INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

Role of the soil-water characteristic curve in unsaturated soil mechanics

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ABSTRACT: The soil-water characteristic curve, SWCC, has been referred to as the key to the implementation of unsaturated soil mechanics into geotechnical engineering practice. Methodologies proposed within soil physics are re-examined in this paper and applied for the geotechnical engineering discipline. Each of the volume-mass variables needs to be taken into consideration when estimating unsaturated soil property functions in geotechnical engineering. The shrinkage curve can be used in conjunction with the (gravimetric) water content versus soil suction relationship (i.e., w-SWCC) to provide a more complete understanding of unsaturated soil behaviour. In this way it is possible to separate the effects of overall volume change from changes in the degree of saturation during the drying of a soil. The estimation of various unsaturated soil property functions, USPFs, requires the use of the saturated soil properties along with mathematical algorithms related to one or more of the volume-mass SWCCs. This paper provides a state-of-the-art synthesis related to the estimation of hydraulic USPFs for geotechnical engineering applications.

1 INTRODUCTION

Saturated soil mechanics defines the engineering description of a two-phase system comprised of solids particles and an incompressible pore fluid (water). The engineering application of saturated soil mechanics started around the 1930s (Terzaghi, 1943; Taylor, 1948). Saturated soil behavior was related to changes in the effective stress state of the soil (i.e., the difference between total stresses, σ_x, σ_y. σ_z, and pore-water pressure, u_w). Saturated soil mechanics principles were applied to commonly encountered geotechnical engineering problems (e.g., seepage, volume change and shear strength related problems). Success related to the application of saturated soil mechanics was strongly related to the ability to measure saturated soil property constants which could be related to stress state and used to describe various physical processes.

Unsaturated soil mechanics emerged later and involved the engineering description of a multi-phase system that had two fluid phases (i.e., water and air) as well as the unique behavior of the contractile skin (i.e., the air-water interphase) and the soil solids. The description of unsaturated soil behavior needed to embrace a wide range of degrees of saturation. The degree of saturation of the soil ranged from saturation in the capillary zone to a discontinuous water phase in the dry soil zone.

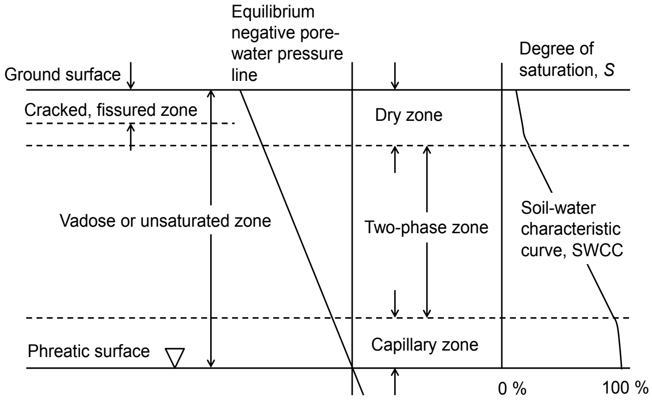

Figure 1. Definition of the vadose or unsaturated soil zone.

Figure 1 defines the subdivisions of the vadose (or unsaturated zone) above the phreatic surface. The zone above the phreatic line has been referred to as the vadose zone in various disciplines. The vadose zone is defined as “that part of the earth between the land surface and the water table (i.e., atmospheric pressure)”, (United States Geological Survey). The definition for the vadose zone is the same as that commonly used for the “unsaturated soil zone” in unsaturated soil mechanics. This is an important definition to retain because of the manner in which the soil-water characteristic curve is measured in the laboratory and used in engineering practice. Disciplines such as hydrology define the unsaturated zone as starting where air becomes present in the pore-water (Freeze & Cherry, 1979); however, this is not the definition commonly used in unsaturated soil mechanics.

The ground surface is subjected to a moisture flux that is continually changing in response to weather conditions. The ground surface forms a new and complex type of boundary condition within soil mechanics. Figure 2 shows the components that combine to give rise to net infiltration or percolation. The relative magnitudes of the upward moisture flux (i.e., evaporation and evapo-transpiration) and downward moisture flux (i.e., precipitation) perturb the equilibrium (negative) pore-water pressure profile giving rise to a trumpet shape for soil suction variations over time.

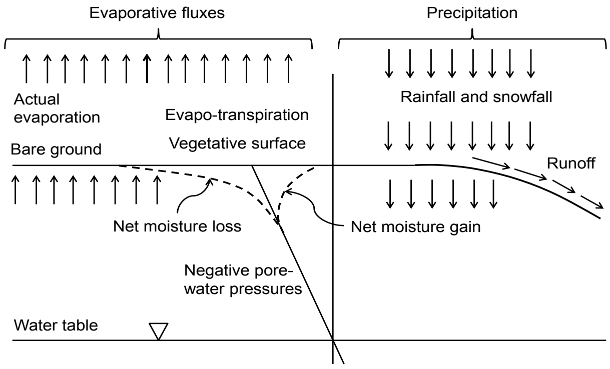

Figure 2. Moisture flux components associated with the calculation of net moisture flux at ground surface.

Figure 2 shows how negative pore-water pressures in the vadose zone are altered by imposed weather conditions. Negative pore-water pressures can also vary due to fluctuations in the groundwater table. Common to numerous geotechnical engineering problems (e.g., soil cover designs) is the assessment of the water balance at the ground surface as a function of time. Ground surface water balance can be assessed based on weather station and soils information data. Published results associated with field case histories would suggest that, in general, the water balance components near ground surface can be adequately predicted for geotechnical engineering purposes (Wilson et al., 1994; Tran et al., 2014; Fredlund et al., 2012).

An unsaturated soil can have three distinct zones of saturation. The wide potential variation in degree of saturation has led to the need to define the soil properties in terms of nonlinear "unsaturated soil property functions, USPFs". The soil properties are mathematical functions that make the engineering analysis (e.g., numerical modeling) the solution of one or more nonlinear partial differential equations. Figure 3 shows what can be referred to as the three pillars of unsaturated soil mechanics; namely, i.) engineering protocols, ii.) laboratory testing procedures and iii.) numerical modeling techniques. Laboratory testing procedures need to either directly or indirectly provide information on the physical soil properties while numerical modeling techniques simulate physical processes.

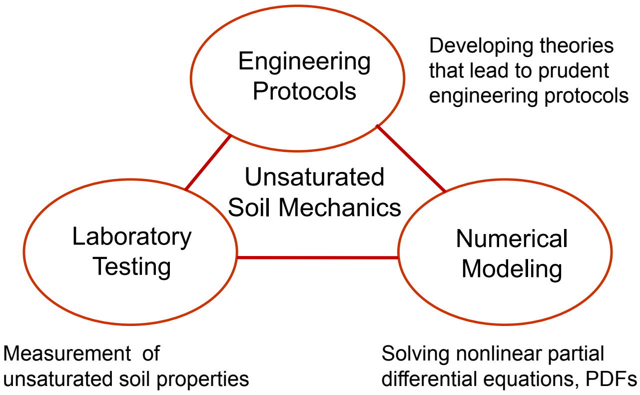

Figure 3. Pillars related to the implementation of unsaturated soil mechanics.

There are three classes of physical processes that have historically formed the core of saturated and unsaturated soil mechanics. The physical processes relate to: (i) flow and storage of water in a porous medium; (ii) shear strength of a particulate material; and (iii) volume change (i.e., compression, distortion and expansion) of soils. An important part of saturated soil mechanics has revolved around the laboratory measurement of appropriate saturated soil property constants. Bringing unsaturated soil mechanics into routine geotechnical engineering practice has involved the estimation of unsaturated soil property function, USPFs, based on the laboratory measurement of soil-water characteristic curves, SWCCs. Estimation procedures for USPFs have been proposed for all application areas of unsaturated soil mechanics. While the above-mentioned processes form the core of soil mechanics, the geotechnical engineer needs to be aware that numerous other physical processes also play an important role when considering near-ground-surface phenomena (e.g., heat flow, air flow, phase changes, chemical transport). Engineering "protocols" for the practice of unsaturated soil mechanics have emerged over time and are being applied worldwide in a relatively consistent manner. The practice of unsaturated soil mechanics has been closely related to the use of the SWCC and for this reason it is prudent to summarize, as far as is possible, aspects that appear to "define" the discipline of unsaturated soil mechanics.

The determination (i.e., measurement and/or estimation) of the soil-water characteristic curve, SWCC, along with saturated soil properties has provided a means of moving unsaturated soil mechanics into routine engineering practice. Consequently, a thorough understanding of the SWCC becomes pivotal to the practice of unsaturated soil mechanics.

The primary objectives of this paper are to (i) describe the primary role played by the "soil-water characteristic curve", SWCC in estimating unsaturated soil property functions; (ii) describe the secondary role played by the "shrinkage curve", SC, in refining the estimation of unsaturated soil property functions; and (iii) illustrate how the SWCC should be analyzed for the computation of unsaturated soil property functions. The application of the SWCC in this paper is limited to the consideration of water flow in geotechnical engineering problems. The manuscript is written in the form of a state-of-the-art and state-of-the-practice paper based on years of research into unsaturated soil behaviour as well as years of experience in putting unsaturated soil mechanics into routine engineering practice. The paper constitutes a generalized template for the analysis of water flow and storage in unsaturated soils.

This paper illustrates the pathway that can be taken from the start of addressing an unsaturated seepage problem to the end of quantifying the required unsaturated soil property functions, USPFs. An assumption is made in this paper that the soil under consideration will be tested in the laboratory to measure the entire soil-water characteristic curve, SWCC. If the soil undergoes volume change as soil suction is increased, then the shrinkage curve, SC, must also be obtained through measurement or estimation.

The following assumptions and conditions are imposed.

(1) The entire SWCC extends over a soil suction range from about 0.1 kPa to 1,000,000 kPa, and it is defined using laboratory measurements.
(2) The drying SWCC is measured over the entire soil suction range.
(3) An assessment is made as to whether or not there is significant volume change as soil suction is increased.
(4) The shrinkage curve for the soil needs to be measured when changes in overall volume need to be separated from changes in degree of saturation as soil suction is increased.
(5) Procedures to address the effects of hysteresis should be assessed in light of the engineering problem being addressed.
(6) All analytical steps are explained in going from the laboratory measurements of the gravimetric water content SWCC and shrinkage curve, (i.e., *w*-SWCC and SC) to the calculation of the unsaturated soil property functions, USPFs (i.e., permeability function and water storage function).
(7) Assumptions associated with all aspects of the analysis are explained along with the significance of each assumption.

The scope of this paper is limited to the consideration of one class of geotechnical engineering problems; namely, the flow of water through a saturated-unsaturated soil system. The soil continuum is assumed to not have significant secondary structure such as fractures, cracks and fissures. The effects of hysteresis and complex stress paths are addressed in a surficial manner due to lack of space.

There are other techniques that have been used to obtain an estimation of the soil-water characteristic curve for a soil such as (i) database mining, (M. Fredlund, 1997); and (ii) calculations based on the grain-size distribution curves (M. Fredlund et al., 2002). There are engineering situations where these methodologies can be used; however, only methodologies based on laboratory measurements of the SWCC and SC are given consideration in this paper. Numerous empirical equations have been proposed to characterize or best-fit SWCCs. There is no attempt in this paper to compare the proposed empirical equations or address the limitations associated with various SWCC equations. Rather, an attempt is made to illustrate the use of a set of mathematical equations that cover the entire range of soil suctions for all soil types. A single pathway is followed in the determination of USPFs for seepage problems in geotechnical engineering practice.

2 HISTORICAL CONTEXT FOR THE DEVELOPMENT OF UNSATURATED SOIL MECHANICS

The study of unsaturated soil behavior has historically emerged on two fronts; one within soil physics (and related agricultural disciplines), and the other within soil mechanics. Figure 4 illustrates how the physics of unsaturated soil behavior found its expression in different applications areas within soil physics and soil mechanics.

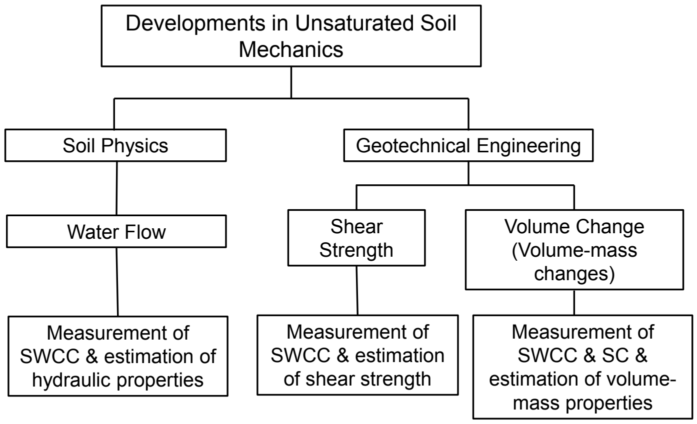

Figure 4. Historical developments for unsaturated soil behaviour.

