The mechanical yield stress in unsaturated and saturated soils

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ABSTRACT: This paper discusses how the variation of mechanical yield stress with matric suction is represented in constitutive models for unsaturated and saturated soils. Particular emphasis is placed on how the mechanical yield stress is modelled across transitions between saturated and unsaturated conditions, highlighting the role of water retention hysteresis and the influence of mechanical behaviour on the water retention response. When the constitutive model used represents the unsaturated condition of the soil solely through matric suction (ignoring any influence of degree of saturation) the variation of mechanical yield stress with matric suction is unique and corresponds to the conventional loading-collapse LC yield curve of the Barcelona Basic Model and many other subsequent models. The incorporation of degree of saturation in modelling unsaturated soil behaviour and, more specifically, inclusion of the hysteretic variation of degree of saturation with suction, suggests that a more realistic representation of the evolution of mechanical yield stress with suction should distinguish between decreasing (wetting) and increasing (drying) variations of suction. These and other relevant implications of incorporating water retention hysteresis in a coupled constitutive model for unsaturated soils are discussed in the paper in the context of the Glasgow Coupled Model.

1 INTRODUCTION

Lloret-Cabot et al. (2015) discussed how the typical increase of mechanical yield stress with matric suction (difference between pore air pressure and pore water pressure) is represented in three constitutive models for unsaturated soils: the Barcelona Basic Model (BBM) by Alonso et al. (1990), the SFG by Sheng et al. (2008) and the Glasgow Coupled Model (GCM) by Wheeler et al. (2003). One of the conclusions of the study was that, even though the three models had rather different formulations, qualitatively similar responses for void ratio e and degree of saturation S_r were predicted by all three models during the unsaturated isotropic loading paths at constant suction that were simulated (for the BBM and SFG models, the coupled relationships for the water retention behaviour proposed by Sheng and Zhou (2011) were used to predict the variations in S_r). The main reason for this similarity between the predictions of the models was that, for the type of stress paths considered (loading at constant suction) and the range of suction investigated, the increase of mechanical yield stress with suction was represented in a similar manner in all three models.

A key aspect not included in the study of Lloret-Cabot et al. (2015), however, was the influence that the different response of S_r during wetting and drying (i.e. retention hysteresis) may have in the mechanical yield stress and, in particular, across the transitions between saturated and unsaturated conditions. Indeed, the difference in suction at saturation (when suction reaches the air-exclusion value, s_{ex}) and at de-saturation (when suction reaches the air-entry value, s_e), as a consequence of the retention hysteresis, should play a role in the representation of mechanical yield stress with suction. Furthermore, air-exclusion and air-entry points are linked to the way a constitutive model couples the mechanical behaviour to the water retention response and, hence, the influence of dry-density on the water retention response should be incorporated in the model in a manner that is consistent with how air-exclusion and air-entry points vary during mechanical yielding (Lloret-Cabot et al. 2017, 2018a,b). This paper incorporates all these effects in discussion of the variation of mechanical yield stress with suction.

2 MECHANICAL YIELDING IN SOILS

This section discusses mechanical yielding in unsaturated and saturated soils. The discussion is presented in four subsections. The first one shows the typical variation of mechanical yield stress with matric suction adopted in the conventional Loading

Collapse (LC) yield curve of the BBM and many other subsequent models. The second subsection shows the consequence of considering a nonzero air-entry/air-exclusion value of suction for the shape of the LC yield curve. The third subsection discusses the implications, for the mechanical yield stress, of hysteretic variation of degree of saturation, and the final subsection incorporates in the discussion the effects of mechanical yielding. It is useful to provide now definitions for mean net stress $\bar{p}$, matric suction s, mean Bishop's stress p^* and modified suction s^*:

$$\bar{p} = p - u_a \tag{1}$$

$$s = u_a - u_w \tag{2}$$

$$p^* = p - \left(S_r u_w + (1 - S_r) u_a\right) = \bar{p} + S_r s \tag{3}$$

$$s^* = (u_a - u_w) n \tag{4}$$

where, p is mean total stress; u_a is pore-air pressure; u_w is pore-water pressure and n is porosity.

