Shakedown behaviour of expansive soils under wetting and drying cycles

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ABSTRACT: To investigate the hydro-mechanical behaviour of unsaturated expansive soils, many laboratory tests have been carried out concerning the effect of suction cycles on expansive soils. All tests show that the soil tends towards an equilibrium state at the end of suction cycles, representing the final shakedown behaviour of expansive soil after suction cycles. In this context, this paper presents a shakedown-based model in numerical formation to predict the volume change in unsaturated expansive soils under wetting and drying cycles. The parameters of this shakedown-based model were calibrated and validated for the different initial states of one expansive soil. The comparisons between the experimental results and the predictions for the different tests, demonstrate the capacity of shakedown modelling to simulate the mechanical behaviour of unsaturated expansive soils.

1 INTRODUCTION

Expansive soils susceptible to be affected by the environmental conditions, expand when water is added and shrink when they dry out, because of their mineralogical composition. This continuous change in soil volume can cause structures built on them to move unevenly and crack. To investigate this hydro-mechanical behaviour of expansive soil, a large number of laboratory tests have been conducted concerning the influence of wetting and drying cycles on unsaturated expansive soils. The test results reported by Alonso et al. (1999, 2005), Nowamooz and Masrouri (2008), Nowamooz et al. (2013) and Sun et al. (2011) show that the differences between two successive wetting and drying paths become smaller as the number of cycles increases. Obviously, the soil tends towards an elastic equilibrium state at the end of suction cycles, representing the final shakedown behaviour of expansive soil after suction cycles.

Shakedown concept was introduced for the first time to the unbound granular materials by Sharp and Booker (1984), which defined shakedown load as the key design load. Whereafter, Habiballah et al. (2005), Allou et al. (2007) and Chazallon et al. (2009) have developed a shakedown model based on Zarka simplified theory (Zarka & Casier 1979; Zarka et al. 1990) with a non-associated flow rule, for unbound granular materials when they are subjected to repeated triaxial loads in order to determine the accumulation of plastic strains due to the long-term traffic loading.

In this context, this paper firstly presents a shakedown-based model in a finite element formation for the future simulation of the volume change problem in expansive soils subjected to seasonal wetting and drying cycles. The required parameters of this model are secondly calibrated by the experimental results obtained for an expansive soil compacted at loose and dense initial states. Finally, the proposed model is validated by the intermediate state of the same expansive soil, and the comparison with the test results shows the capacity of shakedown modelling.

2 NUMERICAL MODELLING OF EXPANSIVE SOILS

In this section, a structure of unsaturated expansive soils with an elasto-plastic behaviour is considered. Its boundary Γ is subjected to the imposed surface forces $F_i(x)$ on Γ_{Fi} partition and the prescribed surface displacements $U_j(x)$ on Γ_{Uj} partition. The body force $X_j(x)$ and the initial strain ε_{ij} $(x,t=0)$ are defined in the volume V. Moreover, the wetting and drying cycles are also defined in the volume V and these cycles are imposed between extremely dry and wet conditions.

2.1 *Mechanical analysis*

In this study, the authors suppose that the studied elasto-plastic structure meets the requirements of small displacements and small deformations.

In-situ stress analysis is firstly performed before shakedown modelling of the structure subjected to wetting and drying cycles. Considering the finite element calculation with the imposed boundary conditions, the stress state can be solved by the linear elasticity:

$$\sigma_{ij}^{*}(x,t)=D_{ijkl}\cdot\left[\varepsilon_{ij}^{*}(x,t)-\varepsilon_{ij}(x,0)\right] \tag{1}$$

where, D_{ijkl} is the matrix of elastic moduli; $\sigma_{ij}^{*}(x,t)$ is the stress tensor and $\varepsilon_{ij}^{*}(x,t)$ is the strain tensor. Here, the superscript * represents the parameters calculated from mechanical analysis. With this calculation, the net mean stress for each point in the structure can be determined, which will be used in the following shakedown modelling.

2.2 *Real response of suction variation*

For the suction variation in the volume of structure, the real response of the studied structure can be written as follows:

$$\varepsilon_{ij}(x,t)=M_{ijkl}\cdot s_{kl}(x,t)+\varepsilon_{ij}^{p}(x,t)+\varepsilon_{ij}^{I}(x,0) \tag{2}$$

where, M_{ijkl} is the compliance elasticity matrix for suction loading, which regresses to $1/E_r$ in terms of one-dimensional case. Additionally, $\varepsilon_{ij}^{p}(x,t)$ is the plastic strain tensor and the strain tensor $\varepsilon_{ij}(x,t)$ is kinematically admissible with $U_j(x,t)$ on Γ_{Uj}.

