Simplified stability analysis of unsaturated soil slopes under rainfall

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ABSTRACT: Stability of an unsaturated soil slope under rainfall is usually assessed by a combined numerical seepage and slope stability analysis. However, such analysis may not be possible in practice due to time and cost constraints. In this paper, a practical framework that consists of simplified infiltration analysis and slope stability analysis using explicit equations is presented. The analysis only requires easily determined parameters (e.g., saturated permeability, soil porosity, initial and final degree of saturation) and the total cohesion approach. Application of the practical framework is illustrated by reanalysing slope stability problems that have been solved by numerical methods.

1 INTRODUCTION

Many soil slopes are unsaturated in nature. The negative pore-water pressures above the groundwater table can increase the soil strength and enhance the slope stability (Fredlund & Rahardjo 1993). The infiltration of rainwater would reduce the matric suction and hence soil strength, which may lead to slope instability. Most of the rainfall-induced landslides were shallow and translational (Toll 2001; Dai et al. 2003; Zizioli et al. 2013), while deep-seated rotational slides were also reported (Toll 2001; Huat et al. 2006; Keaton et al. 2014).

To assess the stability of an unsaturated soil slope under rainfall accurately, a transient seepage analysis needs to be performed first. The calculated pore-water pressure profiles are then integrated into a limit equilibrium analysis (Ng & Shi 1998; Huat et al. 2006; Rahardjo et al. 2007; Cascini et al. 2010) or shear strength reduction finite element analysis (Cai & Ugai 2004; Griffiths & Lu 2005; Huang & Jia 2009; Le et al. 2015) to obtain the global factor of safety. Alternatively, a scalar field of local factor of safety can be obtained by finite element analysis (Alonso et al. 2003; Lu et al. 2012; Robinson et al. 2017). The numerical transient seepage analysis requires that the soil-water characteristic curve (SWCC) and permeability function (PF) of the soil are known. However, the measurement of SWCC and PF is costly, technically-demanding and time-consuming (Leong & Rahardjo 1997a, 1997b). As a result, the measurement is usually reserved for research studies or large projects where substantial risk may be involved (Zhang & Fredlund 2015). The difficulties in obtaining the required input parameters and the complexity involved in the analysis have limited the application of the rigorous numerical methods (Vahedifard et al. 2016).

As a simplified alternative to numerical methods, infinite slope analysis is widely conducted to assess the stability of unsaturated soil slopes under rainfall (Pradel & Raad 1993; Cho & Lee 2002; Lu & Godt 2008; Zizioli et al. 2013). Infinite slope model neglects the boundary resistances. Hence, infinite slope model is only accurate when the depth of the slip surface is much less than the length of the slip surface (i.e., translational failure). The underestimation of factor of safety caused by neglecting the boundary resistances increases with the increase in depth/length ratio (Griffith et al. 2011; Huang et al. 2015).

Huang (2017) proposed a practical framework, which consists of simplified infiltration analysis and stability analysis using explicit equations, to assess the stability of unsaturated soil slopes under rainfall. The analysis only requires easily determined parameters (e.g., saturated permeability, soil porosity, initial and final degree of saturation) and the total cohesion approach. In the following sections, the practical framework is briefly introduced first. The application is then illustrated by reanalysing slope stability problems that have been solved by numerical methods.

2 SIMPLIFIED INFILTRATION ANALYSIS

The geometry of an unsaturated soil slope under rainfall is shown in Figure 1. Under no infiltration condition, matric suction (i.e., the absolute value of negative pore-water pressure) can be assumed to be hydrostatic if the groundwater table is close to the ground surface. However, if the groundwater table is deep, a maximum matric suction ($=\gamma_w h_n^{\max}$, where γ_w is unit weight of water, $h_n^{\max}$ is the maximum suction head in the unsaturated zone) needs to be imposed, otherwise the matric suction at shallow depth could be extremely large and become unrealistic. The infiltration of rainwater can produce a wetting front and change the pore-water pressure distribution in the soil.