2.1 *The Loading Collapse (LC) yield curve*

Figure 1 shows the typical increase of mechanical yield stress (expressed in terms of mean net stress) with matric suction of the conventional Loading Collapse (LC) yield curve first proposed by Alonso et al. (1987), which is included in the BBM and other similar constitutive models for unsaturated soils. In the BBM, the unsaturated condition of the soil is represented solely through matric suction (ignoring any influence of S_r). As a result, the variation of mechanical yield stress with s is unique, so that, for a given value of saturated mechanical yield stress p'_0, the variation of mechanical yield stress with suction $\bar{p}_0(s)$ is the same during wetting and drying (see Figure 1). Transitions between saturated and unsaturated conditions are assumed to occur at s = 0, so that air-exclusion and air-entry values of suction are both zero (i.e. $s_e = s_{ex} = 0$).

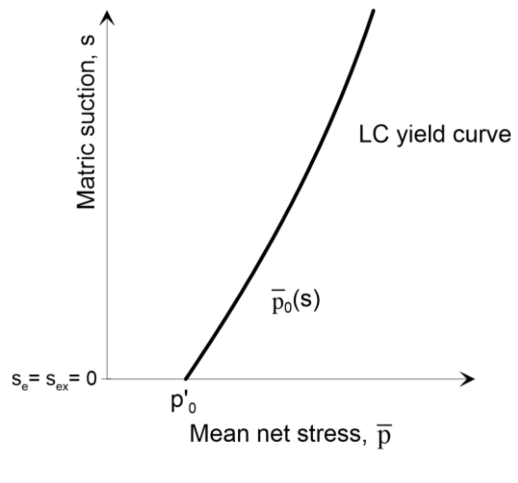

Figure 1. Loading Collapse yield curve (Alonso et al. 1990).

2.2 *Variations of mechanical yield stress with suction, accounting for saturated/unsaturated transitions*

The occurrence of saturated states at values of suction lower than the air-entry/air-exclusion value should influence the shape of the lower section of the LC yield curve plotted in Figure 1. This feature is illustrated in Figure 2, where the value of air-entry and air-exclusion suction is indicated by $s_e = s_{ex}$. If no retention hysteresis is considered, both air-entry and air-exclusion values of suction are the same but, in contrast to Figure 1, s_e and s_{ex} now have a value larger than zero (Figure 2). Against this background, transitions between saturated and unsaturated conditions occur at $s_e = s_{ex}$ and any stress state at suction lower than the air-entry/air-exclusion value will correspond to saturated behaviour. For all these saturated states, the variation of mechanical yield stress with s plots as a 45° degree line in the $s:\bar{p}$ plane (Figure 2), consistent with yield at constant value of saturated mean effective stress ($p' = p - u_w = \bar{p} + s$) (Lloret-Cabot et al. 2017). A number of constitutive models proposed in the literature adopt a yield curve shape similar to Figure 2 (Nuth & Laloui 2007; Sheng et al. 2008; Sheng 2011), although in some of them, the sharp discontinuity of yield curve inclination is smoothed.

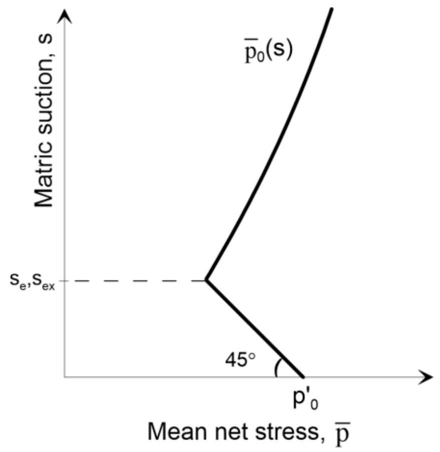

Figure 2. Variation of mechanical yield stress with suction accounting for saturated/unsaturated transitions.

2.3 *Influence of retention hysteresis on the mechanical yield curve*

The occurrence of water retention hysteresis in unsaturated soils (Cunningham 2000; Ng & Pang 2000; Wheeler et al. 2003; Romero et al. 2011; Tarantino 2009) means that de-saturation (air-entry point) and saturation (air-exclusion point) are achieved at different values of suction. Assuming dependency of mechanical yield stress on degree of saturation (Jommi & Di Prisco 1994), the incorporation of retention hysteresis in a coupled constitutive model for unsaturated soils means, as suggested by Tamagnini (2004), that the variation of mechanical yield stress with suction cannot be unique. Indeed, if the mechanical yield stress depends on S_r, hysteretic variation of S_r during wetting and drying means that different variations of mechanical yield stress with suction should be considered for decreasing s (wetting) and increasing s (drying), as illustrated in Figure 3. Lloret-Cabot et al. (2014a) demonstrate that this feature facilitates the unification of plastic compression, not only during loading and wetting (as achieved with the conventional LC yield curve) but, additionally, also during drying.