In this equation, the suction tensor $s_{ij}(x,t)$ can be expressed by:

$$s_{ij}=(u_a-u_w)\cdot\delta_{ij}=s\cdot\delta_{ij} \tag{3}$$

where, δ_{ij} is Kronecker delta whose value is 1 when $i=j$ and is equal to 0 otherwise.

2.3 *Elastic response of suction variation*

The response associated with the elastic part is expressed as follows:

$$\varepsilon_{ij}^{el}(x,t)=M_{ijkl}\cdot s_{kl}^{el}(x,t)+\varepsilon_{ij}^{I}(x,0) \tag{4}$$

where, I_{ijkl} is the identity tensor, $I_{ijkl}=1$ if $i=j=k=l$, and the strain tensor $\varepsilon_{ij}^{el}(x,t)$ is kinematically admissible with $U_j(x,t)$ on Γ_{Uj}.

Thus, the elasticity fields $U_i^{el}(x,t)$ and $\varepsilon_{ij}^{el}(x,t)$ can be calculated by an elastic analysis with the given boundary conditions and the compliance elasticity matrix for suction loading M_{ijkl}.

2.4 *Inelastic response of suction variation*

Because the general problem can be decomposed into elastic part and inelastic part, the inelastic strain can be expressed by the following equation:

$$\varepsilon_{ij}^{ine}(x,t)=\varepsilon_{ij}(x,t)-\varepsilon_{ij}^{el}(x,t) \tag{5}$$

where, $\varepsilon_{ij}^{ine}(x,t)$ is kinematically admissible with 0 on Γ_{Uj}.

Considering Equations (2 and 4), the above equation can be written as:

$$\varepsilon_{ij}^{ine}(x,t)=M_{ijkl}\cdot\rho_{kl}(x,t)+\varepsilon_{ij}^{p}(x,t) \tag{6}$$

The residual field $\rho_{kl}(x,t)$ is obtained by the difference between total suction and elastic suction fields:

$$\rho_{ij}(x,t)=s_{ij}(x,t)-s_{ij}^{el}(x,t) \tag{7}$$

where, $\rho_{ij}(x,t)$ is statically admissible with 0 in V.

As long as the plastic strain tensor $\varepsilon_{ij}^{p}(x,t)$ and the compliance elasticity matrix for suction loading M_{ijkl} are known, the inelastic problem can be solved with null boundary condition and consequently the inelastic fields $U_i^{ine}(x,t)$ and $\varepsilon_{ij}^{ine}(x,t)$ are obtained.

Eventually, the residual field can be derived from equation 6 as follows:

$$\rho_{ij}(x,t)=M_{ijkl}^{-1}\cdot\left[\varepsilon_{ij}^{ine}(x,t)-\varepsilon_{ij}^{p}(x,t)\right] \tag{8}$$

In the following section, this proposed method will be generalized to model the response of the full-scale structures.

2.5 *Structure with kinematic hardening*

The yield surface with a kinematic hardening can be defined by:

$$f=\sqrt{(s_{ij}-y_{ij})\cdot(s_{ij}-y_{ij})}-s_{\alpha} \tag{9}$$

where, s_{α} is the threshold value of elastic limit for suction variation and y_{ij} is a kinematic hardening tensor which can be related to the plastic strain,

$$y_{ij}=h\cdot\varepsilon_{ij}^{p} \tag{10}$$

where, h is the kinematic hardening modulus, a critical important parameter in shakedown modelling.

By rewriting the Equation 7, the suction field can be expressed by:

$$s_{ij}(x,t)=s_{ij}^{el}(x,t)+\rho_{ij}(x,t) \tag{11}$$

Here, the field of transformed structural parameters Y_{ij} (x,t) is defined:

$$Y_{ij}(x,t)=y_{ij}(x,t)-\rho_{ij}(x,t) \tag{12}$$

Considering the above equations, the yield surface can be expressed by:

$$f\left(s_{ij}^{el}-Y_{ij}\right)\leq 0 \tag{13}$$

This equation indicates that the yield surface is centred in s_{ij}^{el} and translates in the transformed structural parameter Y_{ij} plane.

