Solute Movement Analysis on Unsaturated Ground Using Advection-Diffusion Model Considering Growth of Plants

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ABSTRACT: Controlling growth of roots by optimizing ground environment can be one of the solution for geo-environmental problems such as soil contamination, desertification and so on. It is necessary to establish long-term prediction of ground water and solute environment including absorption and growth of roots for conserving geo-environment by utilizing power of plants. Authors have been trying to predict water and solute movement in degraded unsaturated ground with root based on advection-diffusion equation. This paper presents effectiveness of root absorption which changes with its growth through parametric analysis by changing soil-water retention properties followed by experimental values. Two types of measured SWCCs were used for determining unsaturated permeability and coefficient of diffusion. As for the growth parameter of root, length, root length density and diameter were determined. 50 cm width and 100 cm depth of analytical field were supposed. Volumetric water and solute contents in each 10 cm square area were analyzed. Following conclusions were obtained: 1) Absorption of water and water-soluble substance were depended on matrix potential of soils. The higher water retention capacity soil had, the lower root uptake occurred under same effective saturation. 2) In case available moisture of soil was lower, contribution of root uptake was higher.

1 INTRODUCTION

23 % of land in the world is now defined as "degraded ground due to desertification and soil contamination (Bai et al. 2008). Phytoremediation is expected as one of the cost-effective and sustainable solutions for mitigating deteriorated ground (Melinda et al, 2013). It takes advantage of physiologic mechanisms of plant for remedying deteriorated ground. Pollutant solution can be absorbed, fixed or de-toxified during uptake of water and chemical substance by roots.

Water and solute movements in ground strongly affects growth of roots. Controlling growth of roots by optimizing ground environment can be important to utilize phytoremediation. In order to develop technics for conserving geo-environment by using the power of plants, it is necessary to establish long-term prediction method of ground water and solute environment including absorption and growth of roots. However, the suitable way for improving ground environment that enables plant to grow well is tried to clarify majorly experimentally, although it is not clarified analytically. Prediction method for pollutant in unsaturated ground has been developed by Nishigaki et al (1995), Nomura et al. (2011) and so on in Japan. Moreover, Grove et al (1985) suggested an analytical method of suppression of heavy metals. However, these were few considered root absorption of water and chemical substances. As for prediction method of root growth, Yorozu et al. (2010), Guillaume (2014) and some researchers of agricultural field suggested some analytical growth models of plant considering water uptake of roots. Yet, there are few cases of numerical experiment considering pollutants in unsaturated ground.

Authors have been trying to predict water and solute movement in degraded unsaturated ground with root of plant based on advection-diffusion equation. (Fujisawa et al. 2016). It can be calculated roots and their growth that influences in time-space changing of water and solute contents.

This paper presents the effectiveness of root absorption which changes with its growth through parametric analysis by changing soil-water retention properties followed by experimental results. 50 cm width and 100 cm depth of analytical area was made and volumetric water content, and solute compound in each 10 cm square area was analyzed daily. Two types of measured SWCCs such as fine sand and cultivation field soil were used for determining unsaturated permeability and coefficient of diffusion. SWCCs were approximated by Van Genuchten and Mualem (1980). As for the growth parameter of root, length, root length density, diameter and some other parameters were determined.

2 ANALYTICAL METHODS

2.1 *Analytical method for solute movement and root growth*

As I represented in section 1, time-space changings of volumetric water contents and chemical compounds in two-dimensional unsaturated ground were analyzed numerically when root growth and absorption occurred.

2.1.1 *Water transfer with root absorption in unsaturated soils*

Transport of water in the ground illustrates as Equation 1 written below. x is horizontal, and z is vertical direction.

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left\{k_x(\theta)\frac{\partial}{\partial x}\Psi_m(\theta)\right\}+\frac{\partial}{\partial z}\left\{k_z(\theta)\frac{\partial}{\partial z}\Psi_m(\theta)+1\right\}-Y(\theta) \quad (1)$$

where θ = volumetric water content (%); $\Psi_m(\theta)$ = Matrix potential of soils (cm); $k_x(\theta) = k_z(\theta)$ = unsaturated hydraulic conductivity (cm/s); $Y(\theta)$ = amount of absorbed solute per unit area and time ($cm^2/cm^2 \cdot s$).

