Suction-induced dilatancy and stiffness in compacted silty sand via triaxial testing

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ABSTRACT: Several authors have studied the hydro-mechanical behaviour of unsaturated soils. However, few have attempted to investigate on the stress-dilatancy relationship of unsaturated soils. In this paper data collected from a series of suction-controlled conventional triaxial drained tests on silty sand is used to investigate the effect of suction and confining pressure on stiffness, dilatancy and to analyse stress-dilatancy relationship. Test results showed that an increase in suction causes a nonlinear increase in dilatancy rate, peak strength, and critical state strength at constant confining stress. As expected, an increase in confining stress suppressed the tendency of specimen to dilate. In addition, an upward and rightward shift in strength envelope both at critical and peak state was observed with increasing suction.

1 INTRODUCTION

Typically, earth structures are made up of compacted fill and are partially saturated during construction. Furthermore, they are likely being subjected to changes in degree of saturation during their life-time and thereby affecting their long-term hydro-mechanical response. Normally consolidated unsaturated soils tend to show strain-hardening type stress-strain accompanied by volumetric compression type response in triaxial shearing.

On the other hand, densely compacted, over consolidated and cemented soils tend to exhibit distinct peak before reaching critical and residual states strain-softening) during triaxial testing. Such phenomenon is usually accompanied by initial compression followed by subsequent increase in volume of soil specimen (Russel and Khalili 2006; Patil 2014).

Previous findings made via suction-controlled experiments have well established that soil manifests greater shear strength in unsaturated condition than in saturated condition and that the relationship between soil suction and corresponding increase in shear strength in soil is non-linear (Fredlund and Morgenstern 1977; Houston et al. 2008; Patil et al. 2016).

"Dilatancy" refers to an experimentally observed unique property of densely packed granular masses such as sands that describe an increase in volume at failure upon shearing (Reynolds 1885). Such dilation observed in interlocked aggregates while shearing forms the basis of additional energy that is manifested in terms of observed peak strength during triaxial testing. The rate of dilation during triaxial testing also affects the formation and development of shear bands.

Cattoni et al. (2005) conducted suction-controlled triaxial compression tests on "Pazzolana Nera" a silty sand material and presented results from stress-dilatancy analyses. Some of the major conclusions were that increase in matric suction augments the effective stress and hence enhances peak stress; peak strength precedes the onset of maximum dilatancy; and after the peak failure the stress ratio tends to decrease. Fern et al. (2014 and 2015) presented stress-dilatancy analyses on data obtained from direct shear and triaxial testing of two poorly graded sands.

Although, several attempts both theoretical and experimental have been made in past to study stress-dilatancy relationship for dry and saturated soils (e.g., Taylor 1948; Thurairajah 1961; Rowe 1962; Schofield and Wroth 1968; Muhunthan and Olcott 2002; Atkinson 2007; Chen and Kutter 2009), very few researchers have attempted to perform stress-dilatancy analysis of unsaturated soils using experimental data from suction-controlled shear strength testing (e.g., Cattoni et al. 2005; Fern et al. 2014 and 2015).

This paper presents strength and stress-dilatancy analysis of data obtained from suction-controlled triaxial testing of compacted silty sand following conventional triaxial compression (CTC) stress path (Patil 2014).

2 MATERIALS AND METHODS

2.1 *Soil type and compaction properties*

Poorly graded silty sand with 8% of non-expansive clay was used to prepare homogenous and statically compacted triaxial specimens approximately of 71.1mm diameter and 142.2mm height. The compaction procedure produced specimens with over consolidation stress history with initial voids ratio and matric suction, approximately between 0.46-0.49 and 8-10 kPa, respectively.

Various tests including grain size analyses, specific gravity, standard proctor test, and soil water characteristic curve were performed, and more details can be obtained from Patil (2014), and Patil et al. (2016).

2.2 *Unsaturated soil triaxial testing techniques and methods*

A series of s-controlled (i.e., suction-controlled) triaxial tests were conducted at three net confining pressures, $\sigma_3 - u_a$ = 100, 200 and 300 kPa and at matric suction values of 50, 250, 500, and 750 kPa (Patil 2014). Axis-translation technique (Hilf 1954; Patil 2014) was implemented to apply and control matric suction within the test specimens during triaxial testing. The air-entry value of ceramic disc used for testing at s = 50 and 250 kPa was 500 kPa while that used for testing at s = 500 and 750 kPa was 1500 kPa. A suitable shearing rate of 0.0086%/min was determined through a previous series of strain-controlled and s-controlled consolidated drained (CD) triaxial tests (Patil 2014).