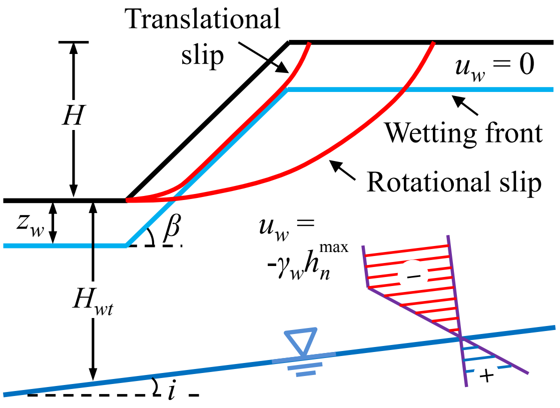


Figure 1. Stability of an unsaturated soil slope under rainfall.

Under a rainfall of high intensity ($I > k_s$, where I is the rainfall intensity, k_s is the saturated permeability of the soil), the depth of wetting front z_w can be estimated by the two-stage model proposed by Mein and Larson (1973). If the intense rainfall ($I \geq k_s$) lasts a long period of time, z_w can be estimated by Equation 1(a) (Lumb 1962), in which t is the rainfall duration, n is the porosity of the soil, S_f is the final degree of saturation after the rainfall and S_o is the initial degree of saturation before the rainfall. If $I < k_s$, z_w can be estimated by Equation 1(b) (Sun et al. 1998).

$$z_w = \begin{cases} \dfrac{k_s t}{n(S_f - S_o)}, \text{if } I \geq k_s & \text{(1a)} \\[2ex] \dfrac{It}{n(S_f - S_o)}, \text{if } I < k_s & \text{(1b)} \end{cases}$$

Pore-water pressure profiles under rainfall infiltration for various soil types and various rainfall conditions can be found in the literature (Zhang et al. 2004; Lee et al. 2009). For an unsaturated homogeneous slope, it is generally reasonable to assume that matric suction reduces to zero for the soil above the wetting front (Huang et al. 2016). Due to the loss of matric suction, rotational or translational failure may occur. The failure mode that gives the lower factor of safety is the governing failure mode.

3 EXPLICIT EQUATIONS FOR SLOPE STABILITY ANALYSIS

For a soil slope with simple geometry and homogeneous soil properties, factor of safety of the slope can be expressed as a function of slope geometry, soil properties and loading conditions. Huang (2017) proposed a series of equations for stability analysis of dry and saturated soil slopes, unsaturated soil slopes under rainfall and seismic loading. The stability equations for unsaturated soil slopes under rainfall based on rotational and translational failure mechanisms are presented below.

3.1 *Rotational failure*

For a dry homogeneous slope, rotational failure is the critical failure mode (Taylor 1948; Chen 1975). The factor of safety F_{rot} can be calculated by:

$$F_{\text{rot}} = A_{\text{est}} \left(\frac{c'}{\gamma H \tan\phi'} \right)^{B_{\text{est}}} \tan\phi' + \frac{\tan\phi'}{\tan\beta} \qquad (2)$$

where, c' is the effective cohesion of the soil; ϕ' is the effective angle of internal friction of the soil; γ is the unit weight of the soil; H is the slope height; β is the slope angle; A_{est} and B_{est} are functions of β and $c'/\gamma H\tan\phi'$. For $0 \leq c'/\gamma H\tan\phi' \leq 3$, A_{est} and B_{est} can be calculated by:

$$A_{\text{est}} = 10.50\exp(-0.009\beta) \qquad (3)$$

$$B_{\text{est}} = \begin{cases} 0.72 - 3.5\times10^{-5}\beta^2 + 0.0031\beta, \text{ if } 0 \leq \dfrac{c'}{\gamma H\tan\phi'} \leq 1 \\[2ex] 0.83 - 2.2\times10^{-5}\beta^2 + 0.0026\beta, \text{ if } 1 < \dfrac{c'}{\gamma H\tan\phi'} \leq 3 \end{cases} \qquad (4)$$

In Equations 3 and 4, β is input as degree (°). In Huang (2017), the accuracy of Equation 2 has been validated with the stability charts developed by Taylor (1937) and Michalowski (2002) for a wide range of slope geometries and soil properties ($15° \leq \beta \leq 90°$, $0 \leq c'/\gamma H\tan\phi' \leq 3$). The difference between the factors of safety obtained by Equation 2 and the stability charts is generally within ±2%.