Van Genuchten (1980), stated as Equation 2, was adopted for describing SWCC of soils.

$$S_e=\left\{1+\alpha\psi_m(\theta)^n\right\}^{-m} \quad (2)$$

where S_e = effective saturation; α, n, m = fitting parameter (m = 1- 1/n).

Unsaturated permeability depends on effective saturation of soils. k_x, k_z were also decided to follow Van Genuchten (1980) (Equation 3).

$$k_x(\theta)=k_z(\theta)=k_{sat}\left\{1-\left(1-S_e^{\frac{1}{m}}\right)^m\right\}^2 \quad (3)$$

where k_{sat} = saturated hydraulic conductivity (cm/s).

$Y(\theta)$ is defined as water uptake of root. It can be decided by Equation 4 (Nakano, 1991).

$$Y(\theta)=\frac{(\Psi_{r1}-\Psi_m(\theta))L_{v2}}{r_{ab}+L_{v2}\cdot R} \quad (4)$$

where Ψ_{r1} = plant water potential (cm); r_{ab} = peameation resistance per length of root (day/ cm); L_{v2} = root length density, which is total length of root in a cube (area) of soil (cm/cm^2); R = resistance to water flow (day/cm^2)

Equation 4 describes root water uptake mainly depends on potentials of soil and root.

2.1.2 *Water transfer with root absorption in unsaturated soils*

Advection-diffusion equation for calculating solute transfer in unsaturated soils with root can be described as Equation 5.

$$\frac{\partial C(\theta)}{\partial t}+\frac{\partial}{\partial x}\left(k_x(\theta)C(\theta)\frac{\partial\psi_m(\theta)}{\partial x}\right)+\frac{\partial}{\partial z}\left(k_z(\theta)C(\theta)\frac{\partial\psi_m(\theta)}{\partial z}\right)+\frac{\partial k_z(\theta)C(\theta)}{\partial z}=\frac{\partial}{\partial x}\left(D(\theta)\frac{\partial C(\theta)}{\partial x}\right)+\frac{\partial}{\partial z}\left(D(\theta)\frac{\partial C(\theta)}{\partial z}\right)-\frac{Y_{cp}(\theta)}{\theta} \quad (5)$$

where $C(\theta)$ = compound of arbitrary chemical substance (g/cm^3); $D(\theta)$ = unsaturated coefficient of diffusion (cm^2/s); $Y_{cp}(\theta)$ = amount of root absorption of chemical substance (g/cm^3).

Unsaturated coefficient of diffusion was also decided to follow Van Genuchten and Mualem (1980) (Equation 6)

$$D(\theta)=\frac{k_{sat}S_e^{0.5}\left\{1-\left(1-S_e^{\frac{1}{m}}\right)\right\}^2}{\alpha(n-1)(\theta_s-\theta_r)S_e^{\frac{1}{m}}\left(1-S_e^{\frac{1}{m}}\right)} \quad (6)$$

Amount of root absorption of chemical substance $Y_{cp}(\theta)$ can be explained as equation (7) (Nakano, 1991).

$$Y_{cp}(\theta)=2\pi rK_pC(\theta)L_{v2} \quad (7)$$

where r = diameter of root (cm), K_p = absorption speed of root (cm/s).

In order to calculate arbitrary area and time of θ, $C(\theta)$, Equation 1 and 5 were differentiated by explicit method.

2.2 *Analytical conditions*

2.2.1 *Analytical field and element*

Conceptual scheme of analytical field was indicated in Figure 1. Width was 50 cm, and depth was 100 cm. Mass of each analytical element was 10 cm width and 10 cm depth. The size of the grids was revealed to guarantee precision of ±10 % in our previous study (Naotsuka, et.al, 2015).