Details on triaxial experimental set-up modified to implement axis-translation technique and step-by-step procedure followed during testing including mounting specimen in triaxial cell, imposing matric suction inside specimen, applying external net confining stress, conducting s-controlled isotropic consolidation, and s-controlled shearing along CTC stress path are published in Patil (2014) and Patil et al. (2016).

3 TYPICAL UNSATURATED TEST RESULTS

Typical stress-strain and volume change curves obtained from s-controlled triaxial testing are shown in Fig.1. Clearly, the shear strength at peak and critical state increased with increasing soil suction and the relationship was non-linear. The soil specimen showed initial compression followed by volumetric expansion, typical of densely compacted soil having over consolidated stress history. The amount of volumetric dilation increased with increasing suction. Although, not shown here, an increase in net confining stress suppressed the amount of dilation (Patil et al. 2016).

Fredlund and Morgenstern's (1977) approach is used to calculate the increase in unsaturated shear strength both at peak and critical state failure and is expressed by Eq. 1 below:

$$\tau = c' + (\sigma - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \qquad (1)$$

where c' = effective cohesion; ϕ' = effective friction angle associated with net confining stress ($\sigma - u_a$); ($u_a - u_w$) = matric suction with u_a = pore air pressure and u_w = pore water pressure; ϕ^b = friction angle associated with a change in suction ($u_a - u_w$). The variation of angle ϕ^b was found to be non-linear with increasing suction and this agrees with previous findings (Fredlund and Morgenstern 1977; Escario and Saez 1986). The increase in shear strength due to increasing suction is simply expressed by the last term in Eq. 1 as below:

$$\tau_{us} = c'' = (u_a - u_w)_f \tan\phi^b \qquad (2)$$

Where, c'' = apparent cohesion due to increase in matric suction and is equal to increase in unsaturated soil shear strength, τ_{us}.

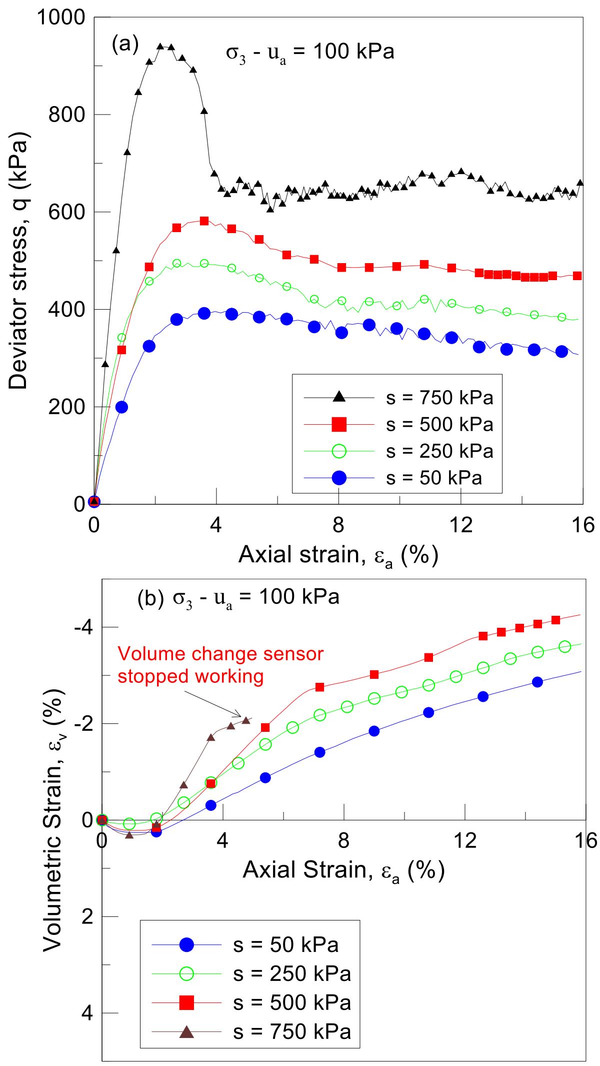

Figure 1. (a) Stress-strain and (b) volume change response of silty sand from s-controlled triaxial testing (Patil et al. 2016).