Some of the early contributors in soil physics are as follows: Haines (1927); Richards (1931); Edelefsen and Anderson (1943); Childs & Collis-George (1950); Klute (1952); Burdine (1952); Gardner (1961); Brooks & Corey (1964); Topp & Miller (1966); van Genuchten (1980); and Mualem (1976). Some of the early research contributors in geotechnical engineering can be listed as follows: Croney (1952); Jennings & Knight (1957); Bishop, Alpan, Donald & Blight (1960); Aitchison (1961); Bishop &Blight (1963); Jennings (1969); Barden (1965); Lytton &Woodburn (1973); Fredlund &Morgenstern (1976); Escario (1980); and Ho &Fredlund (1982). The above-mentioned list is by no means complete; however, the list of references illustrates the different time periods over which basic unsaturated soils research was conducted.

2.1 *Soil physics research in unsaturated soil behavior*

Extensive research studies took place within soil physics in the early 1900s. These studies mainly focused on moisture movement through soils in the vadose zone. Soil suction was introduced as the energy head driving water flow. Diffusive type models were proposed within soil physics with the coefficient of diffusion combining the ease of water flow (i.e., coefficient of permeability) with water storage capacity (i.e., water storage). The diffusive type formulation required the input of a single diffusive soil property (Richards, 1931).

Little attention was given to overall volume change of the soil; in fact, formulations generally assumed that the soil structure was rigid. The earliest research into unsaturated soil behavior was undertaken in soil physics and mainly applied in agriculture-related applications. The water storage capacity of near-ground-surface soils was of interest from the standpoint of plant growth (Buckman and Brady, 1960). The amount of water storage in the soil was measured in terms of volumetric water content, θ_w, and presented as a function of the negative pore-water pressure (or suction) in the soil.

2.2 *Geotechnical engineering research in unsaturated soil behavior*

Geotechnical engineers were interested in water flow through unsaturated soils as well as the shear strength and volume change behaviour of unsaturated soils. Shear strength and volume change problems were common in geotechnical engineering practice and there was a desire for improved engineered solutions.

Several research conferences were held in the 1960s and there was an attempt to transfer research formulations related to physical processes studied in soil physics into the emerging field of unsaturated soil mechanics. The proceedings of the 1961 London, England conference was titled, "*Pore Pressure and Suction in Soils*". In 1965 a symposium-in-print titled "*Moisture Equilibria and Moisture Changes in Soils beneath Covered Areas*" was edited by G. Aitchison (Aitchison, 1965) and presented to the First International Conference on Expansive Soils at Texas A & M, College Station, TX, in 1965. These conferences mainly focused on moisture movement and swelling clay problems encountered in geotechnical engineering. A series of research studies were undertaken at Imperial College, London, in the 1950s and 1960s that identified some of the key aspects of unsaturated soil behaviour and set the course for subsequent research studies in many countries around the world. The research studies at Imperial College were mainly under the supervision of Professor Allan W. Bishop and one of the primary contributing researchers was Geoffrey E. Blight in whose honour this lecture is given.

A number of difficulties became apparent as mathematical formulations were transferred from soil physics into unsaturated soil mechanics. Geotechnical engineers were accustomed to using soil mechanics' principles for water flow through soils. For example, hydraulic head (i.e., $Y + u_w/\gamma_w$ where Y = elevation head, u_w = pore-water pressure, and γ_w = unit weight of water) was used as the driving potential for saturated soils and the desire was to maintain a similar formulation for water flow above the phreatic surface. There was opposition to the use soil suction and volumetric water content as driving potentials for water flow in the unsaturated soil region.

Geotechnical engineers also viewed water flow problems in terms of steady state and transient type analyses. As a result, the hydraulic properties for unsaturated soils were viewed in terms of two independent soil property functions; namely, i.) the water permeability function, k_w, and ii.) the water storage function, m_2^w. These two material property functions served different roles when considering solutions of interest in geotechnical engineering. Each of the hydraulic property functions involved different mathematical operations for their assessment even though both properties were closely related to the soil-water characteristic curve, SWCC.

One of the practical engineering problems facing geotechnical engineers was the prediction of heave in swelling soils. The need to predict total heave in swelling soils provided the primary impetus for a series of international research conferences from 1964 to 1992. The conferences were directed towards better understanding expansive soil behavior. Formulations of moisture movement in soil physics were based on the assumption that the elemental volume under consideration was rigid and therefore, did not undergo volume change. Consequently, it was important to re-derive a more fitting partial differential equation for use in modeling moisture flow and volume change for geotechnical engineering applications.

Studies in soil physics gave little or no consideration to shear strength and volume change problems. The primary problems of interest within soil physics were related to the water storage and its depletion near the ground surface. There was little research consideration given to shear strength and volume change of unsaturated soils. Consideration of shear strength and volume change problems in unsaturated soils meant that the stress state variables for an unsaturated soil needed to be proposed, verified and agreed upon (Fredlund, 2006).

2.3 *Defining soil behaviour in terms of the stress state of an unsaturated soil*

Probably the first research document mentioning the need for two independent stress state variables when describing physical processes in unsaturated soils was published by Biot (1941). Biot derived the theory of consolidation in terms of a partial differential equation for an unsaturated soil. The derivation gave consideration to a pore fluid of water which contained air bubbles. Even for this special case of an unsaturated soil, the use of two independent stress state variables was proposed. Dakshanamurthy et al. (1984) showed that the 1941 Biot theory of consolidation derivation could also be applied to an unsaturated soil with continuous air and water phases. Coleman (1962) also suggested the use of independent stress state variables when considering the volume change behavior of an unsaturated soil.

Bishop (1959) proposed an effective stress equation that related the total normal stresses to the matric suction through use of an empirical χ soil parameter. An extensive experimental study by Bishop and Blight (1963) on the shear strength of several different soils showed that the difference between the shear strength of a saturated soil and an unsaturated soil was related to the degree of saturation of the soil at failure. Figure 5 shows a plot of the test results from four different soils that were tested. These early results show that the researchers were aware that the unsaturated soil shear strength of a soil was related to the degree of saturation of the soils; however, the relationship was not linear, suggesting that the relationship might be somewhat more complex. It was concluded that it is the "stress paths of two components, $(\sigma - u_a)$ and $(u_a - u_w)$ which have to be taken into account." The difference between u_a and u_w was referred to as the matric suction.

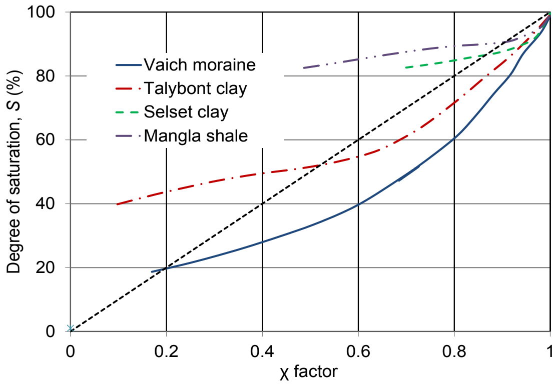

Figure 5. Relationship between shear strength parameters measured on several saturated and unsaturated soils (modified from Bishop and Blight, 1963).

In 1965, Blight presented results on the volume change behavior of an unsaturated soil. The behavior of the unsaturated soil was illustrated in terms of three-dimensional diagrams with the horizontal axes comprised of two independent stress variables and the ordinate being volumetric strain (Figure 6).

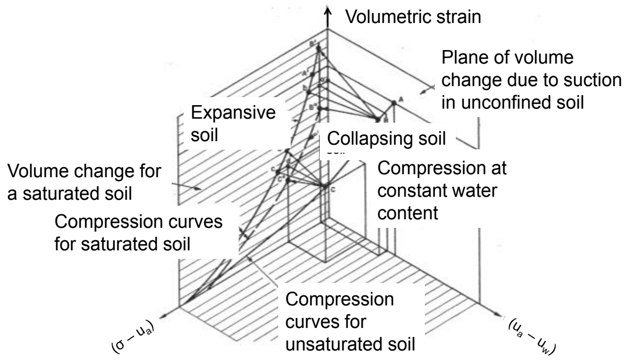

Figure 6. Void ratio versus stress state variables when following stress paths for a stable-structured soil (after Blight, 1961).

Stress paths corresponding to various physical processes were illustrated and it was noted that understanding the collapse phenomena, in particular, turned out to prove challenging. Consideration of other stress paths revealed further stress path dependence (Fredlund & Morgenstern, 1976). Matyas and Radhakrishna (1968) studied volume change and degree of saturation changes while performing isotropic and K_o triaxial tests on a mixture of 80% flint powder and 20% kaolin. The results were presented in terms of two independent stress components, $(\sigma - u_a)$ and $(u_a - u_w)$. Several other studies have been undertaken related to the study of changes in volume-mass behavior of unsaturated soils. In general, the results were presented in terms of the stress paths followed in each of the tests (Barden et al., 1969). In each of the above cases the measured constitutive surfaces proved to be stress path dependent (Pham & Fredlund, 2011).

Fredlund & Morgenstern (1977) presented a theoretical stress equilibrium analysis justifying the components of an unsaturated soil. The analysis was presented within the context of continuum mechanics principles and concluded that two independent stress tensors consisting of $(\sigma - u_a)$ and $(u_a - u_w)$ were best suited for the interpretation and application of unsaturated soil behavior. Research to-date would appear to indicate that two independent stress tensors form an adequate stress state description for physical processes involving unsaturated soils. It is possible that simplifications representing the stress state of an unsaturated soil may prove to be adequate for describing constitutive behaviour in some cases but in general, two independent stress tensors would appear to be the more rigorous and generally acceptable description.

2.4 *Discoveries in soil physics related to the soil-water characteristic curve, SWCC*

The soil-water characteristic curve, SWCC, has been central to water movement modeling from the early studies in soil physics (Klute, 1965). In 1986, Klute identified a series of "findings" related to the SWCC that form important reference points for its usage in both soil physics and geotechnical engineering. Some of these "finding" are paraphrased below. Words in quotation marks are taken directly from Klute (1986).

(1) The relationship between soil water content and soil suction (i.e., herein referred to as the soil-water characteristic curve, SWCC, but also referred to as the water retention curve, WRC, in the research literature) relates a "capacity factor" (i.e., the amount of water in the soil), and "the energy state", (i.e., suction stress state) for the soil-water.

(2) The SWCC "is a fundamental part of the characterization of the hydraulic properties of a soil".

(3) The "energy per unit volume", is equivalent to force per unit area or pressure (i.e., soil suction).

(4) Water content can "be expressed on a weight, volume, or degree of saturation basis". "For analysis of water flow in soil profiles, the volume basis is most useful". While the volume basis for water content has been used in soil physics, other designations for the amount of water need to be given consideration for geotechnical engineering.

(5) The SWCC is "primarily dependent upon the – particle-size distribution of the soil and the structure" (Croney et al., 1958).

(6) The SWCC is hysteretic (i.e., water content at a given suction on the wetting curve is less than that along the drying curve), (Haines, 1927; Topp and Miller, 1966).

(7) The branches of the SWCC are defined as follows: i.) the initial drying curve starting at a degree of saturation of 100%, ii.) the main wetting curve measured after the soil has been dried to near residual water content conditions, iii.) the main drying curve which may have 10 to 20% entrapped air due to incomplete saturation upon wetting, and iv.) there are an infinite set of scanning curves inside the drying and wetting bounding curves. Figure 7 shows a typical set of (initial and main) drying and wetting SWCCs. The compilation of SWCCs defines the boundaries for the relationships between the amount of water in the soil and soil suction.

(8) The SWCC for "rigid structure soils show constant water content up to the air-entry value" for the soil.

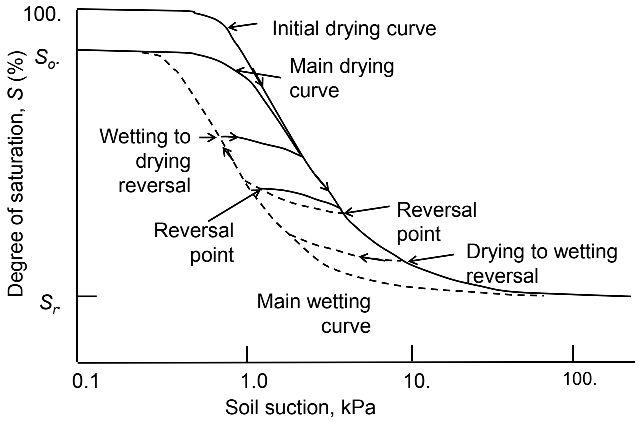

Figure 7. Family of drying and wetting curves of degree of saturation versus soil suction for a rigid structured soil (modified from Klute, 1986).

(9) If the soil structure deforms (i.e., shrinking or swelling), the water content can decrease as soil suction increases without reaching the entry of air, (i.e., no change in degree of saturation).

(10) In some cases, only the drying SWCC is required for modeling water flow. In other cases, only the wetting curve is required. Modeling flow across the ground surface requires the incorporation of hysteretic behavior (Mualem, 1976).

(11) The SWCC is a constitutive material property that relates a stress state (e.g., matric suction and total suction) to the amount of water in a soil.

(12) Water flow occurs in response to gradients of total hydraulic head (i.e., pore-water pressure head plus elevation head).

(13) SWCCs should be measured on "undisturbed core samples". In some cases, remolded and disturbed material may be all that is available for the measurement of the SWCC.

It should be noted that there is an arbitrary division between what is referred to as the "low suction range" and the "high suction range". The arbitrary division occurs at around 1500 kPa; an arbitrary division that is established mainly on the basis of the highest air-entry ceramic disk that can be manufactured. Consequently, soil suction is defined in terms of matric suction from zero to 1500 kPa, while soil suction is defined in terms of total suction in the range from 1500 to 1,000,000 kPa.

