Bearing capacity equations for shallow foundations on unsaturated soils with uniform and linearly varied suction profiles

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ABSTRACT: Shallow foundations on unsaturated soils are of major interest in civil engineering practice since many structures are built on soils for which unsaturated conditions dominate. The conventional bearing capacity equations proposed, for example, by Terzaghi or Vesic, do not consider the effect of suction on the strength and the bearing capacity of the soil and thus may lead to uneconomical design of foundations. This paper presents new bearing capacity equations for shallow foundations on unsaturated soil by modifying the conventional bearing capacity equations to account for the effect of suction. The effective stress principle is adopted. These equations consider both uniform and linearly varied suction profiles. The proposed equations are validated based on laboratory experiments and in-situ plate load test results reported in the literature. Reasonable agreement is obtained between predicted and measured bearing capacities when the general shear failure is the governing mechanism.

1 INTRODUCTION

Shallow foundations are one of the most cost-effective methods for transmitting loads of buildings and other structures to the underlying ground. The bearing capacity is one of the key parameters in the foundation design. It is conventionally estimated using the bearing capacity equations by assuming the soil is fully saturated or completely dry (Terzaghi 1943; Meyerhof 1953; Hansen 1970; Vesic 1973). However, unsaturated soils are widely distributed throughout the world. A large portion of shallow foundations are located in the unsaturated zone above the ground water table. Suction developed within the unsaturated soil significantly increases the bearing capacity of shallow foundations (Costa et al. 2003; Rojas et al. 2007; Oh & Vanapalli 2011; Vanapalli & Mohamed 2013). Therefore, the traditional bearing capacity equations, which do not consider the influence of suction, may lead to uneconomic and overconservative design of foundations.

There is also a need to understand the response of plate load tests on unsaturated soils to be able to evaluate or back-calculate the strength parameters of the ground properly (Costa et al. 2003; Oh & Vanapalli 2013) so that shallow foundations can be designed with confidence. The conventional interpretation of the results of plate load tests performed on unsaturated soils would lead to unrealistically large strength and stiffness parameters if the effect of suction is not considered. Yet, there have been only a few attempts to address the bearing capacity problem in unsaturated soils.

The bearing capacity (q_u) of footings on fully saturated or completely dry soil is generally calculated by Terzaghi's (1943) equation:

$$q_u = c'N_c + q'N_q + 0.5\gamma' B N_\gamma \quad (1)$$

where c' = cohesion; N_c, N_q and N_γ = bearing capacity factors which depend on the internal friction angle; ϕ' of the soil; q' = effective overburden pressure; γ' = effective unit weight of the soil; B = footing width.

Oloo et al. (1997), inspired by the shear strength equation proposed by Fredlund et al. (1978), presented a bearing capacity equation for surface footings on unsaturated soil by extending Terzaghi's equation and considering the effects of suction (s) as an apparent cohesion:

$$q_u = \left[c' + s_e \tan\phi' + \left(s - s_e\right)\tan\phi^b\right]N_c + 0.5\gamma B N_\gamma \quad (2)$$

where s_e = suction value marking the transition from saturated to unsaturated states; γ = unit weight of the unsaturated soil. A constant value of ϕ^b was used in their study and thus the bearing capacity increased linearly with suction. However, experimental data show that this relationship is not linear (Costa et al. 2003; Rojas et al. 2007; Oh & Vanapalli 2011; Vanapalli & Mohamed 2013). Experiments are required to determine ϕ^b for different types of soils and different suction values. To overcome this shortcoming, Vanapalli and Mohamed (2013) proposed the following equation based on the effective shear strength parameters:

$$q_u = \left[c' + s_e \tan\phi' + \left(s_{\mathrm{AVE}} - s_e\right) S_r^{\omega} \tan\phi'\right] N_c \zeta_c + qN_q \zeta_q + 0.5\gamma B N_\gamma \zeta_\gamma \quad (3)$$

where S_r = degree of saturation; ζ_c, ζ_q and ζ_γ = shape factors; ω is a fitting parameter related to the plasticity index of the soil, I_p and is defined as $\omega = 1 + 0.34(I_p) - 0.0031(I_p^2)$; s_{AVE} = average suction in the soil.