4 ANALYSES OF EXPERIMENTAL DATA

4.1 *Initial elastic modulus*

The ratio of deviator stress to axial strain at elastic limit is referred to as modulus of elasticity. In this analysis, this also happens to be a maximum value of modulus of elasticity (E_{max}) and occurs at axial strain less than 1%, irrespective of net confining stress and soil suction applied. The modulus of elasticity (E) was interpreted from stress-strain curves as the bender elements/Hall-effect transducers were not installed in the experimental set-up. The value of E increased with increasing suction at each net confining stress applied. The values of modulus of elasticity and net confining stress for each value of matric and total suction are normalized with atmospheric pressure, P_{atm}, and plotted on log-log scale (Fig. 2).

The experimental data plotted can be fitted well by straight lines (Fig 2) and the equation of the best fit lines on the log-log plot is expressed as below:

$$E_{\max} = KP_a\left(\frac{(\sigma_3 - u_a)}{P_a}\right)^n \tag{3}$$

Where, E_{max} = maximum value of secant elastic modulus; P_a = atmospheric pressure; $(\sigma_3 - u_a)$ = net confining stress; K = the intercept of best fit line at $(\sigma_3 - u_a)/Pa = 1$, net confining stress; n = slope of best fit line.

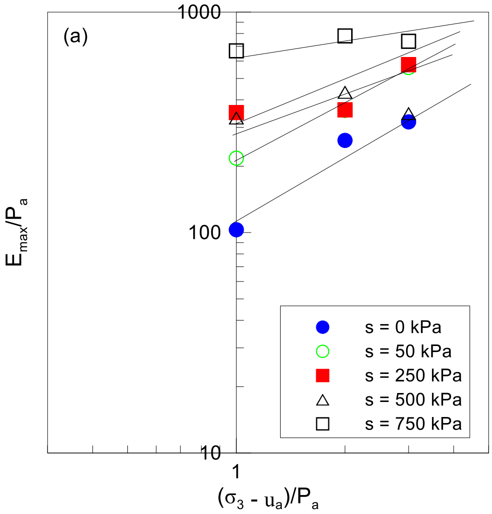

Figure 2. Variation of maximum value of elastic modulus, E_{max}, with confining stress, $(\sigma_3 - u_a)$.

Figure 3 shows the comparison between the experimental and predicted values of E_{max} for three values of net confining stress, $(\sigma_3 - u_a)$ = 100, 200 and 300 kPa; and four values of matric suction s = 50, 250, 500, and 750, kPa. Good correlation is obtained between experimental and predicted values with R^2 = 0.97.

Clearly, there is an upward shift in the best fit lines with increasing suction, indicating an increase in stiffness due to augmentation in effective stress. The value of K and n are calibrated using experimental data at each value of matric suction by plotting E_{max} and net confining stress $(\sigma_3 - u_a)$, both normalized by atmospheric pressure P_a as illustrated by Fig. 2. Furthermore, Eq. 3 is used to predict the value of E_{max} at each value of matric suction and net confining stress. In general, the value of K (unitless) tends to increase while that of n (unitless) decrease with increasing suction and indicate impact of additional cementation from increasing level of suction.

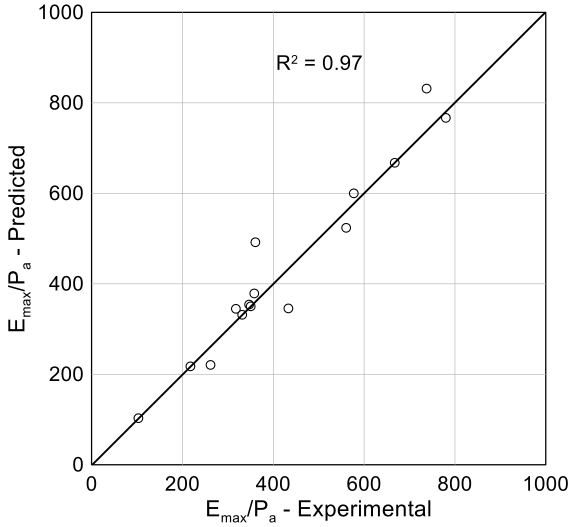

Figure 3. Comparison between experimental and predicted values of E_{max}.