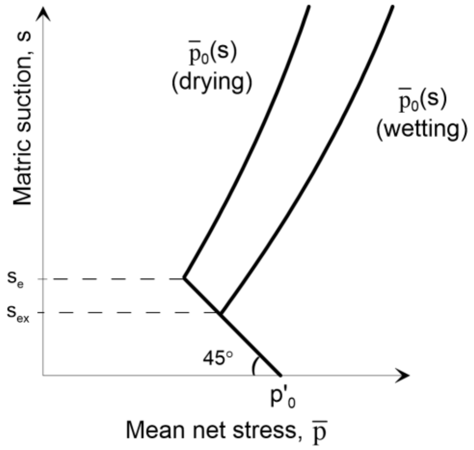

Figure 3. Variation of mechanical yield stress with suction accounting for water retention hysteresis.

2.4 *Influence of mechanical yielding on the mechanical yield curves*

Mechanical yielding causes plastic volumetric compression which, in an elasto-plastic model with volumetric hardening, increases the value of the mechanical hardening parameter used to define the size of the mechanical yield curve. In most models, this mechanical hardening parameter corresponds to the current saturated mechanical yield stress p'_0 and its increase (due to mechanical yielding) expands the size of the mechanical yield curve. The occurrence of mechanical yielding also influences water retention behaviour (Tarantino 2009; Romero et al. 2011). This influence is often represented in a model by a shift of the main wetting and main drying retention curves to higher values of suction when the void ratio decreases (Khalili et al. 2008; Zhou & Sheng 2015; Laloui et al. 2016). Lloret-Cabot et al. (2018a) argue, in the light of the experimental evidence of Tarantino (2009), that this shift in the main water retention curves should also affect the current air-entry and air-exclusion values of suction. Figure 4 illustrates this situation by considering expansion of the mechanical yield curves. The occurrence of mechanical yielding (indicated by an arrow) causes an increase of the saturated mechanical yield stress from p'_{01} to p'_{02}. This increase of p'_0 expands the mechanical yield curves and increases both air-entry and air-exclusion values of suction (Figure 4). A model that includes all these effects is the Glasgow Coupled Model (GCM) of Wheeler et al. (2003) (see also Lloret-Cabot et al. 2017).

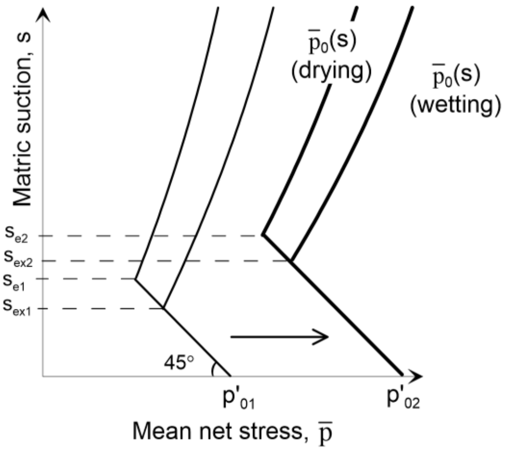

Figure 4. Influence of mechanical yielding (plastic volumetric strain) on the variation of mechanical yield stress with suction.

3 THE GLASGOW COUPLED MODEL

For isotropic stress states, the GCM is expressed in terms of mean Bishop's stress p^* and modified suction s^* (Equations 3 and 4, respectively). Details of the extended formulation to general stress states are given in Lloret-Cabot et al. (2013).