It was decided that same volumetric water content θ and concentration of chemical substance $C(\theta)$ was achieved in one analytical element. The area are homogeneous, which has same SWCC, hydraulic conductivity and coefficient of diffusion. Moreover, there was single root at the center elements of analytical field.

2.2.2 *Soil conditions*

As indicated in section 2.1.1, SWCC (relationships between θ and $\Psi(\theta)$), saturated hydraulic conductivity k_{sat}, initial distribution of effective saturation S_e, and initial distribution of chemical substance $C(\theta)$ were necessary for analysing soil water and chemical transfer. Decomposed granite soil and Cultivated Field soil were chosen for experiment and analysis. The former distributes widely western part of Japan, especially Kyushu region. The latter is originated in cultivated field in Saga prefecture, Japan. It contains more finer grain than Decomposed granite soil, so that its water holding capacity is higher than that of Decomposed granite soil.

SWCCs of two kinds of soils were gotten by centrifugal method. Plots in Figure 2 were measured value of SWCC, and solid lines were approximated value calculated by Van Genuchten (1980). Fitting parameter α, n, m, residual volumetric water content θ_r and saturated volumetric water content θ_s was gotten from the calculation. Wetting process was calculated to change fitting parameters α at drying process to 2α. Broken lines were approximated wetting processes of each soil.

Table 1. shows used parameters for analysis of each soil of SWCC and distributions of initial effective saturation and concentration of arbitrary water-soluble substance. *AM* in Table 1 indicates available moisture, which is one of the indexes of water retention ability of soils. *AM* could be calculated by Equation 8.

$$AM = \frac{1}{2}\sum_{i=1}^{n}\{V_w(i) + V_w(i+1)\} \times \{pF(i+1) - pF(i)\} \tag{8}$$

where, *AM*: available moisture (%); *i*: number of observed data; pF(i): the ith observed data of pF (between pF(1) = 1.8 = 6 kPa, pF(n) = 4.2 = 1617 kPa); *H*: matrix potential (cm); $V_w(i)$: the ith observed data of volumetric water content.

Saturated hydraulic conductivity k_{sat} of each soil was observed by constant water head permeability test.

Initial distributions of effective saturation S_{e0} was assumed condition that ground water level existed deeper part than analytical field shown in Figure 3.

Initial distributions of concentration of arbitrary water-soluble cation C_{z0} was assumed to increase 0.5 (mg/l) /cm. Initial distributions of these were same between both soils. Moreover, there are no validations for horizontal directions.

2.2.3 *Root conditions*

As indicated in Figure 1, there was a root in the center (the analytical element of x = 30 ~ 40 cm) of analytical field.

Table 2 represents growth parameters needed for solving Equation 1, 4, 5 and 7.

Absorption speed K_p, permeation resistance r_{ab}, resistance to water flow R and water potential of root Ψ_{r1} were decided following general values of root uptake of water and water-soluble substances. Their value depends on growth of root. K_p, r_{ab}, and R were known experimentally that it took the order of 10^3 cm/day, $10^8 \sim 10^{13}$ cm/day and $10^8 \sim 10^{13}$ cm/day respectively (Cushman, et al, 1979) (Dunham, et al, 1974, 1976). Therefore, the values of K_p, r_{ab}, and R

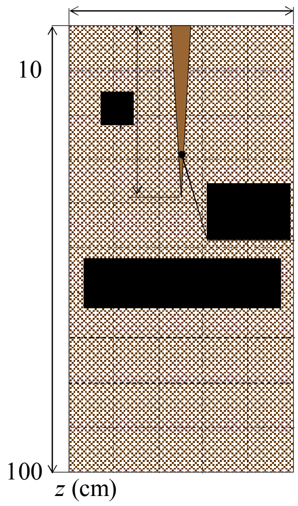


Figure 1. Conceptual scheme of analytical field.