For an unsaturated soil slope under rainfall, by using the "total cohesion" method suggested by Fredlund & Rahardjo (1993), Equation 2 can be extended to:

$$F_{\text{rot}} = A_{\text{est}}\left(\frac{c'}{\gamma H\tan\phi'} - \frac{\gamma_w h_p \tan\phi'}{\gamma H\tan\phi'} + \zeta\frac{\gamma_w h_c \tan\phi^b}{\gamma H\tan\phi'}\right)^{B_{\text{est}}} \times \tan\phi' + \frac{\tan\phi'}{\tan\beta} \tag{5}$$

where, h_p and h_c denote the average positive and negative pore-water pressure heads on the critical rotational slip surface before the rainfall infiltration, respectively; ϕ^b is an angle which indicates the rate of increase in shear strength relative to matric suction; ζ denotes the degree of contribution of matric suction to slope stability. Graphical procedures and charts to determine h_p, h_c and ζ can be found in Huang (2017). Parameter $\zeta = 1$ when there is no infiltration and $\zeta = 0$ when the entire slip surface is saturated by rainwater. Through a large amount of parametric studies, Huang (2017) found that ζ can be approximated by:

$$\zeta = 1 - 1.4\frac{z_w}{H} \tag{6}$$

3.2 *Translational failure*

An innovative kinematically admissible translational failure mode which can consider the boundary resistances was proposed in Huang (2017) and used to generate numerous stability numbers. Through regression analysis of the stability numbers, the factor of safety of an unsaturated soil slope under rainfall based on translational failure mechanism, F_{trl}, can be calculated by:

$$F_{\text{trl}} = \left[\frac{H}{z_w \sin\beta\cos\beta} + 5.0\exp(-0.008\beta)\right]\frac{c'}{\gamma H} + \frac{\tan\phi'}{\tan\beta} \tag{7}$$

Equation 7 can be applied reliably within the range $0 < z_w/H \leq 0.3$ and $15° \leq \beta < 90°$. The difference between the factors of safety obtained by Equation 7 and kinematical approach of limit analysis is generally less than 3%.

4 APPLICATION

The application and effectiveness of Equations 5 and 7 are illustrated by three cases. The given conditions for the three cases are summarized in Table 1.

4.1 *Case 1*

Zhang et al. (2016) conducted stability analysis of an unsaturated soil slope using *SVFLUX* and *SVSLOPE* (example 3.2 in their study). The slope geometry, soil properties and rainfall conditions are shown in Table 1 and Figure 2.

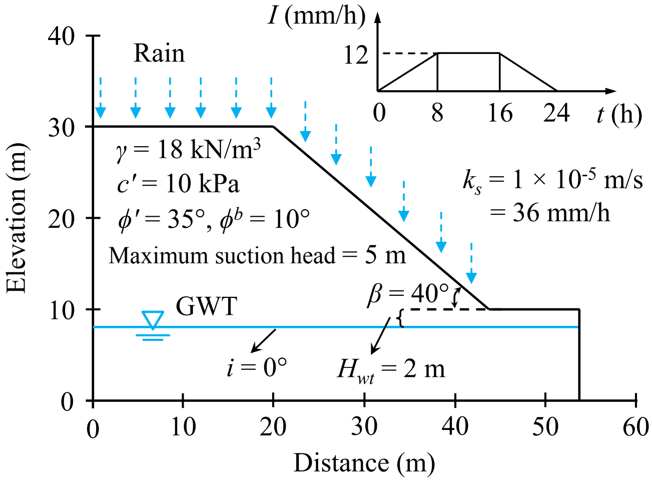


Figure 2. Slope geometry, soil properties and rainfall conditions (Zhang et al. 2016).