The GCM describes the occurrence of plastic volumetric strains (potentially occurring during loading, wetting and drying paths, see Lloret-Cabot et al. 2014) through yielding on a single mechanical M yield surface (mechanical behaviour). Plastic changes of degree of saturation are described by two additional yield surfaces (water retention behaviour). Yielding on the wetting retention yield surface WR corresponds to plastic increases of S_r and yielding on the drying retention surface DR corresponds to plastic decreases of S_r. The current locations of M, WR and DR yield surfaces are defined by current values of the hardening parameters p_0^*, s_1^* and s_2^*, respectively. Figure 5 illustrates two typical configurations of the three yield curves under unsaturated (solid lines) and saturated (dashed lines) conditions. Note that for saturated conditions the positions of the WR and DR yield surfaces are given by the air-exclusion (i.e. $s_1^*=s^*_{ex}$) and air-entry (i.e. $s_2^*=s^*_e$) values of modified suction, respectively. Note also that under saturated conditions (and only under saturated conditions) the stress point in the s^*: p^* plane can lie below the WR yield curve (the consistency condition on the WR curve is relaxed under saturated conditions), as described in Lloret-Cabot et al. (2017). The mechanical yield stress under saturated conditions equals the conventional saturated effective mechanical yield stress p'_0 of the Modified Cam Clay model for saturated soils (Roscoe & Burland 1968).

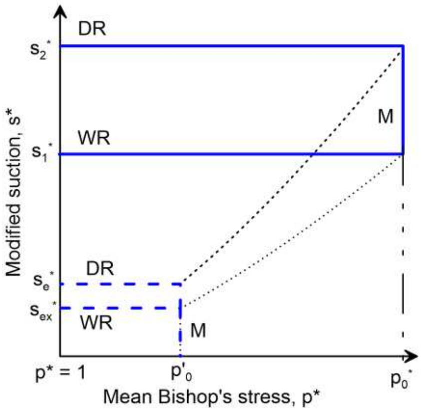

Figure 5. Yield curves for the GCM (Wheeler et al. 2003; Lloret-Cabot et al. 2017, 2018a).

3.1 *Influence of retention hysteresis*

To illustrate how the GCM represents the influence of retention hysteresis on the mechanical yield stress, it is useful to consider a wetting-drying cycle without any occurrence of plastic compression, as discussed in Lloret-Cabot et al. (2017). Figure 6 shows such a wetting–drying cycle ABCDEF involving transitions between unsaturated and saturated conditions during both wetting and drying (at points B and E, respectively). The stress path, shown by the continuous lines, starts on the WR yield surface at A, but remains inside the M yield surface throughout. The stress path shown in Figure 6 in both the s^*:p^* plane (Figure 6a) and the conventional s: $\overline{p}$ plane (Figure 6b) represents a wetting–drying cycle at constant $\overline{p}$, but the discussion presented in this section applies equally well to any general wetting–drying path remaining inside the M yield surface (i.e. no yielding on the M yield surface throughout).

Also shown in Figure 6a is the variation of mechanical yield stress p_0^* predicted by the GCM during the wetting–drying cycle, representing the coupled movement of the M yield surface. The value of p_0^* reduces during the initial unsaturated section AB of the wetting path, due to the coupled inward movement of the M surface caused by the plastic increases of S_r during yielding on the WR surface. During the final saturated section BC of the wetting path, however, the value of p_0^* remains constant, as there are no longer any plastic increases of S_r to produce further coupled movement of the M surface.

The variation of p_0^* with s^* during the wetting process illustrated in Figure 6a is represented by the dashed line ABC and is referred to hereafter as a mechanical wetting MW curve. Lloret-Cabot et al. (2018a) demonstrate the existence of different MW curves for different states of plastic compression (i.e. different values of plastic volumetric strain, $d\varepsilon_v^p$).

During drying path CDEF, the stress path passes back inside the WR surface at point D, but de-saturation only occurs when the stress path reaches the DR surface at E. The value of p_0^* therefore remains constant during the initial saturated section CDE of the drying path and then increases during the final unsaturated section EF. This represents the coupled outward movement of the M yield surface which occurs in the GCM during yielding on the DR surface.

The variation of p_0^* with s^* during the drying process illustrated in Figure 6a is represented by the dashed line CDEF and is referred to hereafter as a mechanical drying MD curve. Similarly, to the wetting process just described, different MD curves exist for different states of plastic compression (see Lloret-Cabot et al. 2018a,b).

Figure 6b shows the variation of mechanical yield stress predicted by the GCM during the wetting–drying cycle ABCDEF re-plotted in the conventional s: $\overline{p}$ plane. The variation of mechanical yield stress during the unsaturated section AB of the wetting path is equivalent to the LC yield curve of the BBM illustrated in Figure 1. From B to C, however, with the soil in a saturated condition, the variation of mechanical yield stress for the GCM plots as a 45° line in the s: $\overline{p}$ plane, consistent with the earlier discussion on Figure 2. During the drying path CDEF, the variation of mechanical yield stress follows a 45° line until the soil de-saturates at E, and then from E to F it forms a curve again. The qualitative form of variation of mechanical yield stress shown in Figure 6b corresponds, exactly, with what has been discussed in Figure 3 for a soil under unsaturated and saturated conditions, where saturation occurs at a nonzero air-exclusion value of suction (point B) and de-saturation occurs at a nonzero air-entry value of suction (point E) that is higher than the air-exclusion value because of hysteresis in the retention behaviour.