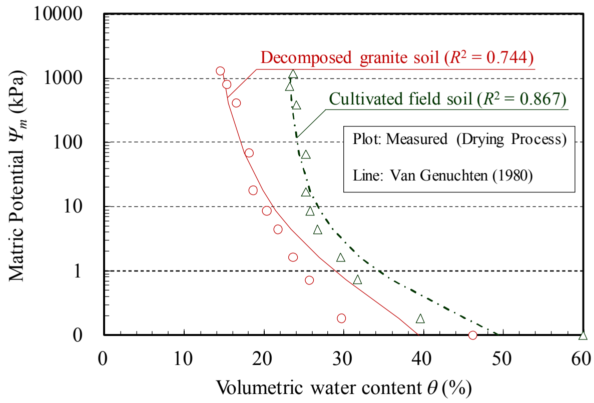


Figure 2. SWCC of soils used for analysis.

were taken 10^3 cm/day, 5×10^{13} cm/day and 5×10^{13} cm/day respectively.

As for growth parameters of root, root diameter r, growth speed of root L_p, root length density L_{v2} should be decided. Upper tip of root diameter r_0 were stabilized. The upper tip was 2 cm, and the bottom r_e were 0.5 cm. Initial length of root L_0 was 30 cm, considering workability for burying root in the ground.

Root length density L_{v2} was generally known that it exponentially decreases with depth (Gerwitz, et al, 1974). Therefore, Exponential function was used for calculating L_{v2} written in Table 2.

Growth speed of root V_R was fixed at 1.0 cm/day which is general speed of plants (Lungrey, 1973).

Root potential Ψ_{r1} depends on health condition of plant and water condition of soil (Kingsley, et al, 1981). However, to clear water and chemical holding capacity of each soil, Ψ_{r1} were set constant in this analysis.

2.2.4 *Analytical interval*

Analytical interval was set 1 day as Δt for finite difference calculation, which was eligible for judging effects of difference of SWCC. Δt was revealed to guarantee precision of ± 10 % in our previous study (Naotsuka, et al, 2015).

3 RESULTS AND DISCUSSIONS

3.1 *SWCC and amount of root water absorption*

Figure 4 and 5 show time-space changing of effective saturation S_e. Figure 4 represents the case of Decomposed granite soil, and Figure 5 represents the case of Cultivation field soil. Numbers in each grid are values of S_e of 0th, 1st, 2nd, 3rd, 4th day respectively. These figures show only grids of root and around the roots in analytical field (x = 10 ~ 40 cm, z = 0 ~ 40 cm).

As shown in these figure, S_e gradually decreased with elapsed time. Moreover, decrease of S_e around the roots were higher than that of other grids. Therefore, absorption of water by roots could be confirmed. On the contrary, S_e of x = 10 cm, z = 30 ~ 40 cm, which were apart from root gained because they were stronger influenced by gravity rather than root uptake. In addition, horizontal movement of water in each ground were very little because absorption ability of root were less than advection current.

Compared with kinds of soils, Cultivation field soil, which had higher water retention ability could hold higher water content than Decomposed granite soil. Decrease of S_e of Cultivation field soil in the grids root existing were lower than those of Decomposed granite soil. As stated in Equation 4, root water uptake was decided by magnitude correlation of matrix potential $\Psi_m(\theta)$. Amount of root water uptake in soil that had higher water retention ability was lower than soil that had lower water retention ability. Therefore, speed of root water uptake in Cultivation field soil was lower than that in Decomposed granite soil.

3.2 *Movement and absorption of chemical substance*

Time-space movement of water-soluble chemical substance were represented in Figure 6 and 7. Figure 6 is the case of Decomposed granite soil, and Figure 7 represents the case of Cultivation field soil. These figures show that condition based on growth parameters of general roots, low available moisture such as Decomposed granite soil, chemical substance in grid root were absorbed. Also, reduction of concentration in grid which root length was gaining (x = 10 ~ 40 cm, z = 0 ~ 40 cm) could be confirmed. In addition, horizontal movement of chemical substance in each ground were very little because absorption ability of root were less than advection and diffusion current.