The arbitrary division between the low and high suction ranges results in two different components of soil suction being used when measuring the SWCC of a soil. The change in the soil suction components along the SWCC would appear to mainly be related to the influence of osmotic suction. However, the inconsistency in the use of two suction components does not appear to created significant difficulties in applications in both soil physics and geotechnical engineering (Fredlund, 2015).

2.5 *Historical contributions from soil physics' research for use of the SWCC*

The historical context for unsaturated soil mechanics reveals that key complimentary findings have emerged within the disciplines of soil physics and geotechnical engineering. Geotechnical engineers have benefited much from the research undertaken in soil physics. At the same time, lessons have been learned about the importance of carefully examining assumptions associated with mathematical formulations (i.e., definition of material properties and physical processes) when moving from one discipline to another.

There has been fruitful and overlapping research undertaken within soil physics and geotechnical engineering. The most significant area of over-lapping research is related to the use of the soil-water characteristic curve, SWCC, for the estimation of unsaturated soil property functions, USPFs. Much of the remainder of this paper is devoted to describing the application of the SWCC in solving typical geotechnical engineering problems. It has been observed that the application of the SWCC requires careful consideration of the assumptions made as part of the estimation procedures used in calculating the USPFs.

The basic measurement of water content versus soil suction, (i.e., soil-water characteristic curve, SWCC, or water retention curve, WRC) originated within the soil physics discipline. The emergence of unsaturated soil mechanics witnessed an attempt to transfer the experience and technology developed in soil physics into geotechnical engineering. The soil physics discipline historically presented unsaturated soil behaviour in terms of a plot of volumetric water content versus soil suction. The use of volumetric water content to designate the amount of water in the soil appears to have been influenced by agriculture related issues (e.g., water storage for plant growth). Overall volume changes related to suction changes were of secondary interest and were not taken into consideration in soil physics applications.

3 APPARATUSES FOR MEASURING THE DRYING SWCC

Laboratory tests in soil physics focused mainly on the measurement of the drying soil-water characteristic curve. Most laboratory test equipment for the agriculture-related disciplines was designed to simultaneously measure the SWCC on several soil specimens that were placed on a single high air-entry ceramic disk (Fredlund & Rahardjo, 1993; Soil Moisture Equipment Corporation, 1983). Following the establishment of equilibrium suction conditions (using the axis-translation technique), each soil specimen was removed from the pressure plate and its water content was measured. Exceptions to the conventional apparatuses were the Tempe cell and the volumetric pressure plate cell which tested a single soil specimen.

The requirements for measuring the SWCC in geotechnical engineering are different from those in soil physics. The amount of volume change experienced by the soil specimen as soil suction is increased is important to the interpretation of the laboratory result (Figure 8). The amount of volume change that occurs as soil suction is increased is one of the first factors requiring a decision when measuring the SWCC for geotechnical engineering applications. It is assumed that the specific gravity of the soil is known prior to analyzing the SWCCs.

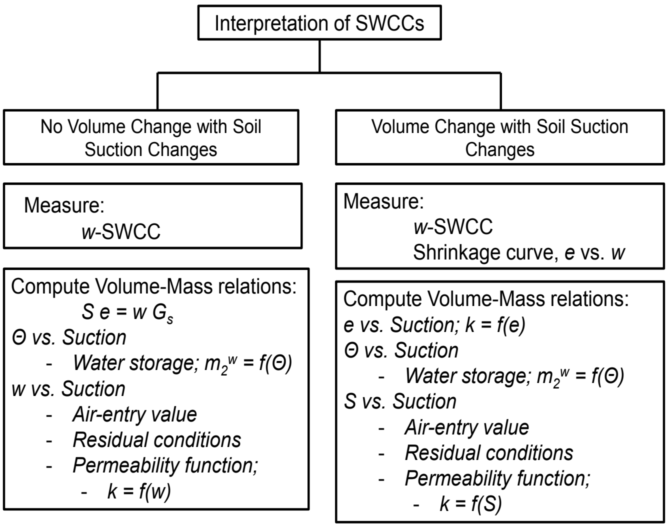

Figure 8. The influence of overall volume change on the volume-mass versus suction relations.

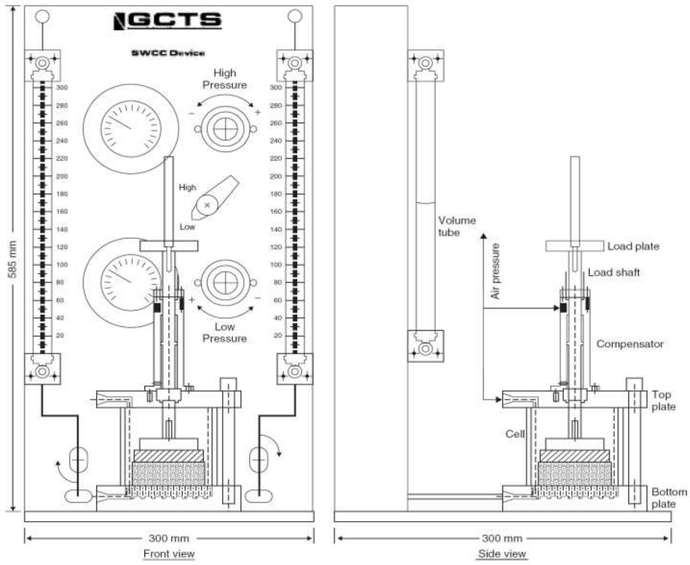

Figure 9. Front and side views of GCTS Pressure Plate cell (Courtesy of GCTS, AZ).

The use of the basic SWCC results (i.e., gravimetric water content versus soil suction) for the estimation of the unsaturated soil property functions is sufficient provided there is limited volume change as soil suction is increased. However, overall volume change as soil suction is increased can significantly affect the calculation of unsaturated soil property functions. Various volume-mass versus soil suction relations need to be computed when the soil undergoes volume change as soil suction is changed.

Apparatuses developed in soil physics have been historically used to measure the SWCC in geotechnical engineering applications. However, over time, various apparatuses have been developed that are better suited to meet the requirements for geotechnical engineering applications.

The equipment developed by GCTS (Figure 9) is typical of a number of K_o type apparatuses that better satisfy the needs within geotechnical engineering.

A single soil specimen is tested in the more recent pressure plate devices without disturbing the soil between suction applications. The pressure plate apparatus is used to establish matric suction in the lower suction range (i.e., suctions less than 1500 kPa). Vapor pressure equilibrium conditions have been used to determine water content conditions for the high suction range (i.e., total suctions greater than 1500 kPa). Figure 10 shows a typical set of test results measured on a silt soil (Pham, 2000). It is possible to measure both the drying and the wetting SWCC; however, it is of primary importance to measure the drying curve in geotechnical engineering applications in order to obtain the most accurate interpretation of the SWCC for the calculation of the unsaturated soil property functions. It is also possible to directly measure overall volume change of the soil specimen under K_o conditions as long as the soil specimen does not separate from the confining metal ring.

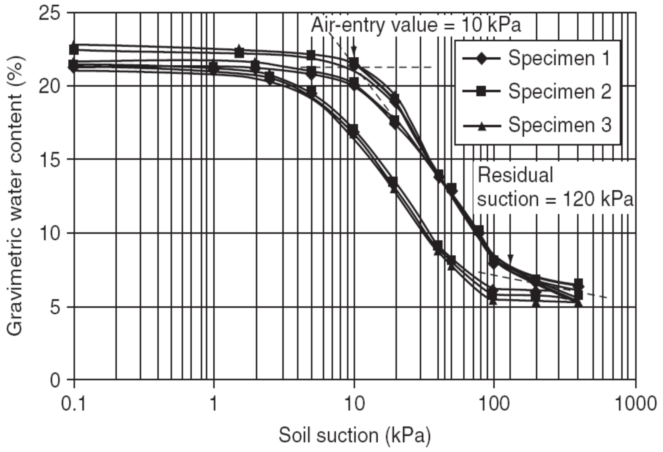

Figure 10. Drying and wetting SWCCs measured on a silt soil using the GCTS Pressure Plate apparatus (Pham 2002).

An accurate interpretation of the SWCC involves the separation of desaturation of a soil specimen from the effects of overall volume change (i.e., changes in void ratio). It is possible to develop triaxial testing equipment that can simultaneously measure both volume change and water content change; however, it is more economical and expedient to independently measure the shrinkage curve, SC, for the soil (i.e., void ratio versus gravimetric water content), and then use this information for the interpretation of the gravimetric water content SWCC. This paper focuses on a detailed analysis of the main drying SWCC. The analysis for the main wetting SWCC is largely outside the scope of this paper although the wetting curve is given some consideration.

3.1 *Steps to estimating unsaturated soil property functions, USPFs*

A dataset for an example soil is generated to illustrate the use of the soil suction stress variable (i.e., matric suction and total suction) to define the water flow and water storage constitutive properties for an unsaturated soil. It is known that unsaturated soil properties are also affected by total applied stresses; however, the assumption is made that soil suction is the dominant stress variable defining hydraulic soil properties. This assumption appears to be adequate for most geotechnical engineering applications.

The steps involved in estimating unsaturated soil property functions, USPFs, through use of the SWCC are listed below. Subsequent sections in this paper address details pertaining to each of the steps in the analysis. The steps of the analysis are outlined for the case where the soil undergoes some volume change as suction is increased. Shortcuts in the analysis are identified for situations where the soil does not undergo significant volume change as suction is increased. The effect of volume change can generally be ignored when dealing with sand and coarse-grained soils.

Step 1: Take note of the assumptions and limitations associated with the analysis for the estimation of unsaturated soil properties.

(1) Consideration is only given to the analysis of the drying (or desorption) measured data (i.e., gravimetric water content versus soil suction).
(2) A decision should be made at the start of the testing program regarding whether or not volume changes related to suction changes needs to be taken into account.
(3) The specific gravity, G_s, of the soil must have been measured.
(4) The following aspects of the analysis apply for soils that behave in a unimodal manner. Bimodal behavior is considered to be outside the scope of this paper.
(5) Terminology: The term "soil suction" or "suction" refers to matric suction in the range of suctions up to 1500 kPa. The term "soil suction" or "suction" refers to total suction in the range of suctions between 1500 kPa and one million kPa. Matric and total suctions are plotted using a continuous logarithmic scale throughout the respective suction ranges.

Step 2: Measure gravimetric water content versus soil suction (w-SWCC) over the entire suction range.

(1) Determine the initial volume-mass properties of the wetted w-SWCC soil specimen.

(2) Combine and plot the data from a low suction value to a high suction value (i.e., starting at 0.01 or 0.1 kPa to a limiting value of 1,000,000 kPa).

(3) Obtain the fitting parameters for an equation that fits the data over the entire range of suction values. The Fredlund & Xing (1994) equation is used in this paper; however, other suitable equations can be used provided it fits the data over the entire suction range. It is sometimes difficult to get a close fit of the data points in cases where the soil undergoes large volume changes during drying. In such cases, it is possible to use a bimodal form of the Fredlund-Xing equation to fit the w-SWCC data points.

Step 3: Measure (or estimate) the shrinkage characteristics of the soil.

(1) The soil specimen is commonly prepared as a saturated paste, placed within a ring and allowed to slowly dry. It is also possible to test undisturbed soil specimens which are initially saturated. Measurements of the specimen volume are taken using micrometer calipers (Figure 11). The soil mass is also measured. Other methods have also been used to measure the volume of the soil specimens during drying (Liu & Buzzi, O., 2014; Liu et al., 2016).

(2) In some cases it is possible to estimate the shrinkage curve with sufficient accuracy.

(3) Obtain the fitting parameters for the shrinkage curve. The M. Fredlund (2000) equation is used in this paper.

(4) The shrinkage curve does not need to be measured when the soil does not undergo volume change upon suction changes.

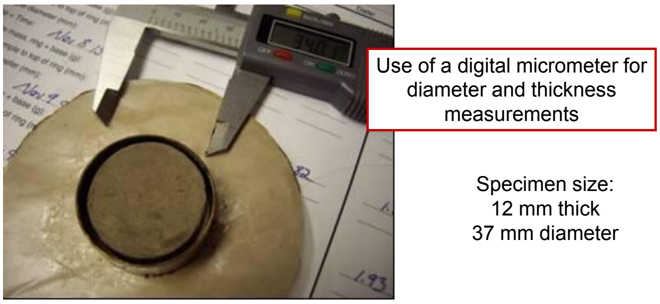

Figure 11: Measurement of the "Shrinkage Curve", SC using micrometer calipers.

Step 4: Calculate and plot the void ratio versus suction relationship.

(1) The combination of the SWCC and the shrinkage curve allow the separation between volume change and desaturation associated with suction increases.

(2) The void ratio plot is not required when the soil does not undergo significant volume change as suction changes.

Step 5: Calculate and plot the degree of saturation SWCC, (S-SWCC).

(1) Obtain the fitting parameters for the degree of saturation SWCC.

(2) Calculate the true Air-Entry Value, AEV, from the S-SWCC.

(3) Calculate the Residual Point (i.e., residual suction and residual degree of saturation).

Step 6: Calculate and plot the volumetric water content SWCC.

(1) The volumetric water content SWCC is computed from the gravimetric water content SWCC and the shrinkage curve.

