A Simple Model for Predicting the Residual Shear Strength of Unsaturated Soils

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Abstract: The shear strength of unsaturated soils can reduce from a peak to a residual shear strength (RSS) value with increasing shear deformation. The RSS can be a crucial parameter for the rational design of geo-structures constructed with or within unsaturated soils undergoing large shear deformation. For investigating the RSS of unsaturated soils, five sets of suction-controlled ring shear tests that were performed on different soils extending axis translation technique were gathered from the literature. Analysis of the experimental results suggest that the RSS envelope of unsaturated soils is linear with respect to net normal stress at any matric suction level. The intercept and slope of the RSS envelopes are functions of matric suction. Based on these experimental observations, a simple model is formulated based on two stress state variables (i.e., the net normal stress and matric suction) for predicting the RSS for a wide range of unsaturated soils. In this model, the contribution of matric suction toward mechanical behaviors of unsaturated soils undergoing large shear deformation is interpreted considering the loss of degree of saturation due to shearing. The prediction capability of the proposed model is validated using the results of the gathered suction-controlled ring shear tests.

Introduction

The shear strength of unsaturated soils can reduce gradually from a peak to a constant low value under a large shear deformation, which is associated with the changes in soil structure and water phase [1-5]. The lowest shear strength value at a large shear deformation is referred to as the residual shear strength (RSS). The shear strength reduction behavior associated with the increasing shear deformation plays an important role in analyzing the performance of geo-structures constructed with or within unsaturated soils, such as the progressive failure of soil slopes [6-8]. Typically, several local sliding zones are likely to develop in the slopes of unsaturated soils due to influence of environmental factors (e.g., the rainfall infiltration, the drying-wetting cycles) and/or construction activities (e.g., excavations) [9-11]. The shear strength of unsaturated soils in the sliding zones will reduce gradually with increasing shear deformation. Due to this reason, the shear stress in the sliding zones that exceeds the post-peak shear strength will be redistributed into surrounding zones. The sliding zones show progressive behavior propagating through the entire slope until a failure condition is reached. During this process, the shear strength of unsaturated soils varies between the peak and the residual value at different locations of the slope depending on the deformation level. Due to this reason, the RSS can be a crucial parameter for a rigorous design and analysis of geo-structures in unsaturated soils undergoing large shear deformation.

The shear strength of soils is typically investigated by using direct shear or triaxial tests. However, these tests can only provide limited shear deformation information, which is not enough for soils to reach the residual shear strength stage. For this reason, the RSS of soils should be investigated by using ring shear testing technique which can provide information of a large shear deformation. For studying the RSS of unsaturated soils, different types of modified ring shear apparatuses have been developed for controlling suction based on the axis translation technique [1-4] and the vapor equilibrium technique [5]. Several researchers summarized results of suction-controlled ring shear tests [1-5] that contribute to a better understanding of the RSS behavior of unsaturated soils. However, studies related to the development of a simple model for predicting the RSS of unsaturated soils are still limited [1, 5] despite various models that have been proposed for predicting the peak and critical shear strength of unsaturated soils [12, 13]. Comparison studies suggest that the models for predicting the peak and critical shear strength of unsaturated soils are not suitable for predicting the RSS of unsaturated soils [14, 15].

Due to this reason, in this study, a simple model is formulated for predicting the RSS for a wide range of unsaturated soils varying from coarse- to fine-grained soils. For this purpose, a series of suction-controlled ring shear tests based on the axis translation technique were gathered from the literature. The behaviors of RSS envelopes of different unsaturated soils were interpreted using the two stress state variables (i.e., the net normal stress and matric suction). The prediction model of RSS of unsaturated soils was formulated and validated based on the suction-controlled ring shear tests results.