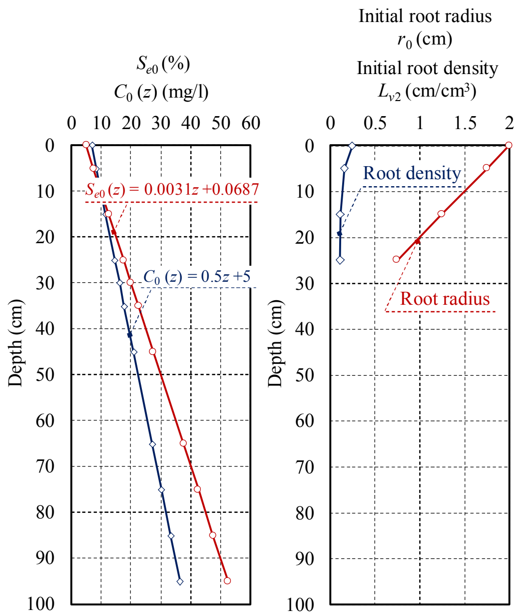
Compared with Figure 6 and 7, decreasing amount of chemical substance of Decomposed granite soil was higher than that of Cultivation field soil. As stated above, in case available moisture was higher, amount of water absorption was lower. Thus, amount of solute absorption was lower in higher-available moisture soils. In addition, there were few changes of effective saturation S_e and concentration C in grids which had no root, It can be assumed that

Table 1. Soil water characteristics used for analysis.

Soil Type	Saturated hydraulic conductivity	Available moisture	Van Genuchten (1980)			
	k_{sat}	AM	θ_s	θ_r	α	n
	cm/day	%	–	–	–	–
Decomposed granite soil	91.584	95.364	45.515	12.74	9.9998	1.2955
Cultivated field soil	22.810	122.139	60.185	22.82	9.9996	1.4906

Table 2. Growth parameters of root.

Parameter		Unit	Value
Diameter of root at z = 0	r_0	cm	2
Diameter of root at bottom tip	r_e	cm	0.5
Speed of absorption	K_p	cm/day	1000
Root length density	L_{v2}	cm/cm^2	$0.1^{1-z/Lp}$
Initial length of root	L_0	cm	30
Speed of growth	V_R	cm/day	1
Water potential	Ψ_{r1}	cm	10.19
Permeation resistance	r_{ab}	day/cm	5E+13
Resistance to water flow	R	day/cm^2	5E+13



(a) effective saturation S_e and chemical concentration C. (b) root diameter and density L_{v2}.

Figure 3. Initial distributions of ground water and chemical environment, root diameter and density.

effective area of single root for absorbing chemical substance was 10 cm around root.

3.2 *Reductions of effective saturation S_e and concentration C*

Figure 8 indicates reduction rates of effective saturation S_e and concentration C at 4th day of analysis. Figure 8 means comparison of reduction rate of these between two types of SWCC. Data in this figure was grids of root and around root (x = 10 ~ 40 cm, z = 0 ~40 cm). Water reduction rate in the figure

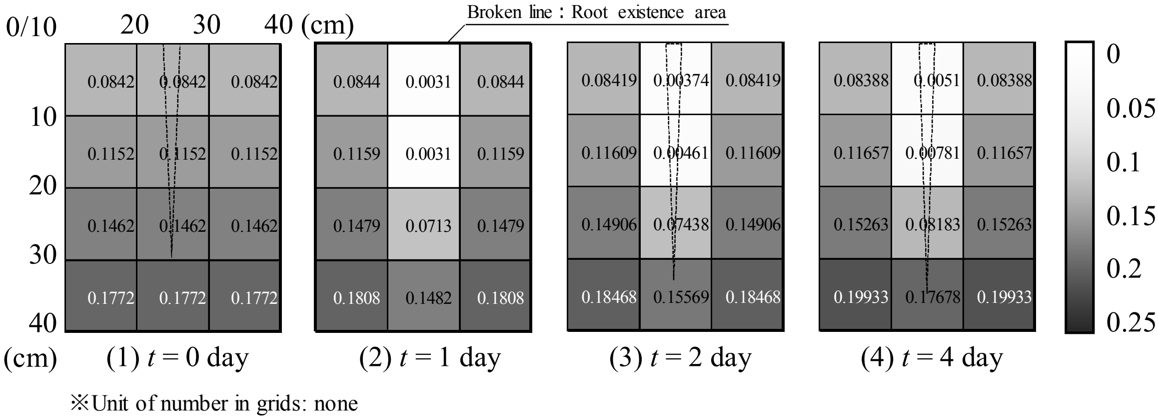


Figure 4. Time-space changing of effective saturation S_e in Decomposed granite soil.