Step 7: Calculate the Unsaturated Soil Property Functions, USPFs.

(1) Calculate the permeability function with respect to void ratio.

(2) Calculate the relative permeability function with respect to soil suction starting from the true air-entry value.

(3) Calculate the water storage function from the volumetric water content SWCC.

(4) Calculate other USPFs (e.g., shear strength function, thermal property functions).

Each of the above-mentioned steps is more clearly detailed in the following sections.

3.2 *Measurement of the gravimetric water content SWCC*

The classification properties of the soil, along with the specific gravity, G_s, should be determined prior to commencing the w-SWCC test. The initial volume-mass properties (e.g., water content, void ratio, and degree of saturation) are required as part of the analysis of volume-mass versus soil suction data. A record should be kept of the initial state of the soil specimen (i.e., slurry, compacted, or undisturbed). The soil specimen is initially allowed free access to water and the soil moves towards saturated conditions.

Data for an "artificial clayey silt" soil is used throughout this paper to illustrate the steps involved in analyzing laboratory measurements. The specific gravity of the soil is 2.68, the initial gravimetric water content is 31.5%, and the initial void ratio is 0.879. The initial degree of saturation is calculated to be 96.08% and the saturated coefficient of permeability is 2.0 x 10^{-6} m/s.

Pressure plate apparatuses with high air-entry ceramic disks have become the most common means of measuring the SWCC in the suction range up to 1500 kPa. Pressure plate apparatuses apply matric suctions using the axis-translation technique.

Vapor pressure equalization methodologies are used to establish total suction environments in the range in excess of 1500 kPa. The suggested methodologies used in soil physics (Klute, 1965) have essentially become the accepted procedures used in geotechnical engineering except for a few exceptions (Fredlund & Rahardjo, 1993). The methodology involves wetting the soil to zero suction at the beginning of the test and then applying a small suction in the range between 0.1 and 1.0 kPa to establish the initial water content corresponding to the start of the *w*-SWCC.

Figure 12(a) shows a typical dataset for the basic drying *w*-SWCC. The diamond-shaped symbols indicate matric suction data obtained from a pressure plate apparatus. The square symbols indicate total suction data obtained by using a vapor pressure equalization procedure (e.g., WP4-T chilled-mirror apparatus; Decagon; 2009).

The entire dataset from a fraction of 1 kPa to one million kPa can be best-fit with a mathematical function that extends over the entire range. Data for the drying SWCC generally takes on the form of a sigmoidal mathematical function (Figure 12b) provided the soil specimen does not undergo excessive volume change. It should be noted that any mathematical equation can be used for the best-fit provided the data can be fit in the low suction range while ending at zero water content at one million kPa.

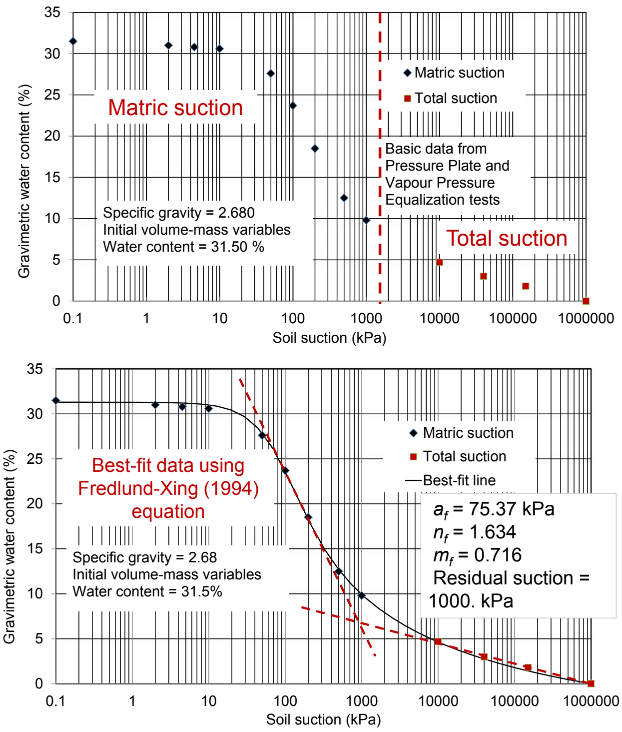

Figure 12. (a) Matric and total suction data for the *w*-SWCC; (b) soil suction data with best-fit Fredlund-Xing (1994) SWCC function.

3.2.1 *Plotting and best-fitting the gravimetric water content SWCC (w-SWCC)*

The Fredlund and Xing (1994) equation (with the applied correction factor for zero water content at one million kPa of suction), is used in this paper to illustrate the steps involved in analyzing gravimetric water content versus soil suction data.

The best-fit regression analysis using the Fredlund and Xing (1994) equation yields 4 fitting parameters that provide a mathematical representation of the *w*-SWCC. The residual suction value can be approximated using the empirical construction procedure shown in Figure 12b. The initial fitting of the gravimetric water content SWCC should not be confused with later best-fitting of the degree of saturation SWCC (*S*-SWCC). It should be noted that the fitting parameters for *w*-SWCC can have slightly different meaning from those obtained for the degree of saturation SWCC, (*S*-SWCC).

Other empirical equations such as that proposed by Pham and Fredlund (2011) can be used at this stage to provide a closer fit of measured data throughout the entire range of suction values. The important guideline is that the fitted equation should closely adhere to the measured (*w*-SWCC) laboratory data.

Following is the proposed Fredlund & Xing (1994) equation applied to the gravimetric water content versus soil suction data for the example soil.

$$w(\psi) = \frac{w_s C(\psi)}{\left(\ln\left(\exp(1) + \left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}} \qquad (1)$$

where $w(\psi)$ = water content at any soil suction, ψ; a_f = fitting parameter near the inflection point on the *w*-SWCC; n_f = fitting parameter related to the maximum rate of gravimetric water content change; m_f = fitting parameter related to the curvature near residual gravimetric water content conditions; ψ_r = suction near residual conditions of the soil, and $C(\psi)$ = correction factor directing the *w*-SWCC towards a suction of 10^6 kPa at zero water content, written as:

$$C(\psi) = 1 - \frac{\ln\left(1 + \psi/\psi_r\right)}{\ln\left(1 + 10^6/\psi_r\right)}$$

Inserting the correction factor, $C(\psi)$, into equation (1) yields the following form for the Fredlund & Xing (1994) equation.

$$w(\psi) = \frac{w_s\left(1 - \ln\left(1 + \psi/\psi_r\right)/\ln\left(1 + 10^6/\psi_r\right)\right)}{\left(\ln\left(\exp(1) + \left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}} \qquad (2)$$

There is close agreement between the Fredlund-Xing (1994) equation and the laboratory data shown in Figure 12b; however, this is not always the case. The fitting parameters are: residual suction, ψ_r = 1000 kPa; a_f = 75.37 kPa; n_f = 1.634 and m_f = 0.716. The *w*-SWCC shows that the mathematical function starts to bend downward in the vicinity of 20 kPa; however, it should be noted that this bend in the curve does not indicate the air-entry value of the soil (except in the case where there is no volume change as soil suction is increased). The "true" air-entry value will be later computed from the degree of saturation SWCC.

When there is considerable overall volume change in the low suction range, it might not be possible to obtain a close fit of the measured data points using the Fredlund-Xing (1994) equation or possibly any other commonly proposed fitting equation. It is important to have a close fit of the data on the *w*-SWCC when the data is combined with the shrinkage curve data for the calculation of other volume-mass SWCCs. Consideration might be given to fitting the *w*-SWCC data using bi-modal forms of the equation or the proposed Pham-Fredlund (2008) equation.

Changes in gravimetric water content versus suction does not allow for the separation of the two independent processes that occur as the soil dries; namely, water content changes associated with volume changes and water content changes associated with changes in degree of saturation. The following basic volume-mass equation illustrates the relationship amongst the volume-mass properties of a soil.

$$S e = G_s w \tag{3}$$

where S is the degree of saturation; and e is the void ratio. Incremental differentiation of the basic volume-mass relationship illustrates that the gravimetric water content change can be due to two processes (Fredlund & Rahardjo, 1993).

$$\Delta w = \frac{S_f \Delta e + e_f \Delta S}{G_s} \tag{4}$$

where the subscript '*f*' refers to the "final" volume-mass states. The separation of the volume change and degree of saturation change can be accomplished through use of a shrinkage curve.

3.3 *Measurement of the shrinkage curve*

Soil suction increases as a soil specimen dries from an initially wet condition (i.e., near zero suction) to a suction of one million kPa. Measurements of changes in mass and volume as the soil dries allows for the determination of the shrinkage curve, SC. Specimens for the shrinkage curve test should have similar initial volume-mass properties (i.e., water content and void ratio) to those used for the *w*-SWCC laboratory test. Figure 13 illustrates how the soil specimens for the *w*-SWCC test and the shrinkage test can be prepared to give similar initial (wet) conditions.

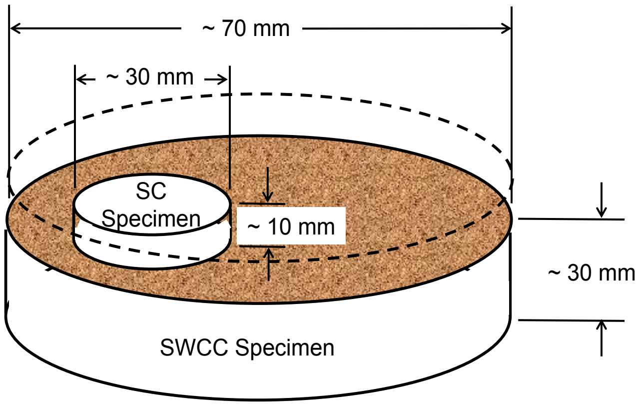

Figure 13. Establishing similar initial volume-mass conditions for the *w*-SWCC and SC tests.

Historical differences in the interpretation of SWCC data centers around an assumption related to the rigidity of the soil structure. The assumption is generally made in soil physics that the soil structure is rigid. Invoking the assumption of a rigid soil structure can introduce significant errors when calculating subsequent unsaturated soil properties functions for geotechnical engineering applications. It is important in geotechnical engineering to quantify the volume change characteristics of the soil with respect to changes in soil suction.

The measurement of the shrinkage curve for a soil provides data on the relationship between gravimetric water content and volume change (i.e., void ratio change) as soil suction is increased from essentially a zero value to one million kPa as shown in Figure 14. The shrinkage curve allows for the calculation of all volume-mass variables with respect to soil suction.

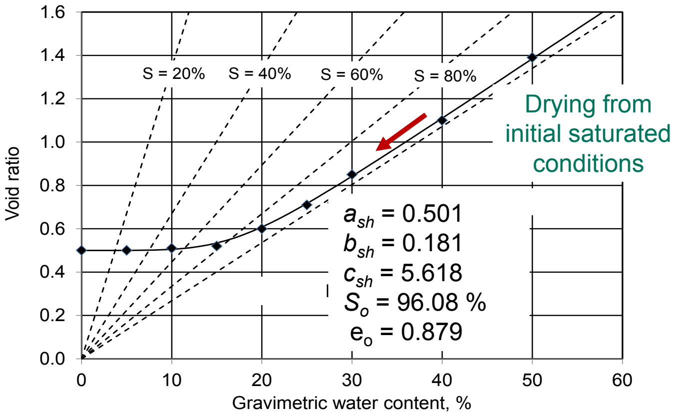

Figure 14. Variables associated with a laboratory shrinkage curve test.

A shrinkage curve equation proposed by M. Fredlund et al., (2002) can be used to best-fit the void ratio versus gravimetric water content drying curve.

$$e(w) = a_{sh}\left(\left(\frac{w}{b_{sh}}\right)^{c_{sh}} + 1\right)^{1/c_{sh}} \tag{5}$$

where a_{sh} is the minimum void ratio upon complete drying; b_{sh} is the variable related to the slope of the drying curve calculated as: $b_{sh} = (a_{sh} \times S_o)/G_s$, and c_{sh} = sharpness of curvature as the soil desaturates.

The best-fit parameters for the shrinkage curve for the example soil are: $a_{sh} = 0.501$, $b_{sh} = 0.181$, and $c_{sh} = 5.618$. The initial degree of saturation, S_o, was 96.08%. The a_{sh} parameter can be calculated from the shrinkage limit of the soil. The b_{sh} parameter is also closely related to the a_{sh} parameter through the above equation. An incompressible soil has a high c_{sh} value (e.g., $c_{sh} = 50$). In general, the shrinkage curve is easy to estimate or measure.