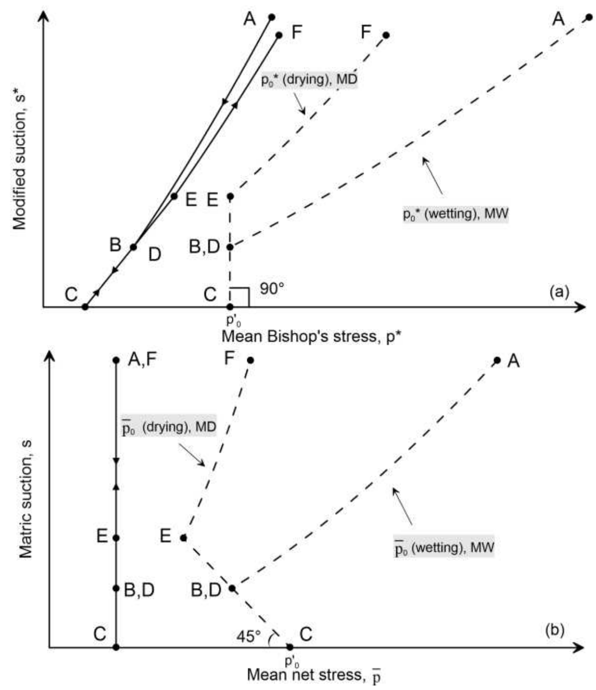

Figure 6. Variation of mechanical yield stress during a wetting-drying cycle (Lloret-Cabot et al. 2017).

3.2 *Influence of mechanical yielding*

Any occurrence of mechanical yielding causes plastic volumetric compression and a corresponding increase in the value of the saturated yield stress p'_0. A consequence of this mechanical yielding in the GCM is that both MW and MD curves expand and also both air-entry and air-exclusion values of suction increase (see Figure 7). The variations of s^*_e with p'_0 and s^*_{ex} with p'_0 predicted by the GCM are given by the equations of a de-saturation line and a saturation line respectively, as presented in Lloret-Cabot et al. (2018a). The qualitative form of behaviour predicted by the GCM during mechanical yielding, and shown in Figure 7, conforms exactly with expected behaviour, as discussed earlier in the context of Figure 4.

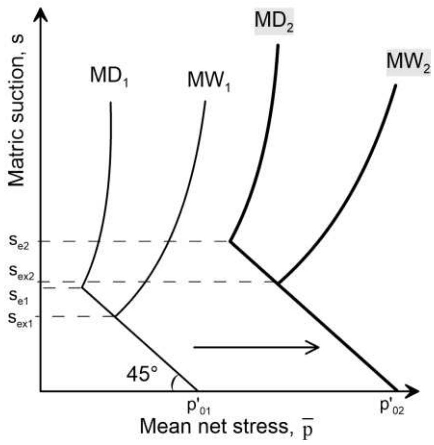

Figure 7. Predicted evolution of MW and MD curves during mechanical yielding.

4 DISCUSSION

As discussed in Lloret-Cabot et al. (2018a), the MW curve acts rather like a yield curve and corresponds to the onset of collapse compression during wetting, although strictly it is the M curve that is the yield curve, and the MW curve simply tracks the coupled movement of the M curve during yielding on the WR surface alone. As discussed in earlier sections, the form of the unsaturated part of the MW curve illustrated in Figure 6 resembles the conventional loading collapse LC yield curve of the BBM (Figure 1). Similar forms of the LC yield curve have also been proposed in many other models, all expressing the variation of the pre-consolidation stress in terms of suction (Khalili et al. 2008; Wheeler & Sivakumar 1995; Laloui et al. 2016) or degree of saturation (Gallipoli et al. 2003; Gallipoli et al. 2008; Jommi & Di Prisco 1994; Tamagnini 2004; Zhou & Sheng 2015). There are, however, important advantages of the MW and MD curves in the GCM over the LC yield curve in most of these constitutive models. Three of these advantages are briefly discussed here and further discussion can be found in Lloret-Cabot et al. (2018a, b).