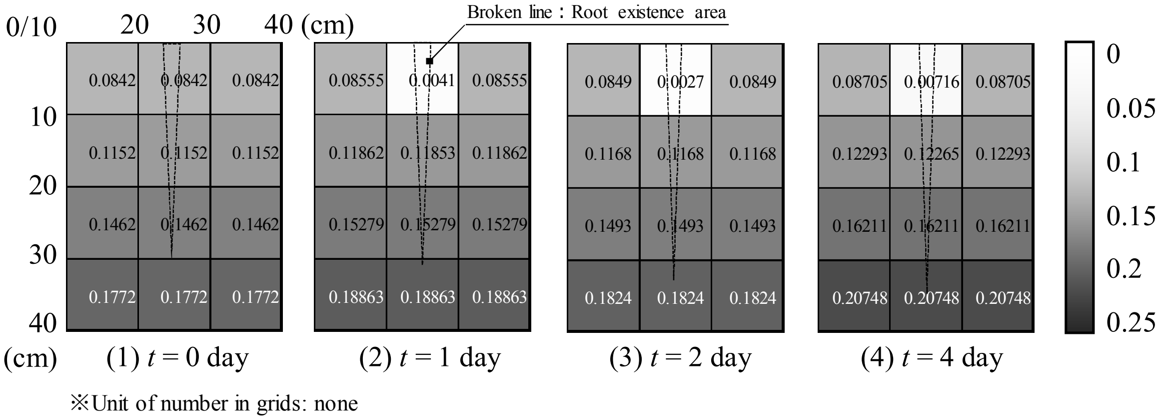


Figure 5. Time-space changing of effective saturation S_e in Cultivation field soil.

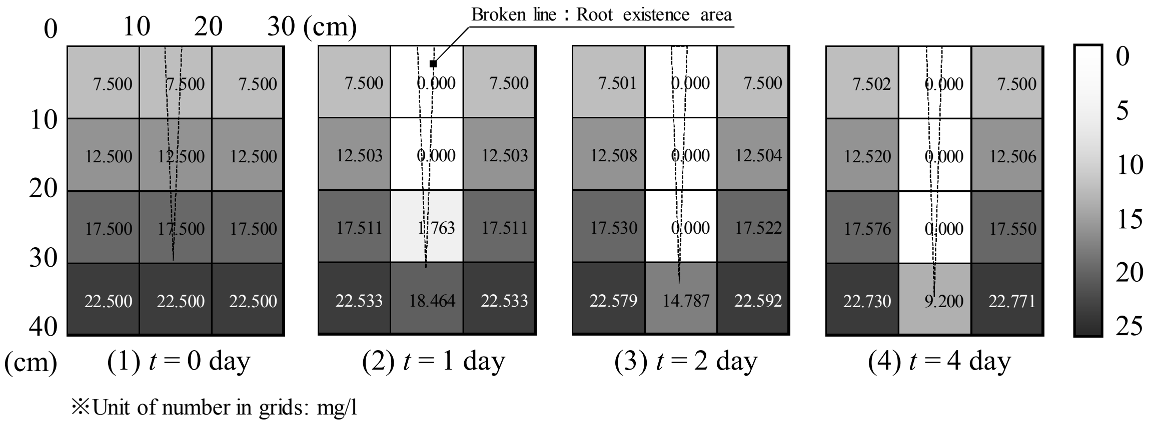


Figure 6. Time-space changing of chemical concentration C in Decomposed granite soil.