The critical rotational slip surface is unlikely to pass below the groundwater table, hence $h_p = 0$. The suction head ranges from 2 m to 5 m, and the average h_c is estimated as 3.5 m. The parameters n and S_o are not explicitly given, but they can be inferred from the soil-water characteristic curve (SWCC) provided in Zhang et al. (2016) (i.e., $n = \theta_s$, $S_o = \theta_o/n$, where θ_o denotes the initial volumetric water content corresponding to the suction $\gamma_w h_c$). The final degree of saturation S_f is assumed to be equal to 1. The rainfall intensity I is a variable that changes with time (Figure 2). Hence, z_w is estimated by a modified form of Equation 1(b):

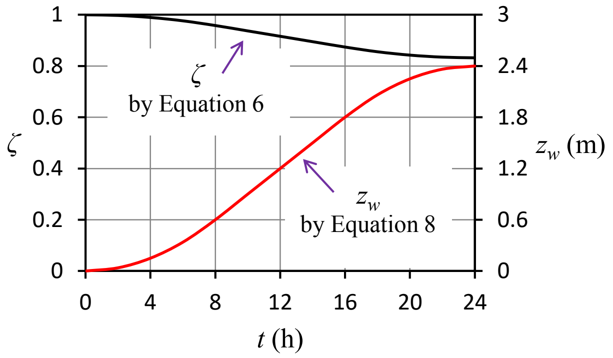
$$z_w = \frac{\int I\,\mathrm{d}t}{n(S_f - S_o)} \tag{8}$$

The parameter ζ can be estimated by Equation 6. The obtained z_w and ζ are shown in Figure 3(a), and they are incorporated into Equation 5 and Equation 7, respectively, for stability analysis. As shown in Figure 3(b), the F values obtained by Equation 5 agree well with the F values given by Zhang et al. (2016) who used General Limit Equilibrium method. At $t = 0$, $F_{\text{rot}} = 1.420$, while F_{trl} tends to infinity as $z_w = 0$. Hence, at no infiltration condition ($t = 0$) rotational failure is more critical than translational failure. With the increase in t, F_{rot} only decreases slightly while F_{trl} decreases dramatically. At $t = 24$ h, $F_{\text{rot}} = 1.391$ and $F_{\text{trl}} = 1.405$, which reveals that translational failure mechanism is almost as critical as rotational failure mechanism. Zhang et al. (2016) also observed that the critical slip surface is deep rotational at no infiltration condition ($t = 0$), but it becomes shallower when the soils near the slope surface are saturated.

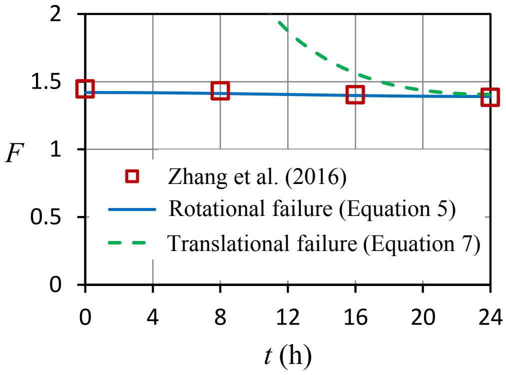
Table 1. Summary of the parameters for unsaturated slope stability analyses

Case No.	Reference	Slope geometry	Soil properties		Groundwater table	Rainfall characteristics
1	Zhang et al. (2016)	$\beta = 40°$ $H = 20$ m	$\gamma = 18$ kN/m^3 $c' = 10$ kPa $\phi' = 35°$ $\phi^b = 10°$	$n = 0.4$ $S_f = 1$ $S_o = 0.8$	$H_{wt} = 2$ m $i = 0°$ $h_n^{\max} = 5$ m	I is a variable $k_s = 1\times10^{-5}$ m/s $t = 24$ h
2	Rahardjo et al. (2007)	$\beta = 45°$ $H = 10$ m	$\gamma = 20$ kN/m^3 $c' = 10$ kPa $\phi' = 26°$ $\phi^b = 26°$	$n = 0.45$ $S_f = 1$ $S_o = 0.84$	$H_{wt} = 5$ m $i = 7°$ $h_n^{\max} = 7.5$ m	$I = 1\times10^{-6}$ m/s $k_s = 1\times10^{-6}$ m/s $t = 24$ h
3	Huat et al. (2006)	$\beta = 56°$ $H = 60$m	$\gamma = 21$ kN/m^3 $c' = 11$ kPa $\phi' = 25°$ $\phi^b = 18°$	$n = 0.5^*$ $S_f = 1^*$ $S_o = 0.2^*$, 0.5^*, 0.8^*	$H_{wt} = 10$ m $i = 0°$ $h_n^{\max} = 10$ m	$I = 1\times10^{-6}$ m/s $k_s = 1\times10^{-6}$ m/s $t = 24$ h

