Data assimilation for geotechnics – exploring the possibilities

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Abstract

With the increasing amount of data being gathered in geotechnics, new methods that quantify and reduce the uncertainties of the numerical forecasting models become available. In geotechnics several researchers have already started to employ Data Assimilation (DA) combining forecasting and monitoring data for parameter inference and state estimation, however, a systematic survey of DA methods in terms of numerical stability and computational efficiency is still lacking. Furthermore, in most geotechnical problems only a limited number of independent physical quantities is measured whilst many model parameters need to be assessed. This high order dimensionality makes parameter inference using augmented system state approach (combining measured state + parameters) a challenging task. In this paper we present a comparison on the potential of different DA methods in geotechnics. A classical problem of soil water coupled one-dimensional settlement of an embankment on soft soil is used as a benchmark demonstration. A general framework incorporating different DA algorithms, namely, the Unscented Kalman Filter, Ensemble Kalman Filter, Ensemble Square Root Filter and Particle filter will be presented and demonstrated for an example case. The initial results show that the Unscented Kalman Filter shows better convergence of parameters for simple elasto-plastic models, however the Ensemble Kalman Filter with perturbed observation performs substantially better compared to its square root variant and all other filters, even for complex elasto-viscoplastic models. Particle Filter is less suitable for geotechnical applications due to the curse of dimensionality although it's accuracy can be increased with an increased sample size or using an appropriate proposal distribution to sample the posterior, but this becomes computationally demanding. This study also shows that the model parameter sensitivity is a function of observation strategy and regardless of the Data Assimilation methodology used, its efficiency depends on an optimally designed monitoring network i.e., optimal sensor location and measurement interval.

Keywords: Unscented Kalman filter, Particle filter, Ensemble Kalman filter, Parameter inference

1. Introduction

The disparity between model predictions and observations from monitoring data at boundary value level in geotechnical engineering emerge from a number of factors, such as an oversimplified description of the system and/or ignoring the inherent heterogeneous nature of the ground condition, as well as uncertainties in the initial and boundary conditions. With the advancement of monitoring technologies in recent years, it is now possible to monitor displacement, pore-water & earth pressures, and temperature in real time, opening up new possibilities for validating optimisation strategies for inverse analysis and lowering model prediction uncertainty. The uncertainties from both the monitoring data and model simulation need to be taken into account, to determine the likely range of behaviour with a practical margin of error. Recent advances in other fields of science have demonstrated a powerful technique known as Data Assimilation (DA), in which observations are rigorously integrated into numerical forecasting models accounting for the aforementioned uncertainties. A variety of DA techniques such as the Unscented Kalman Filter [UKF], Ensemble Kalman Filter [EnKF] and Particle Filter [PF] (Hommels et al., 2009, Murakami et al., 2017) have been applied for geotechnical problems. Each of these methods come with their own set of merits and drawbacks and a valid comparison between these algorithms is yet to be conducted. Hence, a comparative study is performed for a classical geotechnical problem of soil-water coupled one-dimensional settlement of an embankment constructed on soft soil to identify a robust DA algorithm for geotechnical application. Moreover, since the sensitivity of the model parameters is a variable in the spatial-temporal domain, the effect of sensor location and measurement interval plays a significant role on the DA performance which, unfortunately, is often overlooked. This study shall aim to answer (or touch upon) some of the typical questions on the effect of model complexity and sensitivity of the model parameters on the DA performance and the knowledge gained from this analysis is expected to be useful to identify a robust DA method for application to combined state and parameter estimation in geotechnical models.

2. Theory

2.1 Basic principles of Data Assimilation and its different techniques

The state of the system described as $x_k \in R^m$ comprises of the displacements and pore water pressure as shown below. Its evolution in the time window $t \in [0,T]$ is governed by the forward model F: $f(t,x) \rightarrow f(t+\Delta t,x)$, $\forall(t,x) \in [0,T] \times \Omega$ with $f(t_0 = 0,x)$ as the initial condition and process error $q_k \sim N(0,\sigma_q)$. The state vector x_k is augmented in order to estimate the model parameters concurrently with the state evolution (Iglesias et al., 2013). The system is observed via a set of observations modelled by $y_k \in R^n$ in the observation space with an observation mapping operator H: $f(t,x) \rightarrow g(t,x)$, $\forall(t,x) \in [0,T] \times \Omega$ and measurement error $v_k \sim N(0,\sigma_v)$.

$$x_k = \begin{pmatrix} u_k \\ p_k \end{pmatrix} \epsilon\, R^m \quad \rightarrow \quad \tilde{x}_k = \begin{pmatrix} x_k \\ \theta_k \end{pmatrix} \epsilon\, R^{m+p}$$

$$\tilde{x}_{k+1} = F_k(\tilde{x}_k) + q_k \quad ; \quad y_k = \widetilde{H}_k(\tilde{x}_k) + v_k \quad ; \quad \widetilde{H}_k = (H_k \quad 0)\ \epsilon\, R^{n\, X\,(m+p)}$$