3.4 *Void ratio versus soil suction relationship*

The volume change associated with suction change can be calculated by combining the empirical equation for the shrinkage curve, SC, with the gravimetric water content SWCC (*w*-SWCC) (Fredlund and Zhang, 2013). The shrinkage curve relates void ratio changes to gravimetric water content changes as the soil dries and the *w*-SWCC relates gravimetric water content to soil suction. Substituting the *w*-SWCC into the shrinkage curve, SC, equation yields an equation for the void ratio (and overall volume changes), as soil suction increases during drying. The resulting equation for void ratio versus soil suction is shown in Eq. [6] and graphically presented in Figure 15. All variables shown in Eq. [6] have been previously defined.

$$e(\psi)=a_{sh}\left(\left(\frac{w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{b_{sh}\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}}\right)^{c_{sh}}+1\right)^{1/c_{sh}} \qquad (6)$$

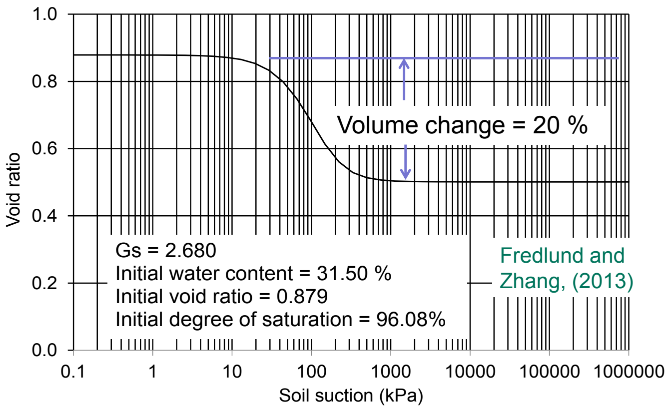

Figure 15. Void ratio versus soil suction showing volume change.

The void ratio plot shows that volume change for the clayey silt starts at a suction near to 20 kPa and continues to occur up to a suction of about 400 kPa. The degree of saturation plot later shows the separation of the desaturation process from the volume change process as the soil dries. Desaturation of the soil takes place when the applied suction exceeds the air-entry value of the soil. There may also be a zone of applied matric suctions where both volume change and desaturation are occurring simultaneously.

3.5 *Designation of water content of a soil*

The amount of water in a soil can be quantified in terms of (i) gravimetric water content, *w*; (ii) volumetric water content, θ_w; or (iii) degree of saturation, *S*. It is important that volumetric water content be defined with respect to the "instantaneous" total volume when the soil changes volume as soil suction is changed. The amount of water in a soil has been historically quantified in the soil physics discipline as the volume of water referenced to the initial total volume of the soil. In geotechnical engineering, the volumetric water content should be defined as the volume of water referenced to the "instantaneous" total volume of the soil when overall volume change occurs. It is noteworthy that in either profession, water content is commonly measured as gravimetric water content in the laboratory and the differences in the terminology revolve around the manner in which the data is reduced and applied.

During early developments in unsaturated soil mechanics there appeared to be little concern with regard to the manner in which the amount of water in a soil was designated. Differences in the designation of water content later became of increased importance as geotechnical engineers increasingly used the soil-water characteristic curve for the estimation of unsaturated soil property functions, USPFs.

3.6 *Volumetric water content versus soil suction relationship*

The water storage function for a soil is calculated as the change in volumetric water content with respect to a change in suction. It is important to first compute the volumetric water content versus suction relationship. The "instantaneous" volumetric water content of the soil, θ_i, can be calculated based on the *w*-SWCC and the shrinkage curve, SC, as shown in equation (7).

$$\theta_i(\psi)=\frac{G_s w(\psi)}{1+e(\psi)} \qquad (7)$$

where $w(\psi)$ = gravimetric water content written as a function of soil suction (*w*-SWCC), and $e(\psi)$ = void ratio as a function of soil suction, ψ.

The Fredlund & Xing (1994) *w*-SWCC equation (i.e., equation (2)), can be substituted into equation (7) along with the M. Fredlund (2000) shrinkage curve equation (i.e., equation (5)) to give an equation for volumetric water content written in terms of basic laboratory data (i.e., equation (8)).

$$\theta_i(\psi)=\frac{G_s w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}\left(1+a_{sh}\left(\left(\frac{w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{b_{sh}\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}}\right)^{c_{sh}}+1\right)^{1/c_{sh}}\right)} \tag{8}$$

It should be noted that other proposed functional forms for *w*-SWCC and SC could also be substituted into equation (7).

The θ_i-SWCC for the sample soil data is plotted as a continuous function in Figure 16. The calculation of the water storage function, m_2^w, is determined from the θ_i-SWCC and is shown later in this paper.

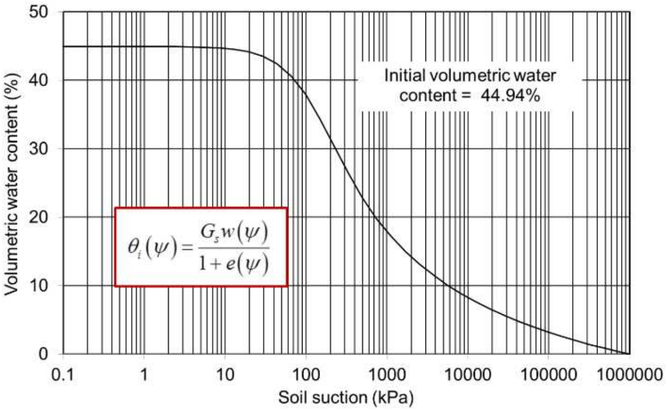

Figure 16: Volumetric water content, θ_i, versus soil suction.

3.7 *Degree of saturation versus suction relationship based on w-SWCC and SC data*

The degree of saturation versus suction relationship can be computed by satisfying the basic volume-mass relationship (i.e., equation (3)) along with the *w*-SWCC and the SC relationships as shown in equation (9).

$$S(\psi)=\frac{G_s w(\psi)}{e(w(\psi))} \tag{9}$$

where $w(\psi)$ = Fredlund-Xing (1994) equation for the *w*-SWCC or any other equation that fits the laboratory data, and $e(\psi)$ = void ratio written as a function of soil suction, ψ.

Substituting the *w*-SWCC equation and the SC equation into equation [9] allows the calculation of degree of saturation versus suction data points. In other words, the *S*-SWCC data points are based on the original *w*-SWCC and SC laboratory data sets. The resulting *S*-SWCC equation has the following form.

$$S(\psi)=\frac{G_s w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{a_{sh}\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}\left(\left(\frac{w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{b_{sh}\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}}\right)^{c_{sh}}+1\right)^{1/c_{sh}}} \tag{10}$$

All variables shown in equation (10) are defined in terms of the original best-fit variables associated with the *w*-SWCC and the SC. Figure 17 shows the computed degree of saturation SWCC data points calculated at arbitrarily selected soil suction values (i.e., equally spaced points on a semi-logarithmic plot).

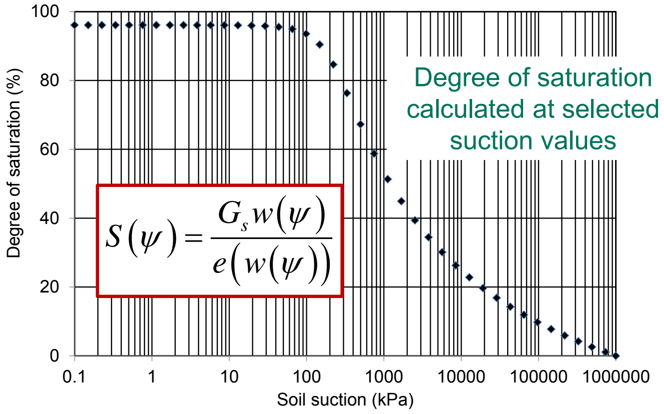

Figure 17. Data points for the degree of saturation SWCC calculated from *w*-SWCC and the SC.

3.7.1 *Plotting and best-fitting the gravimetric water content SWCC (w-SWCC)*

The calculated degree of saturation versus suction data points can once again be best-fit using the Fredlund-Xing (1994) equation. The best-fit provides a new set of fitting parameters for further analysis of the degree of saturation SWCC. The Fredlund-Xing (1994) equation used to fit the degree of saturation SWCC has the same form as previously shown in Eq. [2] for the *w*-SWCC data points. However, the fitting parameters can have a slightly different meaning in the case where volume change occurs as soil suction is increased.

The new fitting parameters are designated as a_{fs}, n_{fs}, m_{fs} and ψ_{rs}. These fitting parameters have the same general meaning as those defined for the *w*-SWCC in the case when the soil does not undergo volume change as soil suction is increased. The '*s*' subscript means the fitting parameters refer to the *S*-SWCC.

$$S(\psi)=\frac{S_0\left(1-\ln\left(1+\psi/\psi_{rs}\right)/\ln\left(1+10^6/\psi_{rs}\right)\right)}{\left(\ln\left(\exp(1)+\left(\psi/a_{fs}\right)^{n_{fs}}\right)\right)^{m_{fs}}} \tag{11}$$

where: $S(\psi)$ is the degree of saturation at any soil suction, S_o is the initial degree of saturation which is generally quite close to 100%; a_{fs} is the fitting parameter near the inflection point on the *S*-SWCC; n_{fs} is the fitting parameter related to the maximum rate of degree of saturation change; m_{fs} is the fitting parameter related to the curvature near residual degree of saturation conditions, and ψ_{rs} is the suction near residual conditions of the soil.

The correction factor directing the *S*-SWCC towards a suction of 10^6 kPa at zero water content is included in equation (11).

The residual soil suction can first be estimated using the empirical procedure previously shown in Figure 12(b). The residual suction, ψ_{rs}, is estimated to be 2000 kPa. The starting degree of saturation is the same as previously calculated from the SC (i.e., $S_o = 96.08$ %).

The next step involves best-fitting the degree of saturation soil-water characteristic curve (*S*-SWCC) with the Fredlund & Xing (1994) equation (Figure 18). A close fit is generally possible over the entire soil suction range as long as the *S*-SWCC is unimodal in character. The new fitting parameters calculated for the *S*-SWCC are as follows: a_{fs} = 261.9 kPa, n_{fs} = 1.922, and m_{fs} = 0.519.

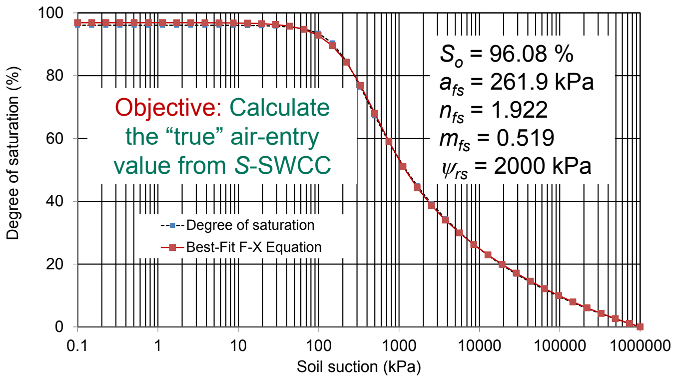

Figure 18. Degree of saturation versus soil suction with best-fit of the Fredlund & Xing (1994) equation.

The *S*-SWCC shows that there is a gradual downward bend in the *S*-SWCC relationship near a suction value of 100 kPa indicating an approximate value for the air-entry of the soil. While an approximate value can be estimated from the *S*-SWCC, it is also possible to determine a more precise and unique air-entry value.

3.8 *Analysis of the degree of saturation SWCC to obtain the air-entry value, AEV*

An analytical procedure was described by Zhang & Fredlund (2015) whereby a unique and reproducible value can be computed and designated as the "true" air-entry value of the soil. It should be noted that the calculated air-entry value is the result of an empirical construction that removes the curvature on the degree of saturation versus logarithm of suction plot. The fitting parameters for the *S*-SWCC can be used to compute the "true" air-entry value for the soil; however, the logarithm suction scale must first be transposed to an equivalent arithmetic scale.

The empirical construction associated with the determination of the air-entry value requires the calculation of the point of inflection on the suction scale. A transformed suction scale can be used for the differentiation step because it is difficult to calculate the correct inflection point directly on the plot of a semi-logarithm relationship for degree of saturation and suction. The logarithmic suction scale can be converted to an arithmetic scale, ξ, by using the following scale transformation.

$$\xi = \log_{10}(\psi) \tag{12}$$

The transformed degree of saturation SWCC takes on the following mathematical form shown in equation (13) and the plot of the transformed degree of saturation SWCC is graphically shown in Figure 19.

$$SS(\xi) = \frac{S_0\left(1 - \ln\left(1 + 10^{\xi}/\psi_{rs}\right)/\ln\left(1 + 10^{6}/\psi_{rs}\right)\right)}{\left(\ln\left(\exp(1) + \left(10^{\xi}/a_{fs}\right)^{n_{fs}}\right)\right)^{m_{fs}}} \tag{13}$$

where $SS(\xi)$ is the degree of saturation as a function of the transformed suction.

The remaining fitting parameters for the transformed degree of saturation equation are the same as calculated by equation (11).

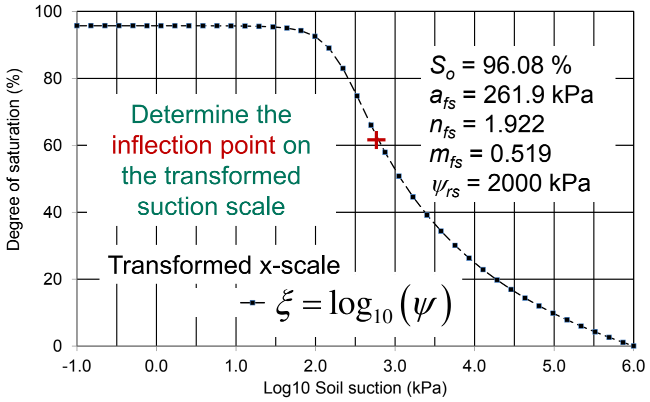

Figure 19: Degree of saturation versus Log10 soil suction.