Formulation of the model for predicting residual shear strength of unsaturated soils

Suction-controlled ring shear tests

The results of five sets of suction-controlled ring shear tests conducted extending the axis translation technique available in the literature were gathered. In these studied tests, the liquid phase flow has a dominant influence on the suction change. The investigated unsaturated soils include two coarse-grained soils, SM [2] and SP-SM [4], and three fine-grained soils, SC-SM [2], CH [3] and Indian Head till (IHT) [4]. The testing results suggested the envelopes of RSS of the investigated unsaturated soils are linear with respect to net normal stress (σ_n - u_a) at any level of matric suction (u_a – u_w). The linear RSS envelopes of unsaturated soils can be described using Eq. 1. Fig. 1 shows three typical stress-strain curves of unsaturated soils corresponding to different net normal stresses under a constant matric suction. The figure also shows how $c_{r,a}$ and ϕ_r^a can be derived from the test results. Essentially, this approach is consistent with the well-known concepts of Mohr-Coulomb envelope in which $c_{r,a}$ represents the intercept while ϕ_r^a represents the slope of the envelope.

$$\tau_r = c_{r,a} + (\sigma_n - u_a)\tan\phi_r^a \qquad (1)$$

where τ_r is the residual shear strength of unsaturated soils, $(\sigma_n - u_a)$ is the net normal stress, $c_{r,a}$ is the apparent residual cohesion, ϕ_r^a is the residual friction angle with respect to net normal stress.

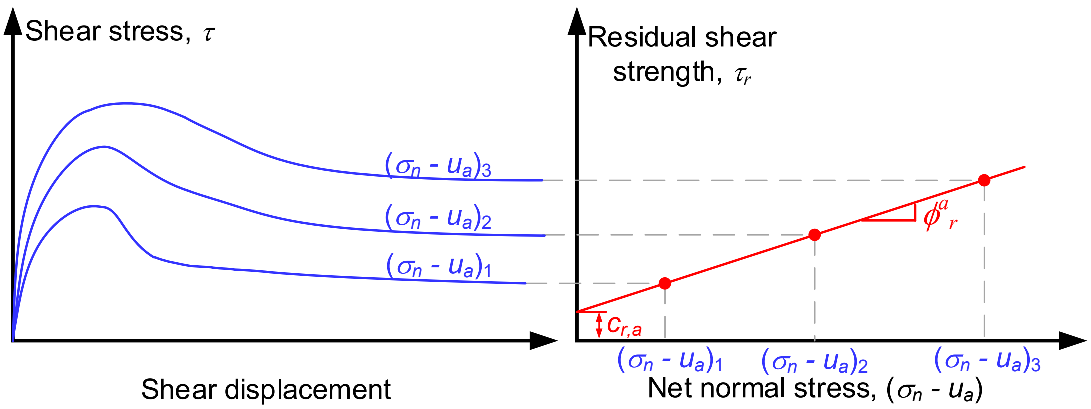


Figure 1: Mohr-Coulomb envelope of residual shear strength of unsaturated soils with respect to net normal stress under a constant matric suction.

The values of $c_{r,a}$ and ϕ_r^a of the studied soils at different suction levels are summarized in Table 1. Fig. 2 compares the values of RSS of unsaturated soils measured from the suction-controlled ring shear tests with their calculated values using Eq. 1 and parameters in Table 1. The comparison suggests Eq. 1 can be used for interpretation and as well as extended for prediction of the RSS of unsaturated soils. However, as shown in Table 1, both $c_{r,a}$ and ϕ_r^a vary with matric suction, which suggests the matric suction can have a significant influence on the values of $c_{r,a}$ and ϕ_r^a. Due to this reason, for the formulation of a prediction model for the RSS of unsaturated soils, the variations of $c_{r,a}$ and ϕ_r^a with matric suction should be investigated, respectively.

Table 1: Results of suction-controlled ring shear tests.