With the transformed structural parameter field (Equations 6, 10 and 12), the inelastic problem can be solved by:

$$\varepsilon_{ij}^{ine}\left(x,t\right)=M'_{ijkl}\cdot\rho_{kl}\left(x,t\right)+1/h\cdot Y_{ij}\left(x,t\right) \tag{14}$$

where, M_{ijkl} with prime symbol is the modified compliance elasticity matrix for suction loading, defined by the following equation:

$$M'_{ijkl}=M_{ijkl}+1/h\cdot I_{ijkl} \tag{15}$$

where, I_{ijkl} is the identity tensor, $I_{ijkl}=1$ if $i=j=k=l$.

Considering Equation 14, the residual field of the elasto-plastic structure is obtained:

$$\rho_{ij}\left(x,t\right)=M'^{-1}_{ijkl}\cdot\left[\varepsilon_{ij}^{ine}\left(x,t\right)-1/h\cdot Y_{ij}\left(x,t\right)\right] \tag{16}$$

Finally, the plastic strain field is given by combining Equation 10 and 12:

$$\varepsilon_{ij}^{p}\left(x,t\right)=1/h\cdot\left[Y_{ij}\left(x,t\right)+\rho_{ij}\left(x,t\right)\right] \tag{17}$$

Consequently, a one-to-one relation exists between the kinematic hardening variable field $y_{ij}(x,t)$ and the transformed structural parameter field $Y_{ij}(x,t)$. At any time t, for a given kinematic hardening variable field $y_{ij}(x,t)$, there is a unique residual field $\rho_{ij}(x,t)$, a unique suction field $s_{ij}(x,t)$ as well as a unique transformed structural parameter $Y_{ij}(x,t)$, and vice versa. Within this framework, the inelastic problem is solved through elastic analysis with a null boundary condition and a modified compliance elasticity matrix for suction loading. Eventually, all unknown fields at the limit state are obtained.

2.6 *Structure response under suction cycles*

During successive wetting and drying cycles, the elastic suction field can be expressed by:

$$s_{ij}^{el}\left(x,t\right)=\left[1-\wedge\left(t\right)\right]\cdot s_{ij_{\min}}^{el}\left(x\right)+\wedge\left(t\right)\cdot s_{ij_{\max}}^{el}\left(x\right) \tag{18}$$

where, $\wedge(t)$ is a monotonic periodic function, varied between 0 and 1.

The local suction at the level of the plastic mechanisms is expressed as:

$$\tilde{s}_{ij}\left(x,t\right)=s_{ij}\left(x,t\right)-y_{ij}\left(x,t\right) \tag{19}$$

In the local suction plane, the plasticity convex domains C_0 is a fixed segment on the isoclinic suction axis. The normality law is written with the Moreau's notation (Moreau 1971):

$$\dot{\varepsilon}_{ij}^{p}\in\partial\phi_{C_0}\left(\tilde{s}_{ij}\right)\quad\text{with}\quad\tilde{s}_{ij}\in C_0 \tag{20}$$

where the plastic strain rate is an external normal to the convex C_0.

At the maximum suction state, the transformed structural parameter at the level of inelastic mechanism is expressed by:

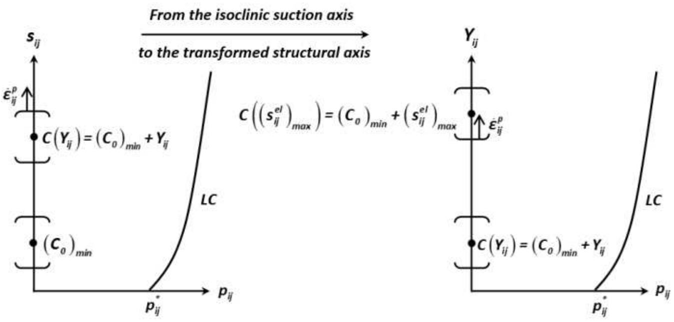
$$Y_{ij}=-\tilde{s}_{ij}+s_{ij_{\max}}^{el} \tag{21}$$

with

$$Y_{ij}\in C\left(s_{ij\max}^{el}\right)\ \text{ and }\ C\left(s_{ij_{\max}}^{el}\right)=\left(C_0\right)_{\min}+s_{ij_{\max}}^{el} \tag{22}$$

This equation implies that Y_{ij} belongs to the convex C obtained from $(C_0)_{min}$ with the translation of maximal value of cyclic suction(Figure 1). The normality law is:

$$\dot{\varepsilon}_{ij}^{p}\in-\partial\phi_{C\left(s_{ij_{\max}}^{el}\right)}\left(Y_{ij}\right)\ \text{ with }\ Y_{ij}\in C\left(s_{ij_{\max}}^{el}\right) \tag{23}$$



where, the plastic strain rate is an internal normal to the convex C.

