., the shrinkage curve). Figure 18 shows the calculated data points for the degree of saturation versus suction relationship.

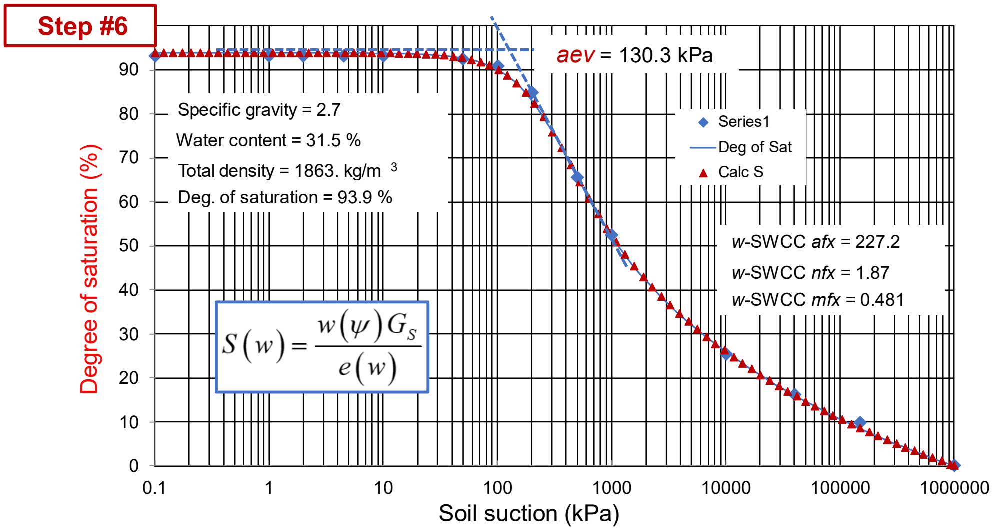


Figure 18: The more rigorous analysis to calculate the *S*-SWCC function and a more precise air-entry value, *aev*.

The "blue" symbols in Figure (18) show the calculated degree of saturation data points based on the *w*-SWCC and the shrinkage curve. These data points are best fit using the Fredlund and Xing (1994) SWCC equation. It should be noted that the same Fredlund and Xing (1994) equation can be used to obtain the best fit parameters for the *w*-SWCC curve and the *S*-SWCC equation; however, the fitting parameters are unique for each of the two functions.

The "red" symbols were generated using a regression analysis on the computed *S*-SWCC function. The "blue" lines represent lines of tangency through the low suction values and the through the inflection point of the *S*-SWCC function. The inflection point, a_{fx} was calculated to be 227.2 kPa. The intersection point represents a more precise air-entry value, *aev*, and was calculated to be equal to 130.3 kPa. The differences between the two *aev* are related to the precision with which the shrinkage curve is represented. It is suggested that there is merit in both the "approximate" determination of the *aev* as well as the "more precise" procedure of analysis.

Table 2 contains a summary of the three interpretations for the *aev* based on the drying soil-water characteristic curves. The first interpretation approximates the *aev* based on sharpest curvature of the *w*-SWCC (i.e., referred to as the *gev*). It should be noted that this is not a true indication of the *aev* because the curvature of the function is the result of both volume change and degree of saturation change. The sharpest "Bend" (i.e., 40 kPa) results in an estimated *aev* that can be quite far below the "*true*" *aev*.

The second interpretation of the *aev* shown in Table 2 makes use of the *w*-SWCC but assumes that the influence of volume change and degree of saturation change can be separated at the shrinkage limit of the soil. This results in an increased *aev* approximation of 200 kPa.

The third interpretation of the *aev* presented in Table 2 makes use of the measured shrinkage curve to calculate the *S*-SWCC. The *S*-SWCC isolates the effect of degree of saturation and provides a more accurate indication of the "true" *aev* for the soil. The calculated *aev* is 130.3 kPa and the inflection point, a_{fx}, occurs at 227.2 kPa.

The three values shown in Table 2 show the influence of the volume change and degree of saturation on the interpretation of the soil-water characteristic curves. The results presented show the importance of either measuring or estimating the shrinkage curve behavior of the soil when attempting to determine the *aev* of a soil.

Table 2: Summary of *aev* and a_{fx} parameters

Soil-water characteristic curves		
w- SWCC	*gev (kPa)*	a_{fx} *(kPa)*
	40	74.2
Shrinkage limit approximation	*aev (kPa)*	a_{fx} *(kPa)*
	200	~ 300
S-SWCC	*aev (kPa)*	a_{fx} *(kPa)*
	130.3	227.2

Conclusions

Following is a list of conclusions and recommendations related to the application of unsaturated soil mechanics into routine engineering practice. It must be recognized that there are a series of assumptions and estimations associated with each of the suggestions advocated. Each of the assumptions and estimation have side effects; however, it is suggested that the benefits far out way the weaknesses associated with invoking the recommendations.

1.) The first and most important variable to assess as part of an engineering project is the air-entry value, *aev*, of the soils. In general, the *aev* must be determined from the drying branch of the degree of saturation soil-water characteristic curve, *S*-SWCC.
2.) The significance of the *aev* needs to be assessed in terms of its effect on the thickness of the "capillary zone". The "capillary zone" can be assumed to behave as saturated soil in terms of its physical and hydraulic soil properties.
3.) It is recommended that the estimation of the unsaturated soil property functions be determined based on the drying branch of the degree of saturation SWCC, *S*-SWCC. It is also recommended that the effects of hysteresis be dealt with independently.

4.) Laboratory measurements of the SWCC can be made by measuring the gravimetric water contents as soil suction is increased (i.e., *w*-SWCC).
5.) Using the gravimetric water content SWCC constitutes an acceptable procedure; however, it is necessary to make one or more assumptions when converting the drying branch *w*-SWCC to *S*-SWCC. The conversion of the *w*-SWCC to the *S*-SWCC can be made by measuring the "shrinkage curve" of the soil or simply estimated from the "shrinkage limit" of the soil.
6.) The measurement of the shrinkage curve along with information on the shrinkage limit of the soil (i.e., *SL*), allows for a separation of the volume change component from the degree of saturation component on the measured gravimetric water content SWCC.
7.) The assumption is also made that the SWCC in the vicinity of the *aev* (and the inflection point of the function) is more important to the geotechnical engineer than the assessment of the function near residual suction.

This paper focuses on the pathways that are plausible at the applied unsaturated soil mechanics level with limited laboratory equipment apart from a "pressure plate" system that uses the axis-translation technique for applying matric suction.

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