The transformed scale for the *S*-SWCC can now be differentiated with respect to transformed suction to find the point corresponding to the maximum slope on the degree of saturation graph. The first derivative can be obtained using *Mathematica* or other comparable software (e.g., *Mathcad* or *MATLAB*). It is also possible to use *Mathematica* to write the second derivative of equation (13). The second derivative can be set to zero, giving rise to the transformed suction value corresponding to the inflection point on the degree of saturation function.

The degree of saturation at the inflection point can be computed by inserting the transformed suction at the inflection point into equation (13). The coordinates of the inflection point are ξ_i for the transformed suction and $SS(\xi_i)$ for the degree of saturation. A tangent line can be drawn through the inflection point using the first derivative of the *S*-SWCC as the slope. The line of tangency can be extended to cross a line passed through the initial degree of saturation. Details of the analysis are shown on Figure 20.

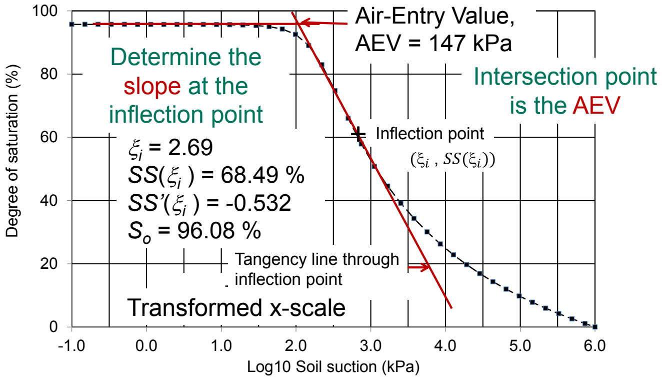

Figure 20. Definition of terms used on the substitution equation for the calculation of the "true" air-entry value.

Following are the definitions of variables used on the transformed suction plot for the computation of the "true" air-entry value for the soil.

S_o = initial degree of saturation at the start of the *S*-SWCC test; ξ = any *x*-coordinate along the transformed scale; ξ_i = *x*-coordinate at the inflection point on the transformed scale (i.e., transformed soil suction); $SS(\xi_i)$ = degree of saturation at the inflection point; $SS'(\xi_i)$ = first derivative of the transformed equation at the inflection point, and $TL(\xi_i)$ = equation for the line of tangency passing through the inflection point.

3.8.1 *Derivation of the "true" air-entry value equation*

The "true" air-entry value, AEV, corresponds to the intersection point between the horizontal line through the initial degree of saturation and the line of tangency through the inflection point (Zhang et al., 2015). The line of tangency through the inflection point is designated as $TL(\xi)$, and can be written as follows:

$$TL(\xi) = SS'(\xi_i)(\xi - \xi_i) + SS(\xi_i) \tag{14}$$

The variable $TL(\xi)$, can be set to the initial degree of saturation in order to calculate the air-entry value. Equation [14] is then solved for the suction on the transformed suction scale equal to the designated degree of saturation (i.e., the ξ value corresponding to the air-entry value, AEV). The derivation steps involved in going from the line of tangency to the determination of the air-entry value are as follows:

$$S_0 = SS'(\xi_i)\xi_{aev} - SS'(\xi_i)\xi_i + SS(\xi_i) \tag{15}$$

where ξ_{aev} is the x-coordinate or the air-entry value on the transformed suction scale at the intersection of the tangency line through the inflection point and a line through the initial degree of saturation.

Equation (15) can be solved for the air-entry value on the transformed suction scale, ξ_{aev}.

$$\xi_{aev} = \frac{SS'(\xi_i)\xi_i + SS(\xi_0) - SS(\xi_i)}{SS'(\xi_i)} \tag{16}$$

where $SS(\xi_0)$ is the degree of saturation at the start of the SWCC test (i.e., equal to S_o).

Equation (16) can be rearranged to the following form.

$$\xi_{aev} = \xi_i + \frac{S_0 - SS(\xi_i)}{SS'(\xi_i)} \tag{17}$$

Equation (17) can now be converted from the transformed suction scale to the original soil suction scale using transform equation (12) ($\xi = \mathrm{Log}_{10}(\psi)$).

$$\log_{10}(\psi_{aev}) = \xi_i + \frac{S_0 - SS(\xi_i)}{SS'(\xi_i)} \tag{18}$$

Therefore,

$$\psi_{aev} = 10^{\xi_i + \frac{S_0 - SS(\xi_i)}{SS'(\xi_i)}} \tag{19}$$

Equation (19) can be used to compute the "true" air-entry value of the soil using: (i) coordinates of the inflection point on the transformed suction scale plot; (ii) line of tangency through the inflection point; and (iii) degree of saturation at the start of the SWCC test. The end result of the above derivation is an empirical procedure that provides a way to calculate a unique value for the "true" air-entry value of a soil. For the artificial clayey silt soil, the "true" air-entry value is computed to be 147 kPa. The "true" air-entry value is used in the integration process when calculating the permeability function.

The information presented thus far is related to the preparation of the *w*-SWCC and the SC data for calculating the unsaturated soil property functions required for undertaking unsaturated soil mechanics simulations; in this case, saturated-unsaturated seepage modeling. The permeability function must be calculated when performing a steady-state seepage analysis and the water storage function must also be calculated when solving an unsteady-state or transient seepage problems.

3.9 *Application of the SWCC for determination of USPFs*

Various forms of the volume-mass soil-water characteristic curves are ready to be used for the estimation of the unsaturated soil property functions, USPFs, for saturated-unsaturated seepage problems. The partial differential equation accounting for two-dimensional unsaturated seepage can be derived in a manner similar to the procedure historically used in saturated soil mechanics. Let us assume that the major and minor coefficients of permeability occur in the *x*- and *y*-directions, respectively. The saturated-unsaturated transient seepage equation can be written as follows (Fredlund & Rahardjo, 1993).

$$\frac{\partial}{\partial x}\left(k_{wx}\frac{\partial h_w}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_{wy}\frac{\partial h_w}{\partial y}\right)=m_2^w\rho_w g\frac{\partial h_w}{\partial t} \tag{20}$$

where k_{wx} and k_{wy} are the major and minor permeability functions, respectively; m_2^w is the water storage function; h_w is the hydraulic head in the water phase (i.e., elevation head plus water pressure head); g is the acceleration due to gravity, and t is time.

If the soil is anisotropic, the saturated coefficients of permeability are different in two orthogonal directions. Let us assume that the major and minor coefficients of permeability occur in the x- and y-directions. Anisotropic soils are commonly assumed to have the same air-entry values in both directions and as a result the unsaturated permeability functions will have the same functional characteristics in the x- and y-directions. Expanding equation (20) and assuming that k_{wx} is equal to k_{wy} (i.e., equal to k_w) results in the following form for saturated-unsaturated transient seepage.

$$k_w\frac{\partial^2 h_w}{\partial x^2}+\frac{\partial k_w}{\partial x}\frac{\partial h_w}{\partial x}+k_w\frac{\partial^2 h_w}{\partial y^2}+\frac{\partial k_w}{\partial y}\frac{\partial h_w}{\partial y}=m_2^w\rho_w g\frac{\partial h_w}{\partial t} \tag{21}$$

Equations (20) and (21) show that two independent soil property functions are required in order to solve transient seepage problems common to geotechnical engineering. The required soil properties are the coefficients of permeability function, k_w, and the water storage function, m_2^w. Both soil properties are nonlinear functions of suction. It has become generally accepted engineering practice (as well as in other related engineering and agricultural disciplines), to estimate the unsaturated soil property functions on the basis of the volume-mass SWCCs and the saturated soil properties. At the same time, there has been some variance with respect to how the estimation procedures are applied. One of the objectives of this paper is to describe in detail the estimation procedures that appear to be most acceptable when solving geotechnical engineering problems.

The permeability function and the water storage function should be calculated as independent soil property functions rather than having the two soil properties combined and used as a single “diffusivity” type variable. It is also important that the estimation of the unsaturated soil property functions be based on w-SWCC and SC test results on the same soil. Both soil property functions are highly nonlinear and dependent on different volume-mass versus soil suction relations. This methodology is particularly important when the soil undergoes volume change as soil suction changes.

The degree of saturation and void ratio are the two main volume-mass variables that influence the estimation of the permeability function, whereas in saturated soil mechanics, void ratio is the only factor that influences the coefficient of permeability.

Techniques used in soil physics for the estimation of the unsaturated permeability function are most commonly based on the assumption that the soil structure is rigid and therefore no volume change occurs during the drying process (Zhang et al., 2015). Only changes in the degree of saturation are assumed to result in changes in the unsaturated coefficients of permeability. Several estimation procedures have been proposed for estimating the unsaturated coefficient of permeability functions, all based on the assumption that the soil does not undergo volume change during the drying process.

3.9.1 *The water storage function*

The water storage property, m_2^w, is defined as the slope of the (instantaneous) volumetric water content versus soil suction relationship. The water storage function is required whenever an unsteady-state seepage analysis is performed. The water storage modulus, m_2^w, can be obtained through the differentiation of any equation that fits the volumetric water content versus suction relationship, (θ_w-SWCC) (Figure 21).

$$m_2^w=\frac{\mathrm{d}\theta_w}{\mathrm{d}(u_a-u_w)} \quad \text{or} \quad m_2^w=\frac{\mathrm{d}\theta_w}{\mathrm{d}\psi} \tag{22}$$

where $(u_a - u_w)$ is the matric suction, and ψ is the total suction in high suction range (i.e., suctions > 1500 kPa) and matric suction in low suction range (i.e., suctions < 1500 kPa).

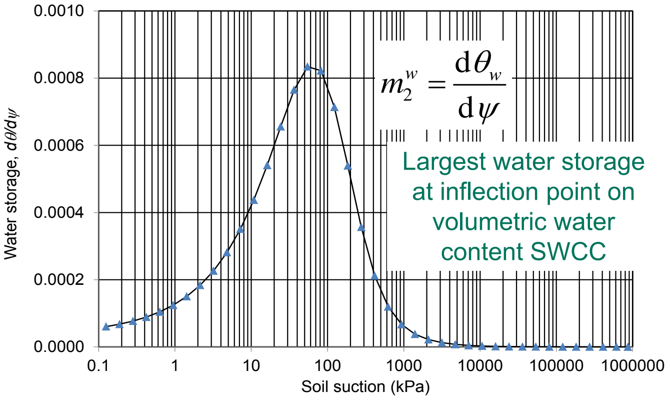

Figure 21. Water storage function for the artificial clayey silt soil.

The water storage of a soil is relatively small as soil suction tends towards zero, becoming equal to the coefficient of volume change, m_v, for the saturated soil (i.e., relative to a change in effective stress). A maximum water storage value is reached in the vicinity of the inflection point along the θ_w-SWCC, then tending towards a low value beyond residual suction conditions. It is preferable for the water storage function and the permeability function to be applied as independent mathematical relations for numerical modeling purposes because of their uniquely different nonlinear characteristics.

3.10 *The permeability function*

A revised methodology is required for the estimation of the coefficient of permeability function when soils undergo volume change as suction changes during the drying process. The estimation procedure for calculating the permeability function is based on the separation of changes in the coefficient of permeability resulting from volume change (or void ratio change) from changes that occur as a result of changes in degree of saturation. In each case, there are existing theories that can be used to estimate the two coefficient of permeability functions.

The term, "relative permeability function" is used when referring to the effect of changes in degree of saturation in a normalized manner. The "relative permeability function" is set equal to 1.0 at the air-entry value for the soil (i.e., $k_{rw}(S)$). The "relative permeability function" can also be defined in terms of soil suction, $k_{rw}(\psi)$, estimated from the degree of saturation SWCC.

$$k_{rw}(\psi) = \frac{k_w(\psi)}{k_{ref}(\psi)} \tag{23}$$

where $k_{ref}(\psi)$ is the reference saturated coefficient of permeability as a function of soil suction. In other words, $k_{ref}(\psi)$ is the saturated coefficient of permeability corresponding to the void ratio at a designated soil suction; $k_w(\psi)$ is the coefficient of permeability as a function of soil suction; $k_{rw}(\psi)$ is the relative coefficient of permeability for changes in the degree of saturation.

The term "reference saturated permeability function" is used when referring to the effect of changes in void ratio or overall volume change, (i.e., $k_{ref}(\psi)$). The reference saturated permeability function is developed based on a saturated permeability function along with the relationship of void ratio to soil suction. The saturated permeability function with respect to void ratio change, $k_{sw}(e)$, can be studied in a dimensionless manner. The "dimensionless saturated permeability function", $k_{sd}(\psi)$, can be written as follows.

$$k_{sd}(e) = \frac{k_{sw}(e)}{k_{sr}} \tag{24}$$

where: $k_{sw}(e)$ is the saturated coefficient of permeability as a function of void ratio, and k_{sr} is the saturated coefficient of permeability at the reference state (i.e., a reference void ratio).

It is possible to calculate the "actual permeability functions" once the reference saturated permeability function for volume change and the relative permeability function for degree of saturation change are known. The two permeability functions can be combined for solving practical seepage problems.

3.10.1 *Estimation of coefficient of permeability function with respect to void ratio changes*

Estimation models for the saturated coefficient of permeability of a porous material are mainly dependent upon the size of the pores and the tortuosity of the flow path (Chapius, 2012). These two factors can be treated in an independent manner by separating the effects of changes in void ratio from the effects of changes in degree of saturation.