SM [2]			SC-SM [2]			CH [3]			SP-SM [4]			IHT [4]		
(u_a-u_w) (kPa)	$c_{r,a}$ (kPa)	ϕ_r^a (°)	(u_a-u_w) (kPa)	$c_{r,a}$ (kPa)	ϕ_r^a (°)	(u_a-u_w) (kPa)	$c_{r,a}$ (kPa)	ϕ_r^a (°)	(u_a-u_w) (kPa)	$c_{r,a}$ (kPa)	ϕ_r^a (°)	(u_a-u_w) (kPa)	$c_{r,a}$ (kPa)	ϕ_r^a (°)
0	0	34	0	15	22	25	8	6	0	0	23	0	0	25
25	11	44	25	23	29	50	12	7	2.5	0	32	25	6	27
50	41	36	50	34	27	100	19	9	7.5	1	26	100	28	27
75	51	50	100	55	23	200	47	5	10	3	23	250	79	26
100	88	36				300	62	8						

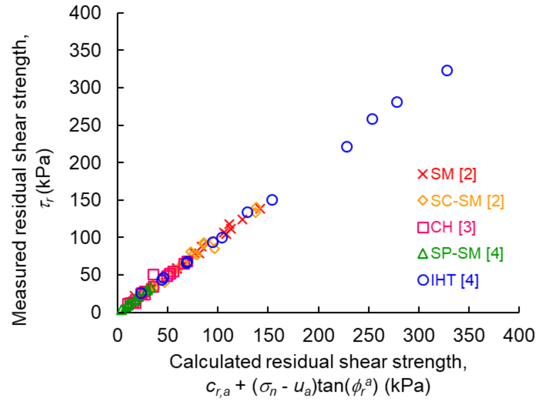


Figure 2: Comparisons between the measured and calculated residual shear strength using the modified Mohr-Coulomb equation.

Prediction of $c_{r,a}$

The variations of $c_{r,a}$ with matric suction of different unsaturated soils were summarized in Fig. 3. Strong linear correlations can be found between $c_{r,a}$ and $(u_a - u_w)$ for all the five studied unsaturated soils. Therefore, $c_{r,a}$ of unsaturated soils can be predicted using the equation below.

$$c_{r,a} = c'_r + (u_a - u_w)\tan\phi_r^b \tag{2}$$

where $(u_a - u_w)$ is the matric suction, c'_r is the effective residual cohesion of soils in saturated state, ϕ_r^b is the residual friction angle with respect to matric suction.

Typically, the friction angle with respect to matric suction for the peak or critical shear strength is considered to decrease gradually with increasing matric suction associated with the reduction of water menisci area. As shown in Fig. 3, however, ϕ_r^b is a constant within the investigated suction range. The mechanisms of the different behaviors of ϕ_r^b and the friction angle with respect to matric suction for peak/critical shear strength are still unclear. Such a behavior may be attributed to the changes in the soil structure and water phase within shear zone under large shear deformation.

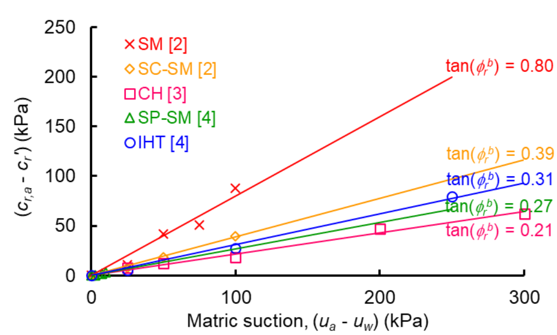


Figure 3: Variations of $c_{r,a}$ with matric suction.

Prediction of ϕ_r^a

The variations of ϕ_r^a with degree of saturation of different unsaturated soils were summarized in Fig. 4. The ϕ_r^a values of the five studied unsaturated soils exhibit a similar variation trend (i.e., ϕ_r^a increases from the effective residual friction angle (ϕ'_r) at first with desaturation and then decreases to ϕ'_r when the degree of saturation is lower than a certain value). This behavior suggests that the variation of ϕ_r^a during desaturation is influenced by the combined effects of suction and degree of saturation. A linear relationship (i.e., Eq. 3) is assumed in this study for the prediction of ϕ_r^a. In the second item of Eq. 3, $(u_a - u_w)f(S)$ represents the combined effects of matric suction and degree of saturation, where $f(S)$ is a function of degree of saturation explaining the contribution of matric suction toward the mechanical behaviors of unsaturated soils undergoing large shear deformation.

$$\tan\phi_r^a = \tan\phi'_r + \alpha_R(u_a - u_w)f(S) \tag{3}$$

where ϕ'_r is the effective residual friction angle of soils in saturated state, S is the degree of saturation, α_R is a material parameter.