(*denotes that the parameters are not given in the literature and they are assumed in this study)



(a) Infiltration analysis

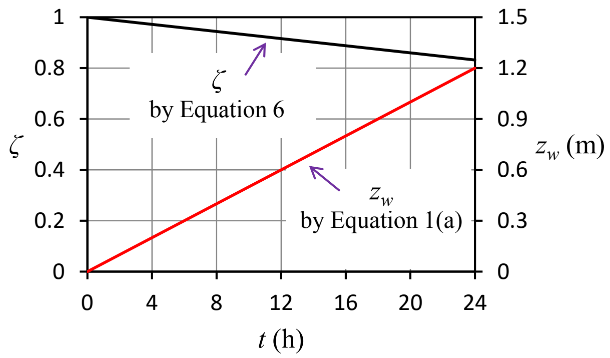


(b) Stability analysis

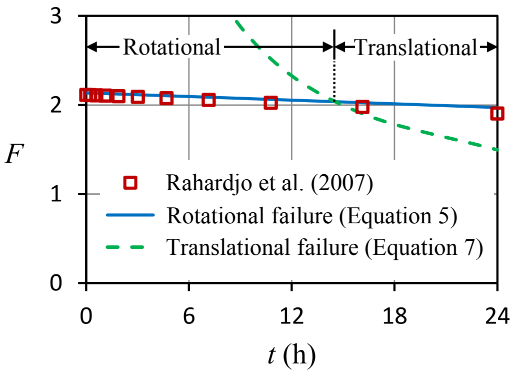
Figure 3. Simplified unsaturated slope stability analysis - Case 1.

4.2 *Case 2*

Rahardjo et al. (2007) conducted stability analysis on an unsaturated soil slope using *SEEP/W* and *SLOPE/W*. According to the given conditions shown in Table 1, $h_p = 0$ m, $h_c \approx 6$ m. Similar to Case 1, the parameters $n = 0.45$ and $S_o = 0.84$ are inferred from the SWCC provided in Rahardjo et al. (2007), and S_f is assumed to be equal to 1. For a specific rainfall duration t, z_w is estimated by Equation 1(a), ζ is estimated by Equation 6, and the results are summarized in Figure 4(a). The obtained z_w and ζ are incorporated into Equation 5 and Equation 7, respectively, for the stability analysis, as shown in Figure 4(b). The F values obtained by Equation 5 agree well with the F values given by Rahardjo et al. (2007) who used Bishop's simplified method with a circular slip surface for the slope stability analysis. Figure 4(b) also shows that rotational failure is more critical than translational failure when $t < 14.6$ h, while translational failure becomes more critical when $t > 14.6$ h. However, Bishop's simplified method fails to search for the shallow critical slip surface when $t > 14.6$ h. The phenomenon that limit equilibrium methods may fail to search for shallow critical slip surface was also reported by Lu et al. (2012).