Vahedifard and Robinson (2016) proposed a bearing capacity equation for unsaturated soil using the effective degree of saturation:

$$q_u = \left[c' + s_e \tan\phi' + \left(s_{\mathrm{AVE}} - s_e\right) S_{eff,\mathrm{AVE}} \tan\phi'\right] N_c \xi_c + qN_q \xi_q + 0.5\gamma B N_\gamma \xi_\gamma \quad (4)$$

where $S_{eff} = (S_r - S_{res})/(1 - S_{res})$ = effective degree of saturation, in which S_{res} = residual degree of saturation; $S_{eff,\mathrm{AVE}}$ = average effective degree of saturation corresponding to the average suction in the stress bulb zone (1.5 times of the width or diameter of the footing).

The effect of varied suction profile is considered in Equations 3 and 4 by using the average value of suction. The effect of the variation of suction with depth may not be accurately included.

This paper, summarizing recent research findings by Tang et al. (2017), presents alternative equations which can be used to estimate the bearing capacity of shallow footings on unsaturated soil with uniform and linearly varied suction profiles. The proposed equations are general in nature, require a smaller number of parameters, and have the added advantage of being based on a sound and robust effective stress principle. The bearing capacity predicted by the proposed equations is compared with data from model footing tests and in-situ plate load tests published in the literature.

2 BEARING CAPACITY EQUATIONS FOR UNSATURATED SOILS WITH UNIFORM AND LINEARLY VARIED SUCTION PROFILES

2.1 *Effective stress principle*

By extending the concept of Terzaghi's (1943) effective stress principle to unsaturated soils, Bishop (1959) proposed an effective stress equation for unsaturated soil:

$$\sigma' = (\sigma - p_a) + \chi(p_a - p_w) = (\sigma - p_a) + \chi s \quad (5)$$

where σ' = effective stress; σ = total stress; p_a and p_w = pore air and pore water pressures, respectively; χ = the effective stress parameter.

The Mohr-Coulomb failure criterion, which is widely applied to saturated and dry soils, may be extended to unsaturated soil using the effective stress principle. The shear strength (τ) of unsaturated soil is expressed as:

$$\tau = c' + \left[(\sigma - p_a) + \chi s\right] \tan\phi' \quad (6)$$

In this equation, the shear strength is governed only by the effective stress and strength parameters c' and ϕ'. It has been found in the shear strength tests that c' and ϕ' are unique at critical state for saturated and unsaturated soils (Geiser 1999; Wheeler & Sivakumar 1995). The contribution of suction on the shear strength of unsaturated soil is taken into account by $\chi s \tan\phi'$. The shear strength can be calculated using Equation 6 although knowledge of χ is required.

The definition of χ has been reported by many researchers (e.g. Bishop 1960; Aitchison 1960; Kohgo et al. 1993; Khalili & Khabbaz 1998). Based on the shear strength test data of different types of unsaturated soils, Khalili and Khabbaz (1998) proposed a unique expression for χ:

$$\chi = \begin{cases} 1 & \textit{for} \quad \dfrac{s}{s_e} \le 1 \\ \left(\dfrac{s}{s_e}\right)^{-0.55} & \textit{for main} \text{ drying curve} \quad \dfrac{s}{s_e} > 1 \end{cases} \quad (7)$$

Definitions for χ like in Equation 7 have been widely adopted to describe the behaviour of a broad range of unsaturated soils (Loret & Khalili 2000; Khalili et al. 2004; Masin 2010).