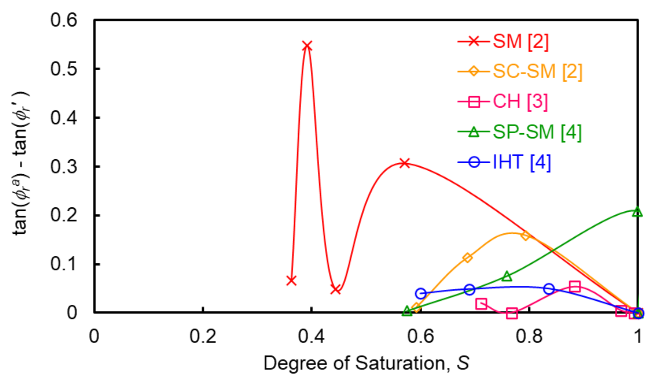


Figure 4: Variations of ϕ_r^a with effective degree of saturation.

The mechanical behaviors of unsaturated soils investigated in this study are mainly influenced by the capillary water, since all the gathered suction-controlled ring shear tests were conducted extending the axis translation technique. For such a scenario, the contribution of matric suction toward the mechanical behaviors of unsaturated soils is typically interpreted by using the concept of effective degree of saturation, $(S - S_r)/(1 - S_r)$, where S is the degree of saturation corresponding to a certain suction and S_r is the residual degree of saturation. S and S_r can be determined from the soil-water characteristic curve (SWCC). When unsaturated soils are sheared to the residual state, however, the microstructure of soils within shear zone experiences significant changes in comparison to the soils in initial state prior to shearing, which include destruction of original soil particle structure, crushing of coarse grains, rupture of aggregates, reorientation of clayey particles, and loss of water menisci. Thus, the S and S_r in the concept of effective degree of saturation used for interpreting the RSS of unsaturated soils should be determined based on the SWCC information consistent with the ring shear test results. However, such a SWCC cannot be measured reliably using the presently available experimental technique. Therefore, an alternative method was proposed in this study based on the conventional SWCC measured using unsaturated compacted specimens that were not subjected to shearing for interpreting the contribution of matric suction toward the mechanical behaviors of unsaturated soils undergoing large shear deformation, $f(S)$. As shown in Fig. 5, the local degree of saturation within the shear zone reduces after unsaturated soils experience large shear deformation. This behavior can be attributed to several factors, such as the dilation and the rupture of water menisci caused by continuous movements and rotations of soil particles. The lost capillary water cannot contribute to the RSS of unsaturated soils. Thus, $f(S)$ can be defined using Eq. 4. In this definition, $(1 - S_r)$ represents the total available volume of capillary water, $(S - S_r)$ represents the available volume of capillary water at a certain suction in initial state prior to shearing and $(S - S_L - S_r)$ represents the remainder volume of capillary water at a certain suction after a large shear deformation.

$$f(S) = \frac{S - S_L - S_r}{1 - S_r} \tag{4}$$

where S_L is the loss of degree of saturation within shear zone caused by large shear displacement, S is the degree of saturation in initial state prior to shearing that is determined using the conventional SWCC, S_r is the residual degree of saturation determined from the conventional SWCC.