(a) Infiltration analysis

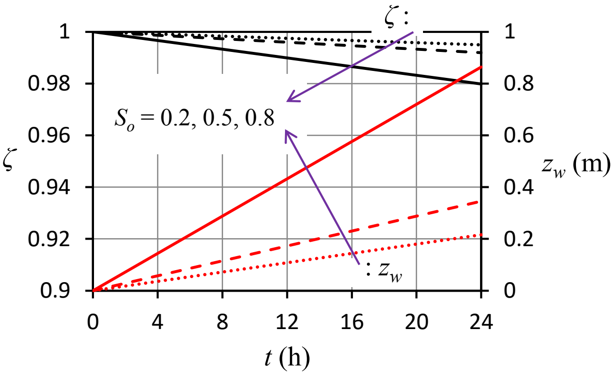


(b) Stability analysis

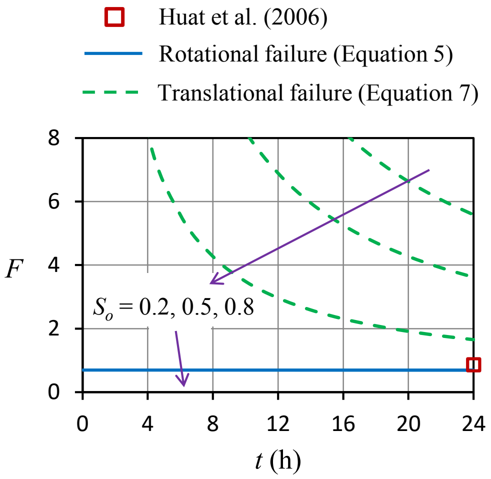
Figure 4. Simplified unsaturated slope stability analysis - Case 2.

4.3 *Case 3*

Case 3 is a back analysis of a rotational slope failure reported by Huat et al. (2006). The residual soil slope failed on 14 November 2003 at the Island of Langkawi, Malaysia after a rainfall event. According to the given conditions shown in Table 1, the groundwater table can be considered as deep, $h_p = 0$ m, $h_c = 10$ m. The parameters n, S_f and S_o are not given in Huat et al. (2006). Lee et al. (2009) summarized the typical properties of the residual soil found in the Malaysian Peninsula and reported that the porosity n typically ranges between 0.4 and 0.6. Hence, in the calculation n is assumed to be 0.5, S_f is assumed to be 1, and a parametric study with $S_o = 0.2$ (dry season), 0.5 and 0.8 (rainy season) is conducted. The infiltration and stability analysis results are summarized in Figure 5.



(a) Infiltration analysis



(b) Stability analysis

Figure 5. Back analysis of Langkawi landslide - Case 3.

For a specific rainfall duration t, the increase in initial degree of saturation S_o greatly increases the z_w [Figure 5(a)] and consequently greatly decrease the F_{trl} obtained by Equation 7 [Figure 5(b)]. The height of the slope $H = 60$ m, which is much larger than the z_w (= 0 ~ 0.86 m). Hence, the ratios z_w/H are very small (0 ~ 0.014). As a result, ζ is only slightly reduced from unity due to the rainfall [Figure 5(a)], and the effects of t and S_o on the F_{rot} calculated by Equation 5 can hardly be observed [Figure 5(b)]. At $t = 24$ h, the F_{rot} (= 0.69) obtained by Equation 5 is generally consistent with the F (= 0.86) obtained by Huat et al. (2006) who used stability charts developed from *SEEP/W* and *SLOPE/W*. Figure 5(b) shows that F_{rot} calculated by Equation 5 is less than F_{trl} calculated by Equation 7 regardless of t and S_o, which reveals that rotational failure is more critical than translational failure. This was confirmed by the occurrence of the deep-seated rotational slide.

5 CONCLUSION

Stability of unsaturated soil slopes under rainfall is commonly assessed by numerical seepage and slope stability analysis. However, time and cost constraints have limited the application of numerical methods in practice. In this paper, simplified procedures for unsaturated slope stability analysis proposed in Huang (2017) are briefly introduced and the application is illustrated with three case studies.

The case studies consistently show that with the increase in rainfall duration, the factor of safety based on rotational failure mechanism only slightly decreases, in contrast to the dramatically decrease of factor of safety based on translational failure mechanism. Hence, if a deep-seated rotational slide occurs due to rainfall infiltration, the slope may be on the verge of failure before the rainfall infiltration. Similarly, if an unsaturated slope has a large safety margin before the rainfall infiltration, rotational failure is unlikely (i.e., translational failure is very likely) to be the governing failure mode of the slope under rainfall infiltration.

Good agreements are generally shown between the simplified stability analysis and the numerical stability analysis. However, the parameters required by the simplified stability analysis (e.g., saturated permeability, soil porosity, initial and final degree of saturation) are more easily determined than the parameters required by the numerical stability analysis (e.g., SWCC and permeability function). Hence, the simplified procedures can readily be used in practice for stability analysis of unsaturated soil slopes under rainfall.

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