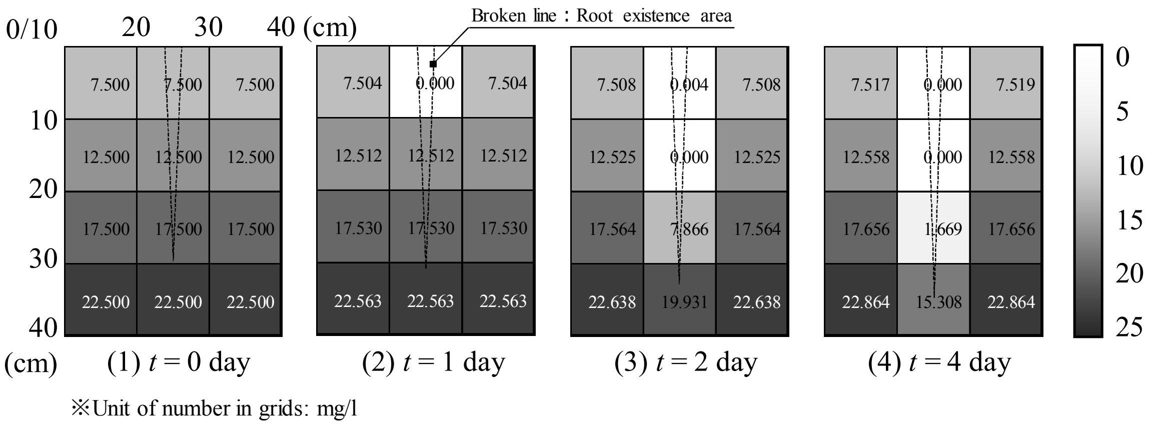


Figure 7. Time-space changing of chemical concentration C in Cultivation field soil.

divided into two parts, those were resulted in root absorption and water flow. Figure 8 can be read that decomposed granite soil had larger reduction rate of water flow than Cultivation field soil, because saturated hydraulic conductivity k_{sat} of Decomposed granite soil was bigger than that of Cultivation field soil.

As for the root water uptake rate, reduction rate of Decomposed granite soil was about 11 %, and that of Cultivation field soil was 0.0013 %. It can be assumed that soil which had high available moisture *AM* had low saturated hydraulic conductivity, so that $\Psi_m(\theta)$ and $Y(\theta)$ became low. Therefore, modification to increase available moisture would be necessary for effective implement of phytoremediation.

4 CONCLUSIONS

For seeking effective ways of phytoremediation, two-dimensional analytical model calculating water movement, solute movement and growth of root in unsaturated ground was developed. In this paper, in order to confirm functions of developed model, influences on time-space changings of water and chemical environment in the soil were considered under two types of SWCCs. The obtained results are as follows;

1) Amount of reduction rate of effective saturation S_e by root water uptake in Decomposed granite soil was 11 % of 25 %, and that of Cultivation field soil was 0.0013 % of 0.22 %. Absorption of water and water-soluble substance were depended on matrix potential of soils. The higher water retention capacity soil had, the lower root uptake occurred under same effective saturation.
2) In case available moisture of soil was lower, contribution of root uptake was higher.

5 ACKNOWLEDGEMENT

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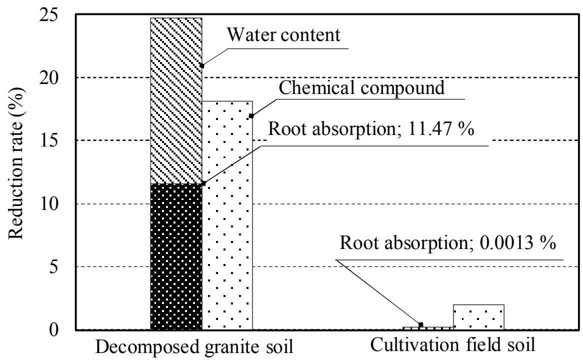

Figure 8. Reduction rate of water and chemical concentration in each soil.

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