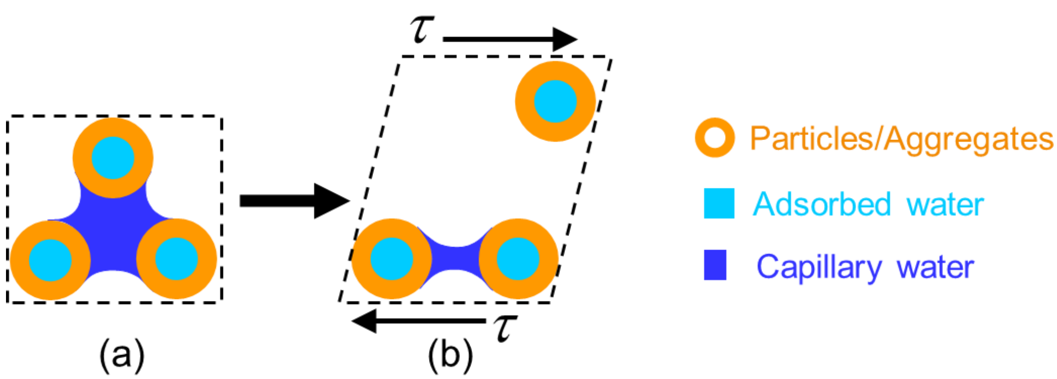


Figure 5: Diagram of microstructure changes of unsaturated soils under large shear deformation: (a) initial state prior to shearing; (b) after large shear deformation.

A valid definition of S_L providing a theoretical justification is difficult due to the limitations of present state-of-the-art experimental techniques for measuring the local degree of saturation within shear zone during shearing. Due to this reason, S_L is expressed as a nonlinear function of degree of saturation based on certain assumptions in this study considering several basic characteristics of S_L. The degree of saturation within shear zone will not be lost during shearing when the specimen is in saturated state (i.e., $S_L = 0$ when $S = 1$). During desaturation, more capillary water will exist as separate water menisci with decreasing degree of saturation, which tend to be ruptured and drained out from shear zone under large shear deformation. As a result,

S_L will increase with decreasing degree of saturation. However, once the degree of saturation is lower than a certain value, less capillary water will exist as separate water menisci since most of capillary water is drained out. Thus, S_L will decrease with a further decrease in degree of saturation. Finally, no capillary water is considered to exist when the degree of saturation is lower than S_r; thus, no capillary water can be lost (i.e., $S_L = 0$ when $S < S_r$). Considering these characteristics, S_L is assumed to be a nonlinear function of degree of saturation (Eq. 5). The shape of the assumed S_L function is shown in Fig. 6.

$$S_L = \text{the smaller value between } \beta_R \left(1 - \frac{S - S_r}{1 - S_r}\right) \ln\left(1 - \frac{S - S_r}{1 - S_r}\right)^{-1} \text{ and } (S - S_r) \tag{5}$$

where β_R is the material parameter, S and S_r are the degree of saturation, and the residual degree of saturation that are respectively determined based on the information from conventional SWCC.

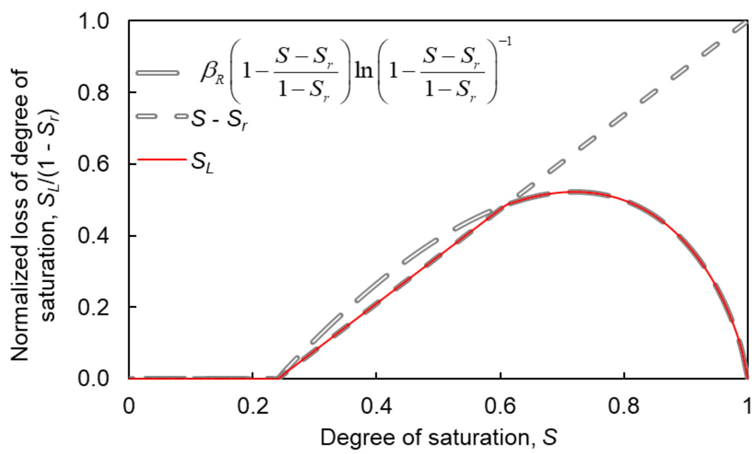


Figure 6: Definition of S_L.