2.2 *Bearing capacity equations*

It has been proved by Vo and Russell (2016) that the cohesion (c') and $\chi s \tan\phi'$ have similar and independent effects on the bearing capacity of unsaturated soil. If suction is uniformly distributed under a footing or the representative average suction in the stress bulb zone is available, a bearing capacity equation can be derived for unsaturated soil of the form:

$$q_u = \left[c' + (\chi s)_{\mathrm{AVE}} \tan\phi'\right] N_c d_c + qN_q d_q + 0.5\gamma B N_\gamma d_\gamma \quad (8)$$

in which $(\chi s)_{\mathrm{AVE}} = \chi s$ value corresponding to the average suction in the stress bulb zone. In this equation, the bearing capacity of unsaturated soil is estimated conveniently by the effective stress parameter, χ, and suction together with the drained shear strength parameters, c' and ϕ'. Equation 8 is consistent with the traditional bearing capacity equation for dry soil when suction is equal to 0. When the soil is fully saturated, the submerged unit weight of soil,

γ', should be used. When the soil is unsaturated, the contribution of suction on the bearing capacity may be simply considered by $\chi s\tan\phi'$. When the suction in the soil is smaller than the air entry value, χ is equal to 1 and Equation 8 is consistent with Equations 3 and 4. Equation 8 provides a smooth transition at the point of air entry value due to the continuous definition of χ.

In practice, suction in the soil above the water table may not be uniformly distributed (Lu & Griffiths 2004; Vo & Russell 2016). At a steady state, suction may be greater at the top of the vadose zone and decrease with depth and vanish at the ground water table. The suction profiles in unsaturated soil at steady state have been derived by theoretical studies (Lu & Griffiths 2004; Vo & Russell 2016). It has been found that linear approximations of χs profiles could be assumed for both evaporation and infiltration and the associated errors are small (Vo & Russell 2016). A linear χs profile in unsaturated soil may be defined as:

$$\chi s = \chi s_0 - \rho z \tag{9}$$

where χs_0 = value of χs at the base of the footing; z = depth below the footing base; $\rho = \partial(\chi s)/\partial z$ = a constant defining the variation of χs with depth.

The effect of the linear variation of χs with depth can be incorporated into the bearing capacity equation similar to a linear cohesion profile in saturated soil. The effect of the constant component of suction, s_0, on the bearing capacity is similar to the effect of apparent cohesion and can be included in the first term of the bearing capacity equation. The gradient of suction plays the same role as soil density in the bearing capacity and is considered in the third term of the bearing capacity equation. Therefore, a bearing capacity equation for unsaturated soil with linearly distributed suction may be expressed as:

$$\begin{aligned} q_u &= \left(c' + \chi s_0 \tan\phi'\right) N_c d_c + q N_q d_q \\ &\quad + 0.5B(\gamma - \rho) N_\gamma d_\gamma \end{aligned} \tag{10}$$

Assuming the water table under a footing is lower than the bottom of the stress bulb, Equations 8 and 10 can be used to estimate the bearing capacity of the footing on unsaturated soil with typical χs profiles which are shown in Figure 1.

A model footing test or an in-situ plate load test is most likely to be performed under a constant moisture content condition due to the relatively low permeability of unsaturated soils. It has been found by Tang et al. (2016) that χs could be assumed constant and the initial value can be used in the interpretation of the test results without loss of significant accuracy. In this study the initial values of χs are used in the calculations of bearing capacity.

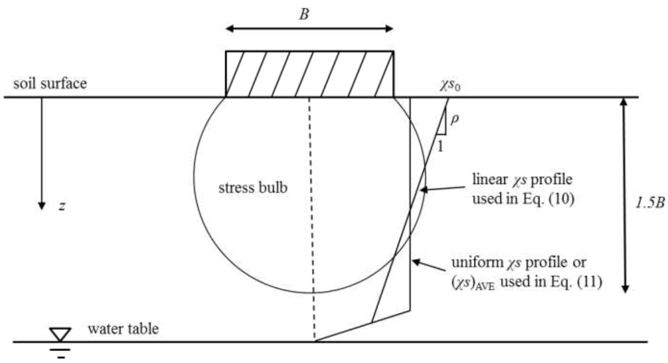

Figure 1. Typical uniform and linear χs profiles in unsaturated soil under a footing.