4.2 *Shear strength failure envelopes*

Strength values of major and minor net principal stress at peak failure obtained from *s*-controlled triaxial testing are used to plot the peak failure envelopes in the form of q-p plot with co-ordinates $[(\sigma_1 - u_a) - (\sigma_3 - u_a)]/2$ against $[(\sigma_1 - u_a) + (\sigma_3 - u_a)]/2$. The experimental data is fitted with straight lines having certain slope and intercept.

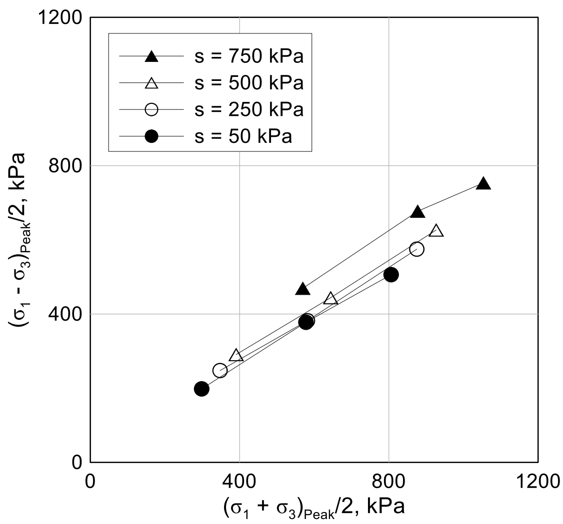

Figure 4. Strength envelopes at peak state failure.

Clearly, an increase in suction causes an upward and rightward shift of failure envelope that explains an increase in strength with increasing suction. However, the slope of failure envelopes remains pretty much constant indicating that the friction angle remains virtually constant despite increase in suction. On the other hand, the cohesion intercept and hence the cementation effect increases with increasing suction. Figure 4 indicates the failure envelopes at peak failure state.

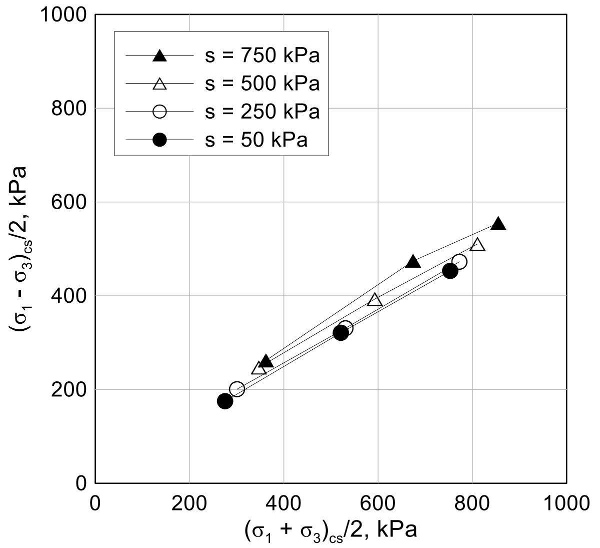

Figure 5. Strength envelopes at critical state failure.

Similarly, failure envelopes were plotted at critical state failure as shown in Figure 5. Although, an upward and rightward shift in failure envelope was observed with increasing suction, the amount of shift was less in magnitude as compared to the one obtained at peak failure.

4.3 *Stress-dilatancy relationship*

The below sections present stress-dilatancy analysis of data obtained from suction-controlled triaxial testing of compacted silty sand following conventional triaxial compression (CTC) stress path (Patil 2014).

4.3.1 *Rowe's (1962) stress-dilatancy relation*

Rowe (1962) proposed Eq. 4 that provides relationship between the peak stress ratio $(\sigma_1/\sigma_3)_{peak}$ and critical state ratio $(\sigma_1/\sigma_3)_{cs}$ via incremental dilatancy rate, D expressed as the ratio $d\varepsilon_v/d\varepsilon_1$; where $d\varepsilon_v$ is the incremental volumetric strain produced during shearing and $d\varepsilon_1$ is the incremental axial strain applied. In past, Fern et al. 2014 conducted constant water content tests via direct shear test on poorly graded Chiba and Cornell sand and Triaxial test on poorly graded Chiba sand (Fern et al. 2015).