Prediction model for residual shear strength of unsaturated soils

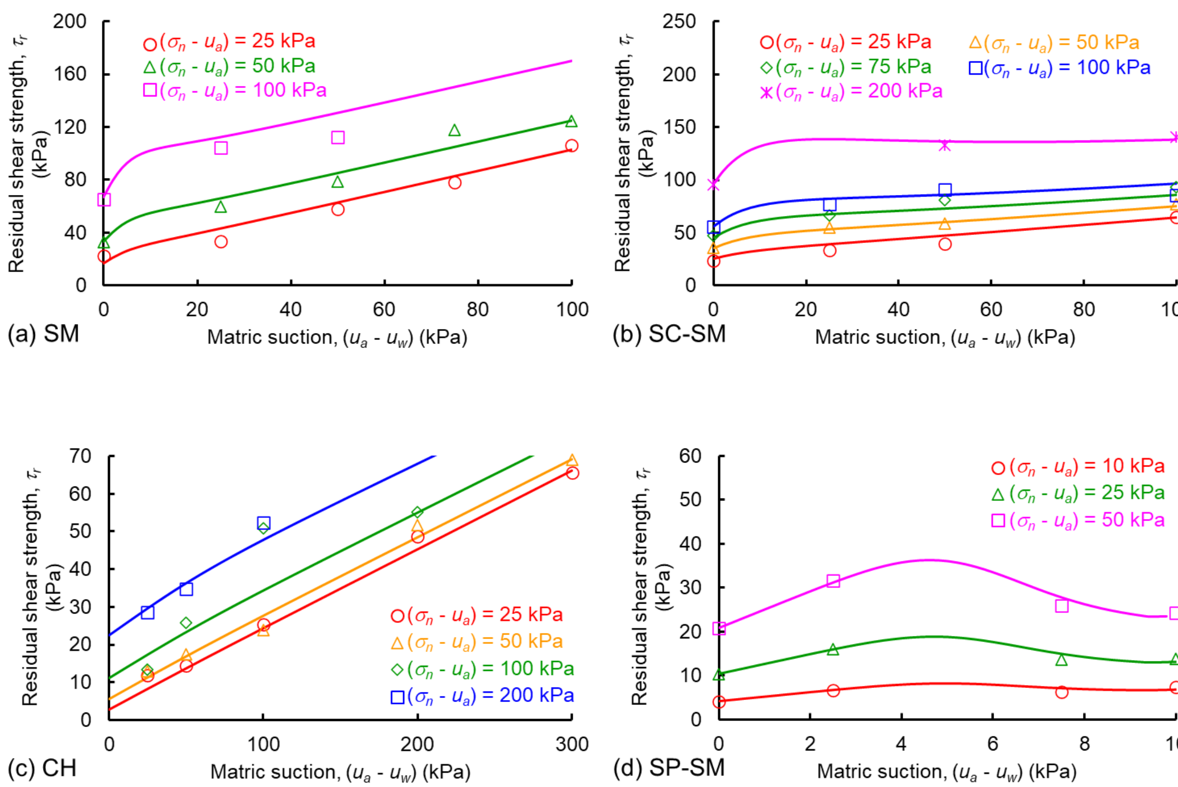
Combining Eqs. 1 through 5, a model can be formulated (Eq. 6) for predicting the RSS of different types of unsaturated soils. In this model, three material parameters (ϕ_r^b, α_R and β_R) are required. The values of ϕ_r^b can be determined by best-fitting the relationship between $c_{r,a}$ and $(u_a - u_w)$ obtained from suction-controlled ring shear tests using Eq. 2. The values of α_R and β_R can be determined by best-fitting the relationship between $\tan(\phi_r^a)$ and $(u_a - u_w)$ obtained from suction-controlled ring shear tests using Eqs. 3-5. In addition, the information of the saturated RSS parameters (i.e., c'_r and ϕ'_r) and SWCC are also required, which can be determined by using drained ring shear tests on saturated specimens and SWCC measurement tests (e.g., pressure plate test), respectively.

$$\tau_r = c_r' + (u_a - u_w)tan\phi_r^b + (\sigma_n - u_a)\left[tan\phi_r' + \alpha_R(u_a - u_w)\frac{max\left\{0, S - S_r - \beta_R\left(1 - \frac{S - S_r}{1 - S_r}\right)ln\left(1 - \frac{S - S_r}{1 - S_r}\right)^{-1}\right\}}{1 - S_r}\right] \quad (6)$$

The results of several suction-controlled ring shear tests based on vapor equilibrium technique [5] suggested suction can still have influence on the RSS of unsaturated soils when $S < S_r$. However, the results of suction-controlled ring shear tests based on the axis translation technique can only highlight the RSS behaviors of unsaturated soils within the suction range where $S \geq S_r$. Therefore, the prediction model proposed in this study for RSS of unsaturated soils can only be used for the suction range where $S \geq S_r$.