Figure 1. Evolution of plasticity convex along the plane of (s_{ij}-p_{ij}) and the plane of (Y_{ij}-p_{ij}).

A loading collapse yield surface is presented in Figure 1, and it shows the increase of the pre-consolidation stress p_{ij} with the suction increase. In this study, no plastic deformation is generated by the mechanical loading during the wetting and drying cycles because the p_{ij} value was selected less than p_{ij}* representing the pre-consolidation stress at the saturated state (suction= 0).

When the suction loading becomes very large, a stationary state can be reached, and plastic shakedown is achieved. In this case, the distance between two extreme positions of the mobile convex in the transformed structural parameter plane can be obtained:

$$\left\|\Delta Y_{ij}\right\|=\left\|s_{ij_{\max}}^{el}\right\|-\left\|s_{ij_{\min}}^{el}\right\|-2\cdot s_{\alpha} \tag{24}$$

Finally, the value of ΔY_{ij} can be written as:

$$\Delta Y_{ij}=\left(1-\frac{2\cdot s_{\alpha}}{\left\|\Delta s_{ij}^{el}\right\|}\right)\cdot\Delta s_{ij}^{el} \tag{25}$$

In these two equations, $\|\bullet\|$ is the notation of 2-norm.

3 MODLE CALIBRATION AND VALIDATION

3.1 *Structure with kinematic hardening*

Nowamooz and Masrouri (2008) and Nowamooz et al. (2013) have performed a series of laboratory tests on an artificially prepared mixture of 40% silt and 60% bentonite. The samples were compacted with an initial water content of 15% under three vertical pressures: 1000, 2000 and 3000kPa and three dry densities were formed: 1.27, 1.48 and 1.55 $Mg \cdot m^{-3}$, corresponding to loose intermediate and dense samples, respectively. The initial suction of all compacted samples is considered to be 20 MPa. The suction-controlled oedometer tests with suction cycles between 8 and 0 MPa under three constant net mean stresses: 15, 30 and 60 kPa, have been carried out, which will be used to calibrate and validate the proposed shakedown model.

3.2 *Required parameters of shakedown-based model for loose and dense samples*

3.2.1 *Elasticity parameters*

The elasticity parameter (E_r) can be estimated at the elastic equilibrium state of suction-controlled tests reached after several wetting and drying cycles. Tables 1 and 2 summarize the E_r value of loose and dense samples for the different net mean stresses 15, 30 and 60 kPa. In Figure 2, the authors propose a linear variation of the inverse of the elasticity parameter ($1/E_r$) with the net mean stress (p):

$$1/E_r = A \cdot p + B \quad (26)$$

where, A and B are constant parameters related to the initial dry density of expansive soil given in Table 3 for the loose and dense samples.

Table 1. Required parameters of shakedown-based model for loose sample.

Applied stress (kPa)	15	30	60
E_r (MPa)	204	103	90
h (MPa)	126	79	60
s_α (MPa)	-	-	-

- In this study, s_α is considered negligible.

Table 2. Required parameters of shakedown-based model for dense sample.

Applied stress (kPa)	15	30	60
E_r (MPa)	53	55	51
h (MPa)	-125	-163	-540
s_α (MPa)	-	-	-

Table 3. Estimated parameters A, B, C and D for the loose and dense initial states.

Parameters	A(MPa^{-2})	B(MPa^{-1})	C(MPa^{-2})	D(MPa^{-1})
Loose	0.125	0.419E-2	0.188	0.589E-2
Dense	0.180E-1	0.182E-1	0.138	-0.101E-1

3.2.2 *Plasticity parameters*

For the application of Zarka's shakedown theory (Zarka & Casier 1979; Zarka et al. 1990) on soil mechanics, the threshold value of elastic limit for suction variation (s_α) may be considered very small and therefore it is neglected in this work.