Furthermore, they conducted stress-dilatancy analyses using an expression similar to the one by Rowe (1962) on experimental data and concluded that an increase in peak strength was a consequence of critical strength and increase in dilatancy rate. Although matric suction was not controlled in previous experiments, in this research the matric suction was controlled during shearing (Patil 2014).

$$\left(\frac{\sigma_1}{\sigma_3}\right)_{peak} = (1+D)\left(\frac{\sigma_1}{\sigma_3}\right)_{cs} \tag{4}$$

The value of dilatancy, D in Eq. 4 is obtained by subtracting dilatancy rate observed at critical state from the dilatancy rate at peak failure. Equation 4 has been modified by replacing σ_1 to σ_1-u_a and σ_3 *to* σ_3-u_a to more appropriately represent the unsaturated triaxial testing conditions as shown in Equation 5.

$$\left(\frac{\sigma_1-u_a}{\sigma_3-u_a}\right)_{peak} = (1+D)\left(\frac{\sigma_1-u_a}{\sigma_3-u_a}\right)_{cs} \tag{5}$$

During s-controlled triaxial testing continuous measurement of incremental stress and strains were recorded and later used to calculate the dilatancy rate at each stage of triaxial shearing. Thus, the dilatancy rate at peak and critical state failure were calculated for each test. The value of stress ratio at critical state failure is then predicted using Eq. 5.

Figure 6 shows the comparison of observed and predicted values of stress ratio at peak failure for three values of net confining stress, $(\sigma_3 - u_a)$ = 100, 200 and 300 kPa; and four values of matric suction s = 50, 250, 500, and 750 kPa. Good correlation was obtained between experimental and predicted values with R^2 = 0.97 (Fig. 6).

Thus, the stress dilatancy relationship (Eq. 4) proposed by Rowe (1962) showed promising application to unsaturated soils.

4.3.2 *Schofield and Wroth's (1968) stress-dilatancy relation*

Based on the experimental work by Thurairajah (1961) and Taylor (1948) analyses, Schofield and Wroth (1968) proposed a stress-dilatancy relationship between the effective mean normal stress (p), the slope of critical state line (*M*) in p-q plane and the incremental plastic shear strain (ratio $d\varepsilon_v^p/d\varepsilon_q^p$) as given below:

$$\eta = \frac{q}{p'} = M - \frac{d\varepsilon_v^p}{d\varepsilon_q^p} \tag{6}$$

Cattoni et al. (2005) used similar expression for calculating dilatancy while analyzing the suction-controlled triaxial test data on silty sand. This stress dilatancy relationship was verified using the experimental data. Eq. 6 was used to predict the stress ratio q/p′ and compared with the one calculated using experimental data. The comparison between experimentally observed and predicted values showed good correlation with R^2 = 0.70 for all the value of matric suction and net confining stress as shown in Fig. 7.