The first advantage relates to the fact that saturated conditions can occur at nonzero values of suction, and yielding should then be governed by the saturated effective stress (Gens 2010a; Gens 2010b; Sheng 2011; Sheng et al. 2008), as discussed earlier in subsection 2.2. This is difficult to capture with a conventional LC yield curve, as it would typically require a dramatic change of yield curve inclination at transitions between unsaturated and saturated conditions (see Figure 2). Lloret-Cabot et al. (2018a) demonstrate that this transition occurs naturally in the GCM, because coupled inward movement of the M surface during wetting ceases when the soil reaches a saturated condition and hence the continuation of the MW curve is simply a line of constant p_0^* once the soil is saturated (see Figure 6).

The second advantage of the MW and MD curves in the GCM over the LC yield curve in more conventional constitutive models for unsaturated soils is the fact that the influence of retention hysteresis on mechanical yielding (as discussed in subsection 2.3) is included through the distinction between MW and MD curves (as described in subsection 3.1). The MD curve is associated with a drying process at a particular value of (ε_v^p) (i.e. without yielding on the M surface), and it describes the onset of plastic compression during drying (shrinkage) or during loading following drying (Lloret-Cabot et al. 2018a). If there is no retention hysteresis, the MD and MW curves coincide, whereas the MD and MW curves are different if retention hysteresis is included (Lloret-Cabot et al. 2018a). Importantly, both MW and MD curves arise in the GCM simply from coupled movements of a single M yield surface.

The third advantage of the MW and MD curves in the GCM over a conventional LC yield curve is that air-entry and air-exclusion values of suction, which define the locations of the discontinuities of gradient in the MD and MW curves respectively, increase during mechanical yielding (as shown in Figure 7), to represent the influence of plastic volumetric strain on water retention behaviour (as discussed in subsections 2.4 and 3.2).

5 CONCLUSIONS

The typical increase of mechanical yield stress with suction observed in a soil under unsaturated conditions has been discussed in the paper, with reference also to the saturated/unsaturated transitions and the influence of retention hysteresis and plastic volumetric strains on these transitions. Early mechanical constitutive models for unsaturated soils (which take no explicit account of degree of saturation) define saturated/unsaturated transitions in terms of matric suction, generally assuming that these transitions only occur at $s = 0$. Some later mechanical models incorporate the possibility to de-saturate or saturate at a constant nonzero air-entry/air-exclusion value of suction. This paper demonstrates that incorporation of the influence of degree of saturation on mechanical behaviour, together with hysteretic variation of S_r, within coupled mechanical/retention constitutive models reveals several advantages. Firstly, conventional saturated mechanical yield behaviour can be predicted when $S_r = 1$, even if this occurs when suction is not zero. Secondly, due to the consideration of retention hysteresis, saturation (air-exclusion point) and de-saturation (air-entry point) do not necessarily occur at the same value of suction. Finally, the variation of air-entry and air-exclusion values of suction with the previous history of mechanical yielding can be consistently represented. The paper shows how all these issues affect the representation of transitions between saturated and saturated conditions and how the mechanical yield stress behaves across these transitions. All these effects are handled consistently in the Glasgow Coupled Model (GCM).

6 ACKNOWLEDGEMENTS

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7 REFERENCES

Alonso, E.E., Gens, A. & Hight, D.W. 1987. Special problem soils: general report. *Proc. 9th Eur. Conf. Soil Mech. Found. Engng* 3: 1087-1146.

Alonso, E.E., Gens, A. & Josa, A. 1990. A constitutive model for partially saturated soils. *Géotechnique* 40(3): 405-430.

Cunningham, M.R. 2000. The mechanical behaviour of a reconstituted, unsaturated soil. *PhD Thesis*, Imperial College of London, UK.

Gallipoli, D., Gens, A., Sharma, R. & Vaunat, J. 2003. An elasto-plastic model for unsaturated soil including the effect of saturation degree on mechanical behaviour. *Géotechnique* 53(1): 123–135.

Gallipoli, D., Gens, A., Chen, G. & D'Onza, F. 2008. Modelling unsaturated soil behaviour during normal consolidation and at critical state. *Comp. Geotech.* 35(6): 825–834.

Gens, A. 2010a. Soil–environment interactions in geotechnical engineering. *Géotechnique* 60(1): 3–74.