Kozeny (1927) developed an estimation model for the coefficient of permeability based on applying Poiseuille's law to laminar flow through straight circular pipes. The model was later modified by Carman (1937) and is generally referred to as the Kozeny-Carman model. The model took several factors into consideration in calculating the coefficient of permeability; however, it is the form in which void ratio changes are characterized that is relevant to the development of a permeability estimation model based on the soil-water characteristic curve, SWCC. The effect of changes in void ratio is shown in equation [25] where all other factors represented in the Kozeny-Carman equation are treated as a single constant (Taylor, 1948).

$$k_{sw}(e) = \frac{C e^3}{1+e} \tag{25}$$

where C is the constant representing all factors (other than void ratio) affecting the calculation of the saturated coefficient of permeability.

The constant, C, can be used as a fitting parameter provided the void ratio of the soil is known. The coefficient of permeability can be measured using a one-dimensional consolidation test or a permeameter test. The Kozeny-Carman relationship is used to quantify the changes in the coefficients of permeability over the range of possible void ratio changes. Incorporating Eq. [24] and Eq. [25], coefficients of permeability due to increases or decreases in void ratio are then estimated as a proportionality as shown in the following equation.

$$k_{sd}(e) = \frac{k_{sw}(e)}{k_{sr}} = \left(\frac{e^3}{1+e}\right) \Big/ \left(\frac{e_r^3}{1+e_r}\right) \tag{26}$$

where k_{sr} is the saturated coefficient of permeability at a known reference void ratio; k_{sw} is the saturated coefficient of permeability at another selected void ratio of e; e_r is the void ratio as a reference point at which the permeability is known, and e is the void ratio at which the permeability is to be calculated.

A single permeability measurement can be used along with equation (26) to compute the "actual saturated coefficient of permeability" with respect to void ratio. The dimensionless saturated permeability function, $k_{sd}(e)$, has a value of 1.0 at its reference state (i.e., the initial void ratio corresponding to saturation in the w-SWCC test). In other words, $k_{sd}(e_r) = 1.0$. Figure 15 shows the maximum void ratio (i.e.,

e_{max} = 0.871) and minimum void ratio (i.e., e_{min} = 0.500) that form the limits for the artificial clayey silt soil being analyzed.

Figure 22 shows plots of the “dimensionless saturated coefficient of permeability” function along with the “actual saturated coefficient of permeability” function for the artificial clayey silt soil being analyzed.

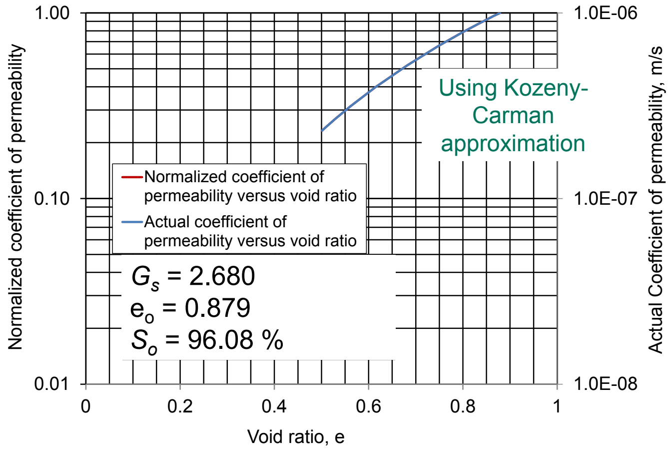

Figure 22. Dimensionless and actual saturated coefficients of permeability versus void ratio.

The “actual saturated coefficient of permeability” function can also be written as a function of soil suction through use of the basic laboratory test results; namely, the *w*-SWCC and the shrinkage curve data. The “actual saturated coefficient of permeability” function (i.e., $k_{sw}(e)$) written as a function of soil suction can be referred to as the reference saturated coefficient of permeability function (i.e., $k_{ref}(\psi)$).

Figure 23 shows a plot of the reference coefficient of permeability versus soil suction. The upper line is the reference coefficient of permeability function in a dimensionless form reflecting the effect of void ratio change. The lower plot in Figure 23 shows the reference saturated permeability function assuming that the saturated coefficient of permeability used for non-dimensionalization was 1.0 x 10^{-6} m/s at zero suction. The lower permeability function shows how the permeability function can be scaled up or down depending on the saturated coefficient of permeability used for non-dimensionalization.

The relationship between void ratio and soil suction is mathematically described in equation [6].

The “reference saturated coefficient of permeability” function for void ratio change can be written in terms of soil suction, $k_{ref}(\psi)$, as shown in equation (27).

$$k_{ws}(\psi)=\frac{C\left(a_{sh}\left(\left(\frac{w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{b_{sh}\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}}\right)^{c_{sh}}+1\right)^{\frac{1}{c_{sh}}}\right)^{x}}{1+a_{sh}\left(\left(\frac{w_s\left(1-\ln\left(1+\psi/\psi_r\right)/\ln\left(1+10^6/\psi_r\right)\right)}{b_{sh}\left(\ln\left(\exp(1)+\left(\psi/a_f\right)^{n_f}\right)\right)^{m_f}}\right)^{c_{sh}}+1\right)^{\frac{1}{c_{sh}}}} \qquad (27)$$

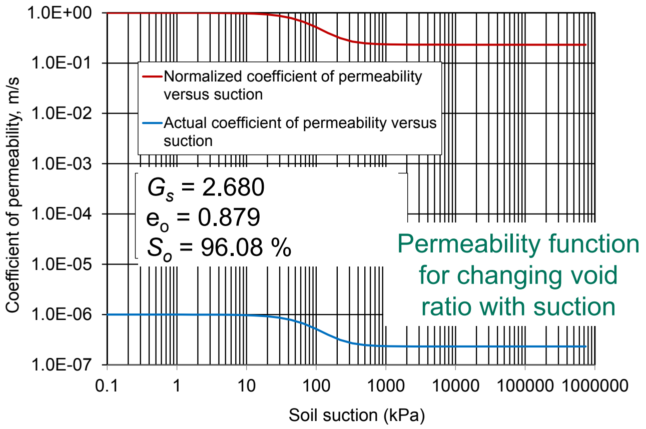

Figure 23. Changes in the permeability function as a function of soil suction due to void ratio changes.

Kozeny-Carman equation has been found to be better suited for sand soils than for clay soils (Taylor, 1948). However, it should be noted that only an approximation of the effect of void ratio changes is required in most cases because the major changes in the coefficient of permeability are due to changes in the degree of saturation. The following Somogyi (1980) equation can also be better suited for computing the effect of void ratio changes on the saturated coefficient of permeability.

$$k_{sw}(e) = Ae^{B} \qquad (28)$$

where $k_{sw}(e)$ is the saturated coefficient of permeability, *A* is one of the fitting parameters for the void ratio versus coefficient of permeability measurements, and *B* is the second exponential fitting parameter for the void ratio versus coefficient of permeability measurements.

The use of the Somogyi (1980) equation requires that there be a series of measurements of void ratio versus coefficient of permeability, possibly from a one-dimensional laboratory consolidation test.

3.10.2 *Estimation of coefficient of permeability function with respect to changes in degree of saturation*

The permeability function that takes changes in degree of saturation into consideration can be formulated independent of void ratio changes and referred to as the relative coefficient of permeability function. Various forms of integration along the *S*-SWCC have been used for the estimation of the coefficient of permeability function with respect to changes in the degree of saturation. The individual permeability functions taking void ratio changes and degree of saturation changes into consideration can then be combined for solving seepage problems where the soil undergoes volume change and desaturation as drying occurs.

Childs & Collis-George (1950) proposed a model for estimating the coefficient of permeability based on a random variation in pore sizes. The permeability function was derived based on Poiseuille’s equation and the assumption was made that the overall volume change of the soil was negligible as soil suction increased. The model was later improved by Marshall (1958) and further modified by Kunze et al. (1968).

Fredlund, Xing &Huang (1994) used the Fredlund & Xing (1994) soil-water characteristic curve equation along with the Childs & Collis-George (1950) physical model to compute a water permeability function. The procedure involved starting at saturated soil conditions and integrating numerically along the volumetric water content SWCC to completely dry conditions. The Fredlund et al. (1994) relative permeability function took the following form when using the degree of saturation SWCC.

$$k_{rw}(\psi)=\frac{\displaystyle\int_{\ln(\psi)}^{b}\frac{S(e^y)-S(\psi)}{e^y}S'(e^y)dy}{\displaystyle\int_{\ln(\psi_{aev})}^{b}\frac{S(e^y)-S(\psi_{aev})}{e^y}S'(e^y)dy} \qquad (29)$$

where b is the upper limit of integration (i.e., ln (1,000,000)), y is a dummy variable of integration representing the natural logarithm of suction, S' is the first derivative of the soil-water characteristic curve equation; and e^y is the natural number raised to the dummy variable power.

Equation (29) shows that integration commences at the air-entry value and continues to an upper limit of 1,000,000 kPa.

The relative coefficient of permeability, $k_{rw}(\psi)$, is the ratio of the unsaturated coefficient of permeability at soil suctions in excess of the air-entry value for the soil to the reference saturated coefficient of permeability, $k_{ref}(\psi)$, as shown in equation (23). The reference saturated coefficient of permeability, $k_{ref}(\psi)$, corresponds to the relevant void ratio at the soil suction under consideration.

The integration along the SWCC should take place from the air-entry value of the soil to at least residual water content conditions. It is commonly assumed that the coefficient of permeability of a soil is essentially zero when its water content is below the residual water content. Kunze et al. (1968) concluded that the accuracy of the prediction is significantly improved when the soil-water characteristic curve extended at least to residual conditions. Fredlund et al. (1994) continued the integration process beyond residual condition to near zero water content. The authors were aware that there was no confirmation of the accuracy of the lower portion of the permeability function. However, the intent was to provide a continuous permeability function over the entire possible suction range. The assumption was made that liquid flow of water would tend towards zero as vapor flow commenced at some point near residual conditions. The relationship between liquid and vapor flow is dealt with later in this manuscript. To avoid numerical difficulties, integration was performed over the soil suction range from ψ_{AEV} to 10^6 kPa on an arithmetic scale. The "relative permeability function" for the artificial clayey silt soil data used in this paper are presented in Figure 24.

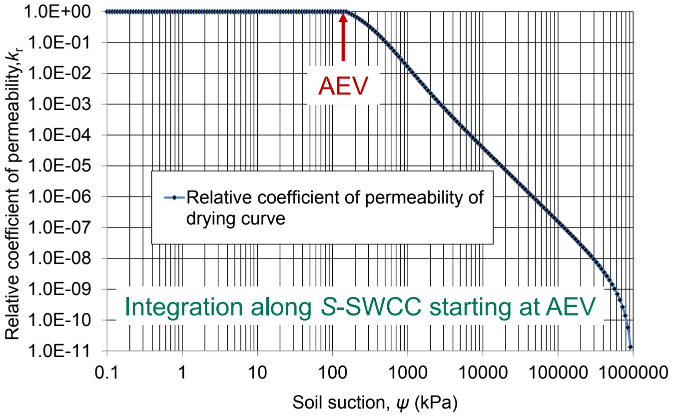

Figure 24. Permeability functions for the artificial clayey silt as soil suction is increased beyond "true" air-entry value of the soil.

The "relative permeability" function starts at 1.0 corresponding to suctions up to the "true" air-entry value. The logarithm of the coefficient of permeability then decreases almost linearly with the logarithm of soil suction beyond the air-entry value.

The relative permeability function calculated using the Fredlund et al. (1994) integration procedure is shown as a series of data points corresponding to the suctions at which the coefficient of permeability was calculated. The lower limit for the coefficient of permeability function can be set to either 2.0 x 10^{-14} m/s or a coefficient of permeability value corresponding to a soil suction of 10,000 kPa, whichever is larger.

The permeability function calculated using the Fredlund et al. (1994) integration procedure consists of a series of discrete data points that can be best-fit using the Fredlund & Xing (1994) SWCC equation or the Gardner (1958) equation. In so doing, the permeability function becomes a closed-form, continuous function.

3.10.3 *Combining the effects of volume change and degree of saturation change on the permeability functions*

The overall permeability function for a soil is the product of the coefficient of permeability with respect to volume change (i.e., equation [27]) and the coefficient of permeability with respect to changes in degree of saturation (i.e., equations [29]).

$$k_w(\psi)=k_{rw}(\psi)\times k_{ref}(\psi) \qquad (30)$$

where $k_w(\psi)$ is the coefficient of permeability at a particular suction ψ; $k_{rw}(\psi)$ is the relative coefficient of permeability as the soil desaturated at suctions beyond the air-entry value; and $k_{ref}(\psi)$ is the reference saturated coefficient of permeability.

The reference saturated coefficient of the permeability, $k_{ref}(\psi)$ refers to the coefficient of permeability of a saturated soil at a particular void ratio when the overall porous skeleton subjected to a suction of ψ. The relative coefficient of permeability is 1.0 at the saturated state. When soil suction is less than the air-entry value, the soil remains saturated and changes in permeability are related to changes in void ratio.

When suction exceeds the air-entry value, desaturation starts and the relative coefficient of permeability decreases from 1.0 to a value approaching zero as the soil dries. The coefficient of permeability $k_w(\psi)$ for soil in an unsaturated state is always smaller than the reference saturated coefficient of permeability. The overall coefficient of permeability $k_w(\psi)$ is equal to the product of the reference permeability associated with desaturation and the saturated coefficient of permeability at the suction corresponding to a particular void ratio.