Since knowing the accumulated plastic deformation ($\Delta\varepsilon_{vs}^{p}$) as well as the transformed internal parameter (Δy_{ij}) at a given net mean stress, the hardening modulus (h) can be deduced according to Equation 10. Because of the volumetric shrinkage strains for the loose samples during the suction cycles, a positive sign was attributed to h values. Tables 1 and 2 summarize these calibrated parameters of both samples. Figure 3 illustrates also the evolution of the inverse of the hardening modulus ($1/h$) with the net mean stress (p). The larger the applied vertical pressure, the larger the inverse of the hardening modulus ($1/h$). The authors propose additionally a linear variation of the inverse of the hardening modulus ($1/h$) with the net mean stress (p):

$$1/h = C \cdot p + D \quad (27)$$

where, C and D are constant parameters related to the initial dry density of expansive soils, given in Table 3 for the loose and dense samples. In contrast, this modulus is a constant for steels and it depends on the loading paths for unbound granular materials.

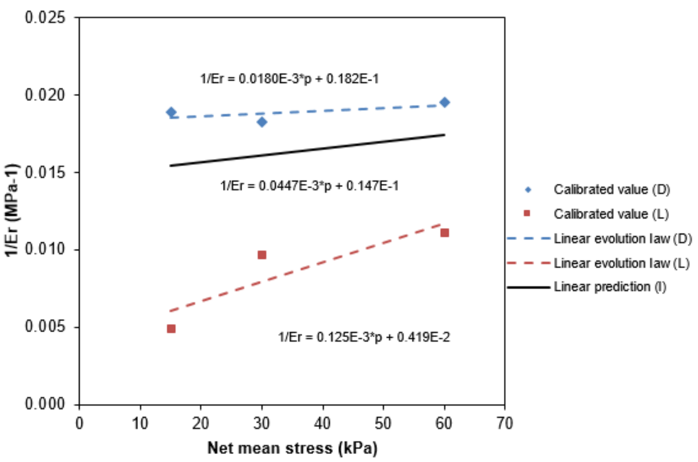


Figure 2. Variation of the inverse of the resilient modulus ($1/E_r$) with net mean stress for the loose and dense samples as well as the prediction for the intermediate samples.

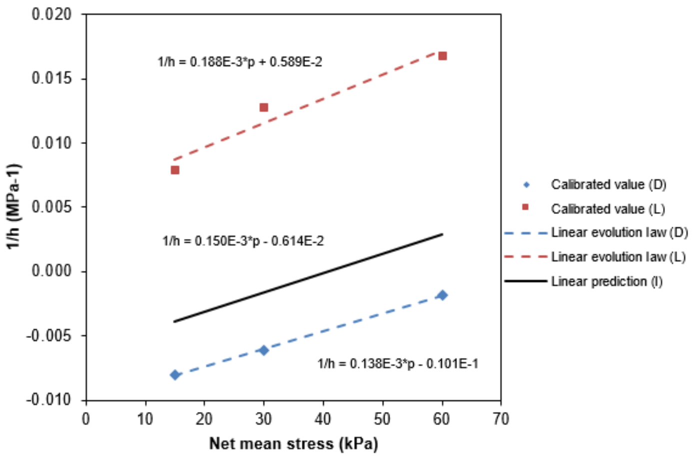


Figure 3. Variation of the inverse of the hardening modulus ($1/h$) with net mean stress for the loose and dense samples as well as the prediction for the intermediate samples.

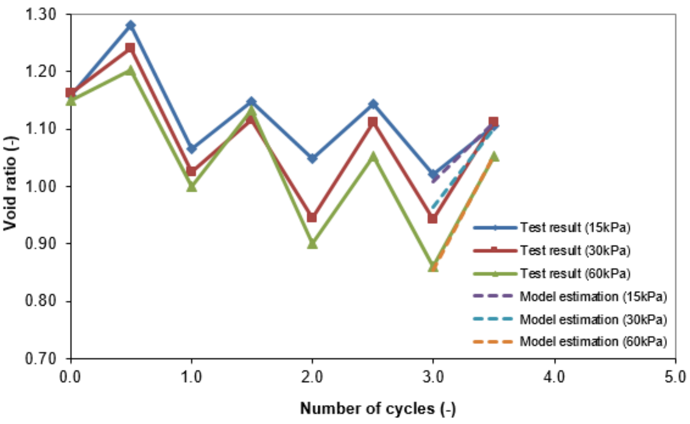


Figure 4. Comparison of results with model estimations for the loose samples at different net mean stresses.

Figures 4 and 5 present the model results compared with the experimental results for the loose and dense samples at the net mean stresses of 15, 30 and 60 kPa. It can be observed that the wetting and drying cycles generate the shrinkage strain accumulation for the loose samples and the swelling strain accumulation for the dense samples. It can be also noted that model estimations produce a good agreement with the test results at the different net mean stresses.