Gens, A. 2010b. Some issues in constitutive modelling of unsaturated soils. In: *Unsaturated soils - theoretical & numerical advances in unsaturated soil mechanics:* 613–626.

Jommi, C. & Di Prisco, C. 1994. A simple theoretical approach for modelling the mechanical behaviour of unsaturated granular soils (in Italian) Il ruolo dei fluidi in ingegneria geotecnica. *Proc. of Italian conf. Mondovi*: 167–188.

Khalili, N., Habte, M.A. & Zargarbashi, S. 2008. A fully coupled flow deformation model for cyclic analysis of unsaturated soils including hydraulic and mechanical hysteresis. *Comp. Geotech*. 35(6): 872–889.

Laloui, L., Ferrari, A., Li, C. & Eichenberger, J. 2016. Hydro-mechanical analysis of volcanic ash slopes during rainfall. *Géotechnique* 66(3): 220–231.

Lloret-Cabot, M., Sánchez, M. & Wheeler, S.J. 2013. Formulation of a three-dimensional constitutive model for unsaturated soils incorporating mechanical-water retention couplings. *International Journal for Numerical and Analytical Methods in Geomechanics* 37: 3008-3035.

Lloret-Cabot, M., Wheeler, S.J. & Sánchez, M. 2014. Unification of plastic compression in a coupled mechanical and water retention model for unsaturated soils. *Canadian Geotechnical Journal* 51(12): 1488-1493.

Lloret-Cabot, M., Pineda, J.A, Sheng, D. & Cordao-Neto, M.P. 2015. Some remarks on the mechanical yielding of unsaturated soils. *Unsaturated soil mechanics – from Theory to Practice* 1: 197–203.

Lloret-Cabot, M., Wheeler, S.J. & Sánchez, M. 2017. A unified mechanical and retention model for saturated and unsaturated soil behaviour. *Acta Geotechnica* 12(1): 1-21.

Lloret-Cabot, M., Wheeler, S.J., Pineda, J.A., Romero, E. & Sheng, D. 2018a. From saturated to unsaturated conditions and vice versa. *Acta Geotechnica* 13(1): 15-37.

Lloret-Cabot, M., Wheeler, S.J., Pineda, J.A., Romero, E. & Sheng, D. 2018b. Reply to "Discussion of "From saturated to unsaturated conditions and vice versa" *Acta Geotechnica* 13(1): 15-37.

Ng, C.W.W. & Pang, Y.W. 2000. Influence of stress state on soil-water characteristics and slope stability. *J. Geotech. Eng. ASCE* 126(2): 157–166.

Nuth M. & Laloui L. 2007. New insight into the unified hydro-mechanical constitutive modelling of unsaturated soils. *3rd Asian Conference on Unsaturated Soils*: 109-125.

Romero, E., Della Vecchia, G., & Jommi, C. 2011. An insight into the water retention properties of compacted clayey soils. *Géotechnique* 61(4): 313–328.

Roscoe, K.H. & Burland, J.B. 1968. *On the generalised stress-strain behaviour of wet clay*. Engineering Plasticity (eds Heyman J & Leckie FA), Cambridge University Press, Cambridge: 535-609.

Sheng, D. 2011. Review of fundamental principles in modelling unsaturated soil behaviour. *Comp. Geotech*. 38(6): 757–776.

Sheng, D., Fredlund, D.G. & Gens, A. 2008. A new modelling approach for unsaturated soils using independent stress variables. *Canadian Geotechnical Journal* 45: 511-534.

Sheng, D. & Zhou, A. 2011. Coupling hydraulic with mechanical models for unsaturated Soils. *Canadian Geotechnical Journal* 48: 826-840.

Tamagnini, R. 2004. An extended Cam-clay model for unsaturated soils with hydraulic hysteresis. *Géotechnique* 54(3): 223–228.

Tarantino, A. 2009. A water retention model for deformable soils. *Géotechnique* 59(9): 751–762.

Wheeler, S.J. & Sivakumar, V. 1995. An elasto-plastic critical state framework for unsaturated soil. *Géotechnique* 45(1): 35-53.

Wheeler, S.J., Sharma, R.S. & Buisson, M.S.R. 2003. Coupling of hydraulic hysteresis and stress-strain behaviour in unsaturated soils. *Géotechnique* 53(1): 41-54.