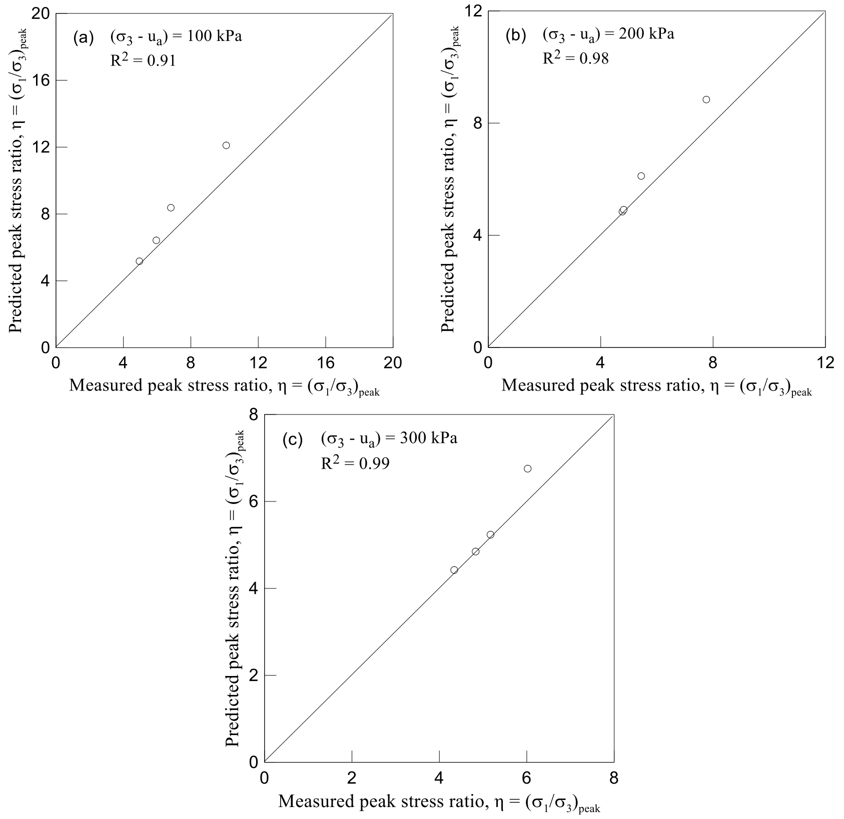

Figure 6. Comparison between experimental and predicted values of peak stress ratio at (a) (σ_3-u_a) = 100 kPa; (b) (σ_3-u_a) = 200 kPa; (c) (σ_3-u_a) = 300 kPa (Rowe 1962).

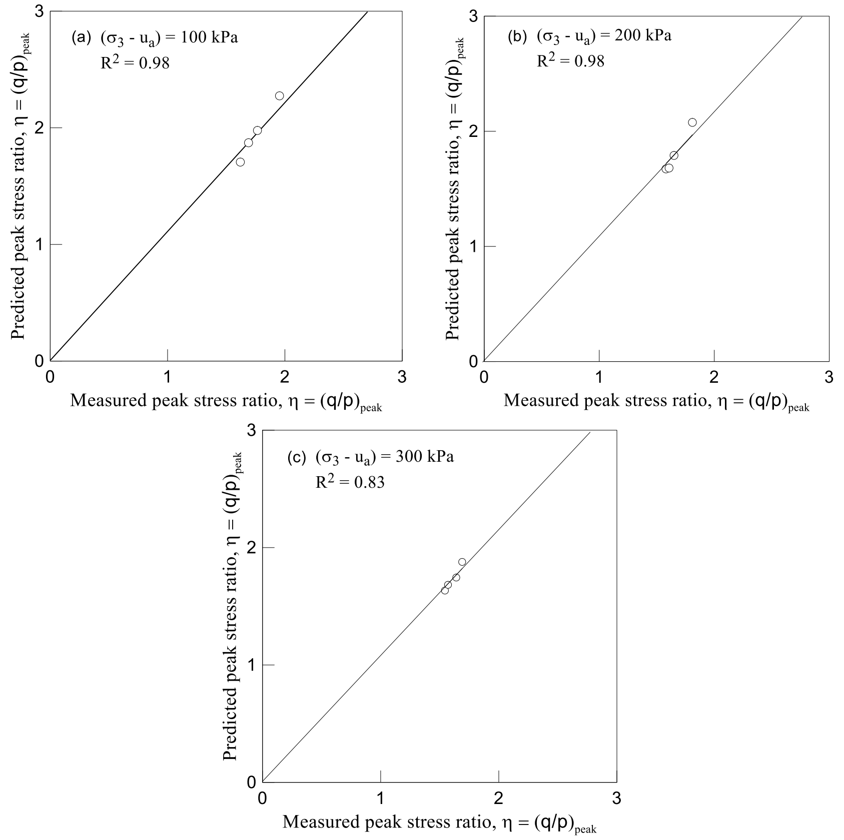

Figure 7. Comparison between experimental and predicted values of peak stress ratio at (a) (σ_3-u_a) = 100 kPa; (b) (σ_3-u_a) = 200 kPa; (c) (σ_3-u_a) = 300 kPa (Schofield and Wroth (1968)).

5 CONCLUSIONS

Strength, stiffness and stress-dilation behavior of statically compacted silty sand soil is studied at three different net confining pressures, ($\sigma_3 - u_a$) = 100, 200 & 300 kPa and four different matric suction values (s = 50-750 kPa). As expected, an increase in matric suction and net confining stress caused an increase in effective stress and hence increase in shear strength and stiffness of unsaturated soil. Augmentation in peak stress ratio is found to be dependent on the magnitude of matric suction imposed, observed maximum dilatancy and critical stress ratio.

Increase in matric suction caused an upward and rightward shift in critical and peak state failure envelopes. Predictions from popular stress-dilatancy models proposed by Rowe (1962) and Schofield and Wroth (1968) were compared with experimental data obtained from s-controlled triaxial testing of compacted silty sand. Good correlation was obtained between experimental and predicted values and hence it showed promising applications of both stress-dilatancy models to unsaturated soil.

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