3 APPLICATION AND VALIDATION

The proposed bearing capacity equations are applied to interpret the model footing tests and plate load tests presented in the literature. The calculated bearing capacities are compared with the experimental data. This study considers the tests performed on the surface of dense soils where the general shear failure happened. Details of the parameters of the tested soils are summarized in Table 1.

Table 1. Details of the parameters of the tested soils.

Tests	s_e (kPa)	c' (kPa)	ϕ' (°)	ω	γ_{dry} (kN/m^3)
Rojas et al. (2007)	18	3	26	4.6	15.7
Oh & Vanapalli (2011)	7.5	3.5	21	5.5	15
Oh & Vanapalli (2013)	3	0.6	39	1	16.1
Vanapalli & Mohamed (2013)	3	0.6	39	1	16

Rigorous values of the bearing capacity factors N_c, N_q and N_γ for rough circular and strip footings, presented by Martin (2004), are used in the calculation of the bearing capacity in this study using Equations 8 and 10. As suggested by Vanapalli and Mohamed (2013) and Vahedifard and Robinson (2016), the values of N_c and N_q from Terzaghi (1943), the values of N_γ from Kumbhokjar (1993) and the values of shape factors from Vesic (1973) are used in the calculation using Equations 3 and 4. The effective stress parameter, χ, is calculated using Equation 7.

Rojas et al. (2007) carried out in-situ plate load tests on unsaturated lean clay using a circular steel plate of diameter 0.31 m. The test pits were excavated up to 1.4 m under the ground level. The variation of suction was measured by four tensiometers at depth 0.1 m, 0.3 m, 0.6 m and 0.9 m below the plate.

Loading of the plate was stopped when either a settlement equal to 10% of the plate diameter or a maximum pressure of 650 kPa was achieved. For the latter case the load-settlement curves were extrapolated to obtain the bearing capacity of the plate, defined as the capacity at a settlement equal to 10% of the plate diameter. Figure 2 compares the predicted and measured bearing capacities. For the bearing capacity calculated using Equations 3, 4 and 8, the average value of the measured suctions at depth 0.1 m and 0.3 m below the plate were used. Also shown in Figure 2 is the bearing capacity calculated using equation 10 for the linear χs profile. For this case, it is assumed that suction at the surface is equal to the one measured at a depth of 0.1 m and varies linearly to that measured at a depth of 0.9 m. It is shown in Figure 2 that the bearing capacities estimated using Equations 8 and 10 agree well with the measured values.

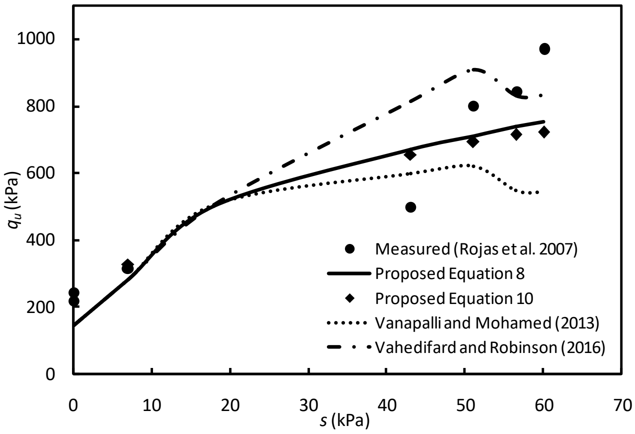

Figure 2. Comparison between measured and predicted bearing capacities of plate on lean clay (Rojas et al. 2007).