Validation of the proposed model

The capability of the proposed prediction model for the RSS of unsaturated soils can be validated by predicting the RSS envelopes of unsaturated soils obtained from the five sets of suction-controlled ring shear tests gathered from the literature [2-4]. Fig. 7 compares the predicted RSS envelopes of unsaturated soils with the measured results from the suction-controlled ring shear tests. The comparisons suggest the proposed model can provide a reasonable prediction of the RSS of a wide range of unsaturated soils varying from coarse- to fine-grained soils.



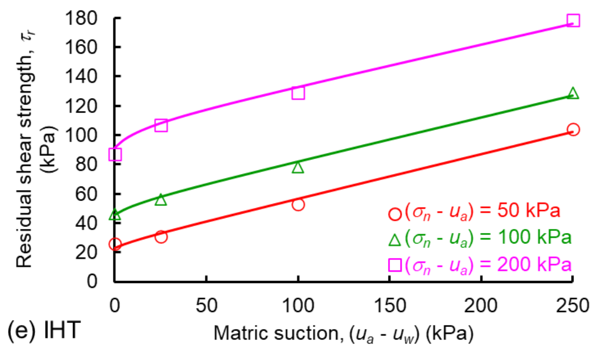


Figure 7: Comparison between the measured and predicted envelopes of residual shear strength of five unsaturated soils.

Summary and conclusions

A simple model is formulated in this study for predicting the residual shear strength (RSS) for a wide range of unsaturated soils varying from coarse- to fine-grained soils. Five sets of suction-controlled ring shear tests based on the axis translation technique available in the literature were gathered as a database for proposing the model. The materials include two coarse-grained soils (i.e., SM and SP-SM) and three fine-grained soils (i.e., SC-SM, CH and Indian Head till (IHT)). The RSS behaviors of unsaturated soils are discussed based on the suction-controlled ring shear tests results. The RSS envelopes of unsaturated soils are linear with respect to net normal stress (σ_n - u_a) at any level of matric suction (u_a – u_w). The linear RSS envelopes of unsaturated soils can be described using Eq. 1. This approach is consistent with the well-known concepts of Mohr-Coulomb envelope in which the apparent residual cohesion ($c_{r,a}$) represents the intercept while the residual friction angle with respect to net normal stress (ϕ_r^a) represents the slope of the envelope. In addition, the experimental results suggest the matric suction has significant influence on $c_{r,a}$ and ϕ_r^a.

Strong linear correlations can be found between $c_{r,a}$ and (u_a – u_w) for all the five studied unsaturated soils. Thus, a linear function (Eq. 2) can be used to predict $c_{r,a}$ based on the suction value. ϕ_r^a increases from the effective residual friction angle (ϕ'_r) at first with desaturation and then decreases to ϕ'_r when the degree of saturation is lower than a certain value, which suggests ϕ_r^a is influenced by the combined effects of suction and degree of saturation. The traditional concept of effective degree of saturation cannot be used for interpreting the RSS of unsaturated soils undergoing large shear deformation due to the difficulties in the measurement of SWCC consistent with the ring shear test results. Therefore, an alternative method (Eq. 3-5) was proposed for interpreting the ϕ_r^a of unsaturated soils based on the loss of degree of saturation within shear zone caused by large shear displacement (S_L) and the conventional SWCC measured using unsaturated compacted specimens not subjected to shearing. S_L was assumed to be a nonlinear function of degree of saturation considering several basic characteristics of S_L. Finally, a simple model was formulated combining Eqs. 1 through 5 for predicting the RSS for different types of unsaturated soils. The prediction capability of the proposed model was validated by comparing the predicted RSS envelopes of unsaturated soils with the measured results of the gathered suction-controlled ring shear tests.

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