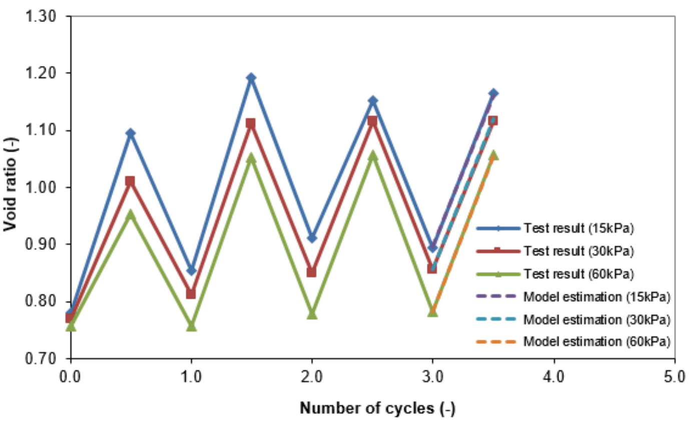


Figure 5. Comparison of results with model estimations for the dense samples at different net mean stresses.

3.2.3 *Model validation*

The validation of the model is carried out with the test results obtained for the samples compacted at the intermediate initial state (Nowamooz et al. 2013). The linear variation of the inverse of the elasticity parameter ($1/E_r$) as well as the hardening modulus ($1/h$) with the net mean stresses is interpolated in Figures 2 and 3 for these samples. The estimated parameters A, B, C and D of equations 26 and 27 are presented in Table 4. For intermediate samples, the threshold value of elastic limit for suction variation (s_α) is also considered negligible.

Table 4. Predicted parameters of shakedown-based model for intermediate sample.

Parameters	A(MPa^{-2})	BMPa^{-1})	CMPa^{-2})	DMPa^{-1})
Intermediate	0.447E-1	0.147E-1	0.150	-0.614E-2

Based on these predicted model parameters, the model validation is conducted for the intermediated samples. Figure 6 represents the comparison between the test results and the model predictions at the different net mean stresses. For these samples, the initial state is closer to the reversible line which need less suction cycles to obtain the equilibrium state. The relative tolerance varies between 5% and 8% confirming the capacity of the proposed model to estimate the accumulated plastic strains.

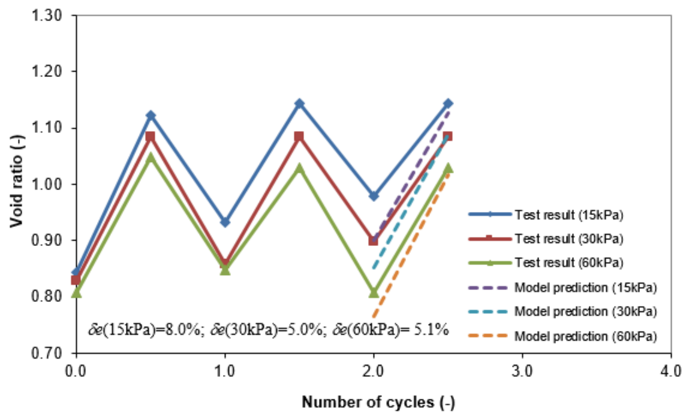


Figure 6. Comparisons of test results with model predictions for intermediate sample at the different net mean stresses.

4 CONCLUSIONS

With the framework of shakedown theory, a finite element model is proposed for the unsaturated expansive soils subjected to wetting and drying cycles. The proposed shakedown model requires five parameters to be defined: parameters A and B for the linear evolution law of elasticity parameters ($1/E_r$) determined from the equilibrium state at the end of the wetting and drying cycles; parameters C and D

for the linear evolution law of hardening modulus ($1/h$) calibrated from the accumulated plastic strains during the wetting and drying cycles; and the threshold value of elastic limit for suction variation (s_{α}), taken zero for the sake of simplification.

Calibrations of model parameters with laboratory tests have been performed with loose and dense samples. The prediction of the intermediate state is carried out and good results have been obtained. The proposed model is able to take into account the mechanical behaviour of unsaturated expansive soils with a wide range of density. The future work is to implement the proposed numerical shakedown model into a finite element code to simulate the mechanical behaviour of different structures constructed on expansive soils submitted to wetting and drying cycles.

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