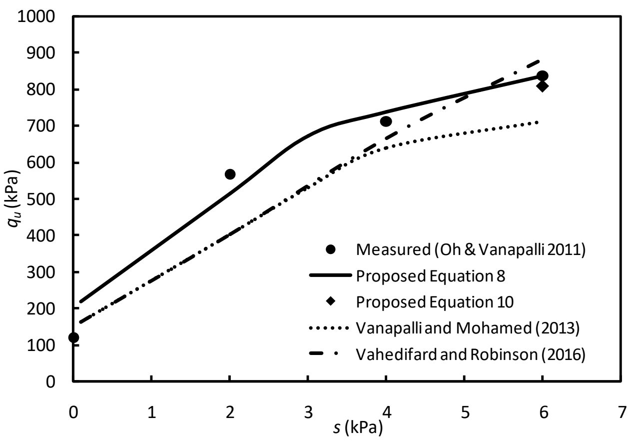

Figure 3. Comparison between measured and predicted bearing capacities of square footing on coarse sand (Oh & Vanapalli 2011).

Oh and Vanapalli (2011) presented results of model footing tests using a square footing of 0.1 m $\times$0.1 m on unsaturated coarse sand. The air entry value of the soil reported by Oh and Vanapalli (2011) is 3 kPa. This value is used in the calculations here except for Equation 4 since Vahedifard and Robinson (2016) assumed the air entry value to be of 5.7 kPa. The bearing capacities calculated using Equation 4 are based on this assumption. The variation of suction with depth was presented for one of the tests, which is used here to validate Equation 10. Figure 3 compares the experimental data with the bearing capacity predicted by different equations. The predictions by the proposed equations match the experimental data very well.

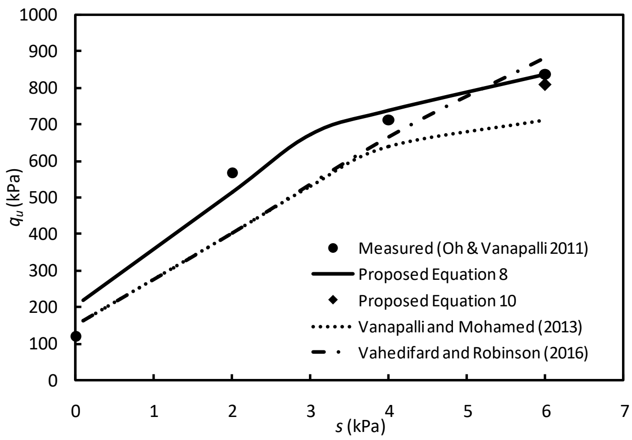

Figure 3. Comparison between measured and predicted bearing capacities of square footing on coarse sand (Oh & Vanapalli 2011).

Oh and Vanapalli (2013) performed load tests on unsaturated fine grained soil using a 0.05 m $\times$ 0.05 m square footing. Figure 4 presents the measured bearing capacities corresponding to 0.1 B settlement for suction values from 55 kPa to 160 kPa together with the predicted values. It can be seen in Figure 4 that Equation 8 provides good estimations. Compared to the experimental data, Equation 3 underestimates and Equation 4 overestimates the bearing capacity for unsaturated conditions.

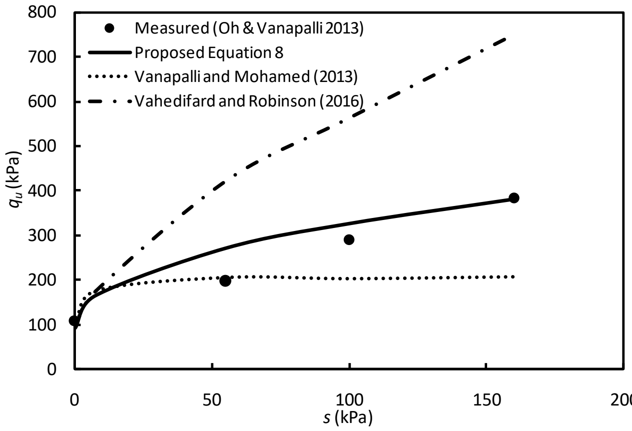

Figure 4. Comparison between measured and predicted bearing capacities of square footing on fine grained soil (Oh & Vanapalli 2013).