Changes in the degree of saturation and void ratio of a soil are the two main factors that result in changes in the coefficient of permeability for a soil. Equation (30) has two components; namely the degree of saturation effect on the relative coefficient of permeability, $k_{rw}(\psi)$ and the void ratio effect on the reference saturated coefficient of permeability $k_{ref}(\psi)$. Decreases in the degree of saturation significantly change the tortuosity of the flow path within the porous media, and as a result, the coefficient of permeability is changed. In an unsaturated soil, the degree of saturation and void ratio combine to govern the overall coefficient of permeability for a soil that undergoes volume change as soil suction changes during a drying process.

3.10.4 *Lower limit for the liquid coefficient of permeability*

Laboratory measurements and proposed permeability models show that the coefficient of permeability decreases logarithmically as soil suction increases beyond the air-entry value. It is also suggested that at some point the water coefficient of permeability becomes so small that more moisture can be moved in the vapor phase than in the liquid phase (Ebrahimi-Birang et al., 2004). Tran et al. (2014) showed that there was a "shut-off" suction where the liquid flow of water essentially ceases.

There is limited research on the transition point where liquid and vapor flow becomes essentially equal. Ebrahimi-Birang et al. (2004) suggested that there should be a lower limit for the water coefficient of permeability and that the lower limit should be related to the vapor diffusion value. Lai et al. (1976) summarized various tortuosity models associated with vapor flow. All vapor flow models gave similar flow patterns when plotted versus soil suction for various soil types (Figure 25). The results showed that the maximum vapor flow occur near the residual state of the soil and remains essentially constant up to and beyond a suction of 10,000 kPa.

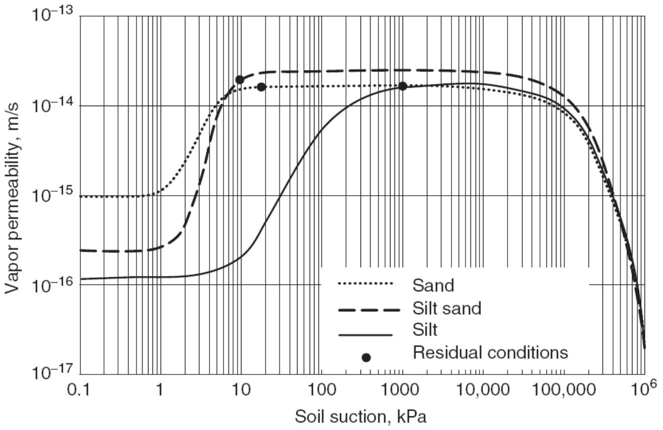

Figure 25. Vapor permeability for three soil types (after Lai et al., 1976).

The average vapor permeability for the three soil types shown was 2.0 x 10^{-14} m/s. It is suggested that the vapor permeability be used as a lower limit for the water coefficient of permeability. The water coefficient of permeability function then becomes a continuous function over the entire suction range. The lack of a lower limit for water coefficient of permeability can give rise to numerical convergence issues when modeling saturated-unsaturated seepage problems.

It is also possible that the permeability function may not reach 2.0 x 10^{-14} m/s before soil suction reaches 10,000 kPa. In this case, it is suggested that the permeability at a suction of 10,000 kPa be used as the lower limit of permeability.

4 HYSTERESIS ASSOCIATED WITH DRYING AND WETTING

Figure 10 showed that soils have hysteresis with the main drying and wetting curves forming boundaries for the water content versus soil suction relationship. However, it is only the drying *S*-SWCC (or the desorption curve) that has been generally used to estimate the permeability function. Another permeability function can be calculated corresponding to the wetting (or adsorption) *S*-SWCC.

Pham (2002, 2005) analyzed the drying and wetting curves for 34 datasets for a variety of soils reported in the literature. The difference between the hysteresis loops at the inflection points was used as the primary indicator of the magnitude of the hysteresis loop. The laboratory measurements of the drying and wetting SWCCs showed that the bounding drying curve tended to be approximately congruent, (i.e., parallel on a semi-log plot), to the bounding wetting curve (Pham et al., 2003, 2005). The distance between the main drying and wetting curves varied between 0.15 and 0.35 of a log cycle for sands (i.e., 15 to 35% of a log cycle). The bounding curve spacing for well-graded clayey silt soils varied between 0.35 and 0.60 of a log cycle. On average, the approximate spacing between the drying and wetting SWCCs was about 25% of a log cycle for sands and 50% of a log cycle for well-graded clayey silt. The overall average shift between the drying and wetting bounding curves was approximately 35%.

Pham (2002) measured the drying and wetting bounding curves for a sand soil and a processed silt soil. Results for the processed silt soil were shown in Figure 10. A total of three specimens were tested with each test showing essentially the same drying and wetting SWCCs. The drying curves were measured up to residual suction conditions and showed wetting curves that were essentially congruent with respect to the drying SWCC. It is not always practical to measure both the drying and the wetting SWCCs when solving practical engineering seepage problems. However, it would appear to be reasonable to estimate the wetting SWCC based on the assumption that the drying curve is congruent with the wetting curve and an estimate is made regarding the magnitude of the hysteresis loop.

The fitting parameters for the drying curve are the same as for the wetting curve with the exception that the a_{fs} fitting parameter must be reduced by a magnitude dependent upon the size of the hysteresis loop. The a_{fs} fitting parameter for the wetting curve can be calculated based on equation (31) (Fredlund et al., 2011).

$$\xi = 100\left(\log\left(\psi_{ad}\right) - \log\left(\psi_{aw}\right)\right) \tag{31}$$

where ξ is the percent shift between the drying and wetting hysteresis loops; ψ_{ad} is the suction corresponding to the a_{fs} fitting parameter on the drying SWCC; and ψ_{aw} is the suction corresponding to the a_{fs} fitting parameter on the wetting SWCC.

Equation (31) can be rearranged and solved for the suction on the wetting curve that corresponds to the a_{fs} fitting parameter.

$$\log\left(\psi_{aw}\right) = \log\left(\psi_{ad}\right) - \xi/100 \tag{32}$$

Let us make the assumption that the shift between the drying and wetting SWCCs is 35%. The a_{fs} fitting parameter changes from 261.9 kPa for the drying curve to 117.0 kPa for the wetting curve for the arbitrary soil being analyzed in this paper. The other fitting parameters remain the same and the wetting SWCC can be calculated and plotted as shown in Figure 26.

The wetting SWCC can now be used to compute a permeability function corresponding to a wetting process.

It is recognized that the suggested procedure for handling hysteresis is approximate. The outlined procedure is meant to illustrate reasonable assumptions that can be made to accommodate the hysteretic behavior of soils.

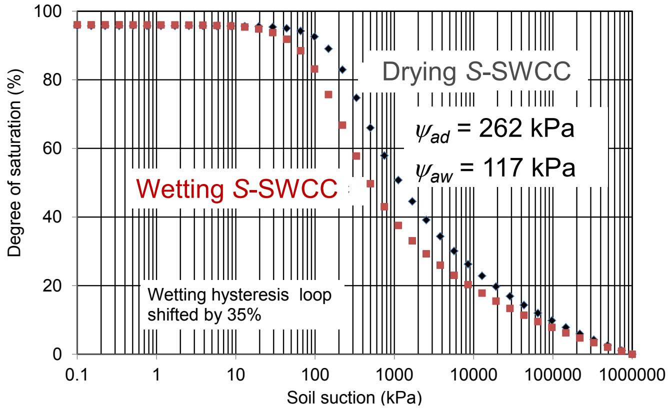

Figure 26. Drying and wetting hysteresis loops for the artificial clayey silt soil.

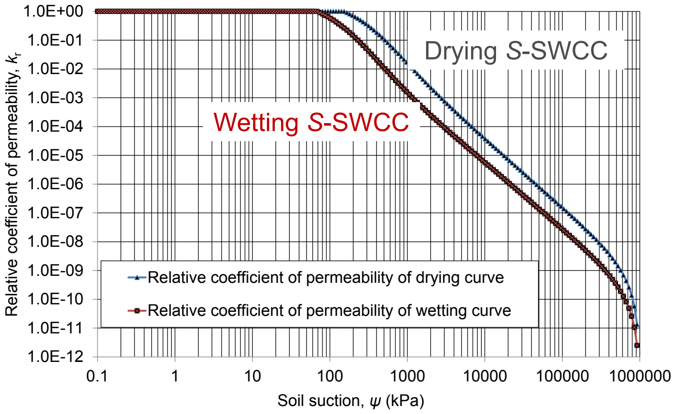

Figure 27. Drying and wetting permeability functions for the artificial clayey silt soil.

Figure 27 shows the drying and wetting permeability functions for the artificial clayey silt soil. Some computer codes can take hysteresis effects into consideration by using the appropriate drying and wetting permeability functions. Bashir et al. (2016) suggested cross-plotting the drying and wetting relative permeability functions versus the degree of saturation and thereby producing essentially a unique permeability relationship that can be used in numerical modeling. The SWCC versus degree of saturation plot is unique as long as there is congruency between the drying and wetting SWCCs (Figure 28).

There are numerous assumptions that have been made during the analysis of the data from the measurement of the drying curve soil-water characteristic curve and the shrinkage curve. These assumptions are recognized and the intent is to provide the geotechnical engineer with the best possible protocols for interpreting and applying data associated with unsaturated soil behaviour. The proposed protocols are meant to provide the geotechnical engineer with a thorough understanding of present theories for applying unsaturated soil mechanics in engineering practice.

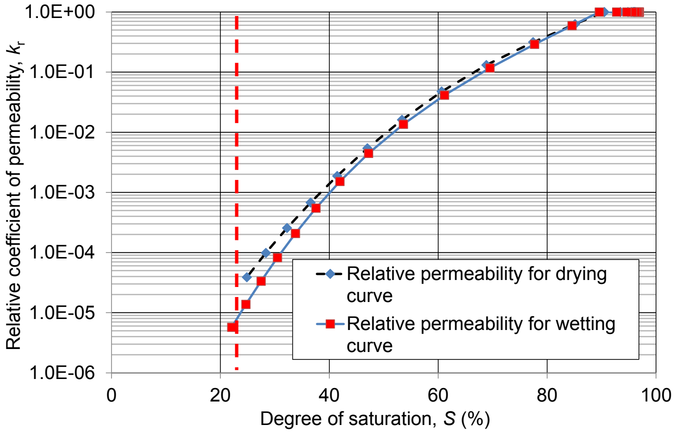

Figure 28. Cross-plot of the drying and wetting relative permeability functions with the degree of saturation for a plot of relative permeability versus degree of saturation for the artificial clayey silt soil.

The use of estimation procedures in geotechnical engineering practice has found increasing acceptance over the past couple of decades. The acceptance is mainly due to (i) reduced costs associated with indirectly estimating the permeability function; and (ii) realization that estimation techniques provide adequate information for most engineering design purposes. Success in applying unsaturated soil mechanics in engineering practice has been closely related to the use of the soil-water characteristic curve for estimating the water coefficient of permeability and the water storage functions.

5 CONCLUSIONS

The engineering protocols being used in geotechnical engineering are closely tied to the earlier research undertaken in soil physics. The use of the pressure plate apparatuses focused on measuring the drying SWCC and the procedures and protocols developed for engineering applications have been built on earlier findings in soil physics. The early research in soil physics has been valuable in geotechnical engineering applications but it has been necessary to carefully review the assumptions associated with use of the SWCCs.

Unsaturated soil mechanics can be applied in geotechnical engineering practice; however, the manner in which it is applied differs from saturated soil mechanics. The required soil properties for analysis purposes take the form of nonlinear functions. The costs associated with directly measuring the nonlinear functions in the laboratory are prohibitive. This paper has focused on the assessment of the material properties associated with unsaturated seepage modeling.

In recent years there has been worldwide research focused on the indirect determination of the unsaturated soil property functions through use of the soil-water characteristic curve, SWCC. In particular, the drying SWCC can be used in conjunction with the measurement of the shrinkage curve, SC, to provide increased accuracy in the estimations of unsaturated soil property functions. The use of the shrinkage curve in conjunction with the SWCC allows for the separation of effects of volume change from the effects of changes in degree of saturation of the soil. The SWCC and SC data allow for a more rigorous analysis of laboratory measurements for a wide range of soil conditions encountered in geotechnical engineering.

The concept of indirectly estimating unsaturated soil property functions, (e.g., coefficient of permeability and water storage) has become acceptable as part of prudent geotechnical engineering practice.

6 RECOMMENDATIONS

There are numerous important issues that still require further research. Unsaturated soils are near to the ground surface and as such are heterogeneous and randomly cracked with secondary structure. There is need for further integration of geotechnical engineering with other disciplines such as meteorology and surface hydrology for improved methods to estimate ground surface boundary conditions, actual evaporation, runoff, and infiltration. These topics are also of importance to soil physicists and the ongoing sharing of information is important for progress in both areas.

7 ACKNOWLEDGEMENTS

The authors wishes to acknowledge the significant contribution of Feixia Zhang in assembling the theoretical components involved with this study as well as confirming the reliability of the regression analyses. This paper reports on a small amount of the extensive confirmatory laboratory testing that she has undertaken and reported in other research papers.

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