Vanapalli and Mohamed (2013) presented the results of loading tests using a square plate of 0.15 m ×0.15 m on the surface of saturated and unsaturated coarse sand. The air entry value of the tested sand is 3 kPa. Figure 5 compares the experimental data with the bearing capacities predicted by different equations. The bearing capacities predicted by the Equation 4 are based on their assumption that the air entry value is 5.7 kPa. The prediction of Equation 10 is presented in Figure 5 for only one of the tests for which the suction profile was available. It is shown that the proposed equations can capture the increase in the bearing capacity due to suction.

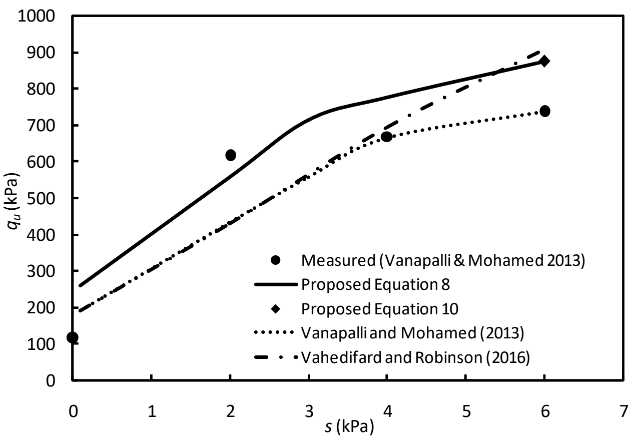

Figure 5. Comparison between measured and predicted bearing capacities of square plate on coarse sand (Vanapalli & Mohamed 2013).

Figure 6 compares the measured bearing capacities and those values predicted by the proposed equations for all the tests presented in this paper. It can be seen that, the errors associated are less than 25% for most cases.

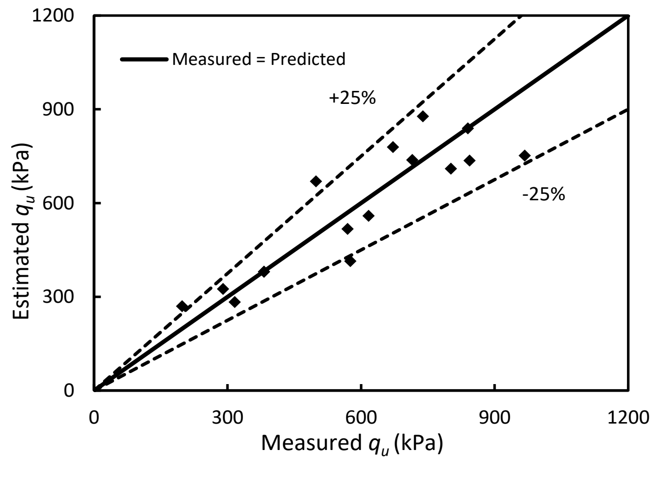

Figure 6. Comparison between measured and predicted bearing capacities based on the proposed equations.

Compared to the equations available in the literature it can be seen from Figure 2 to Figure 5 that the proposed equations provide better estimations of bearing capacity. It can be found that, the equation proposed by Vahedifard and Robinson (2016) may overestimate the bearing capacity when the suction is greater than the air entry value. In this equation, the contribution of suction from zero to the air entry value is always accounted by $s_e \tan\phi'$. In the proposed equations, the effect of air entry value has been included in the definition of χ. When the suction is greater than the air entry value, the value of χ is less than 1 and the contribution of suction up to the air entry value should be accounted by $\chi s_e \tan\phi'$.

4 CONCLUSIONS

Alternative equations were proposed in this paper, which can be used to predict the bearing capacity of shallow footings on unsaturated soil. These equations are based on the effective stress principle and their validity was examined by comparing their predictions with the data of plate load tests on unsaturated soils. These comparisons showed a satisfactory predictive capacity of the proposed equations. An advantage of these expressions is that they are simple for practical use since only the extra parameters required for unsaturated soil are the suction and the SWCC of the foundation soil. These equations provide preliminary methods for the practical engineers to estimate the bearing capacity in the foundation design.

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