Where u_k is the displacement vector and p_k is the porewater pressure vector at the nodes for time step 'k' of the discretised system. The persistence model is chosen for the parameters meaning the parameters remain constant during the state evolution and gets modified during the assimilation process. The prior distribution at time 'k' is obtained by projecting the posterior at time 'k-1' based on all available observations until time 'k-1'. Then using the likelihood from observation at time 'k', the prior is then updated to get the posterior.

$$p(\tilde{x}_k|y_{1:k-1}) = \int p(\tilde{x}_k|\tilde{x}_{k-1})\, p(\tilde{x}_{k-1}|y_{1:k-1})\, d\tilde{x}_{k-1}$$

$$p(\tilde{x}_k|y_{1:k}) = \frac{p(y_k|\tilde{x}_k)\, p(\tilde{x}_k|y_{1:k-1})}{p(y_k|y_{1:k-1})}$$

Solution for these equations is usually intractable especially in geotechnical applications, hence, algorithms that can approximate the exact solution are necessary. For this purpose, the Unscented Kalman Filter (Julier & Uhlmann 1997), the ensemble extensions of the traditional Kalman filter, i.e Ensemble Kalman Filter (Evensen 2007) and Ensemble Square Root Filter (Whitaker & Hamill 2002) and Particle Filter (Murakami 2013) are considered. The details of the formulation are not repeated here for the sake of brevity and the interested readers are directed to their corresponding references.

2.2 Geotechnical forward model

In this study, an elasto-plastic constitutive formulation representing the simple forward model and an elasto-viscoplastic constitutive formulation representing the advanced forward model coupled with consolidation to calculate the settlements and excess pore water pressure under an embankment loading in one dimensional vertical section (2-way drainage) is considered. The total strain rate is decomposed as shown below.

$$\dot{\varepsilon}_z = \dot{\varepsilon}_z^e + \dot{\varepsilon}_z^p \text{ (elasto-plastic)} \qquad \dot{\varepsilon}_z = \dot{\varepsilon}_z^e + \dot{\varepsilon}_z^{vp} \text{ (elasto-viscoplastic)}$$

$$\text{where,} \quad \dot{\varepsilon}_z^e = \kappa^* \frac{\dot{\sigma}'_z}{\sigma'_z} \qquad \dot{\varepsilon}_z^p = (\lambda^* - \kappa^*) \frac{\dot{\sigma}'_z}{\sigma'_z}$$

$$\dot{\varepsilon}_z^{vp} = \frac{\mu_i^*}{\tau} \left\{ \frac{\sigma'_z\,(1 + \chi_0)}{[1 + \chi_0 \exp(-\rho \varepsilon_z^{vp})]\sigma'_{po} \exp\left(\frac{\varepsilon_z^{vp}}{\lambda_i^* - \kappa^*}\right)} \right\}^{\frac{\lambda_i^* - \kappa^*}{\mu_i^*}}$$

The 1D constitutive models are coupled with the continuity equation (Yin and Graham 1996) and the hydraulic conductivity is made state dependent on the current void ratio.

$$\frac{k_v}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = -\frac{\partial \varepsilon_z}{\partial t} \qquad C_k \log\left(\frac{k_{vo}}{k_v}\right) = e_0 - e$$

The coupled partial differential equation is solved using the finite difference procedure implemented in Python. The external load is made time dependent to better represent the embankment construction process and the settlement (below the embankment) along with the excess pore water pressure is calculated numerically serving as synthetically generated measurement data retrieved from certain nodes and superimposed with an additional Gaussian noise. The parameters of different constitutive models along with the chosen feasibility space (in parenthesis as blue) are summarised in *Table* 1. The excess porewater pressure is not considered in the measurement dataset although it can improve the convergence rate. In our study, only the settlements are used to investigate the performance of different Data Assimilation procedures. The synthetic noisy measurements are sampled at time steps corresponding to standard practice of retrieving settlements, i.e. from frequent intervals at the start of the embankment construction with a progressively reduced measurement interval with increasing time-period.

3. Results

3.1 Performance with simple elastoplastic model

In this Section, the characteristics of different DA procedures are compared with the synthetic dataset generated from the simple elasto-plastic model. Halton sequencing (Murakami et al., 2017) is used to generate samples for the ensemble variants (EnKF & EnSRF) and particle filter. EnKF and EnSRF with a fixed ensemble size of 100 has been used. The chosen range of parameters are summarised in Table 1 (in the parenthesis). Figure 1 shows the performance of UKF, EnKF and EnSRF for the elasto-plastic model. All 3 parameters converge toward their synthetic true value but with different convergence rates depending on their sensitivity (shown in Figure 1d with normalized variance of each parameter) showing that the modified compression index (λ^*) and preconsolidation pressure (σ'_p) achieve faster convergence than the modified swelling index (κ^*). In Table 1, the final accuracy and an indication of the precision (measured as $\sigma_{\vartheta\vartheta}^2/\sigma_{\vartheta\vartheta 0}^2$) are mentioned in the parenthesis (in red) along with the computational cost for each DA method. As expected, the UKF is efficient in computational demand due to its limited ensemble size, but the EnKF is relatively superior in performance.

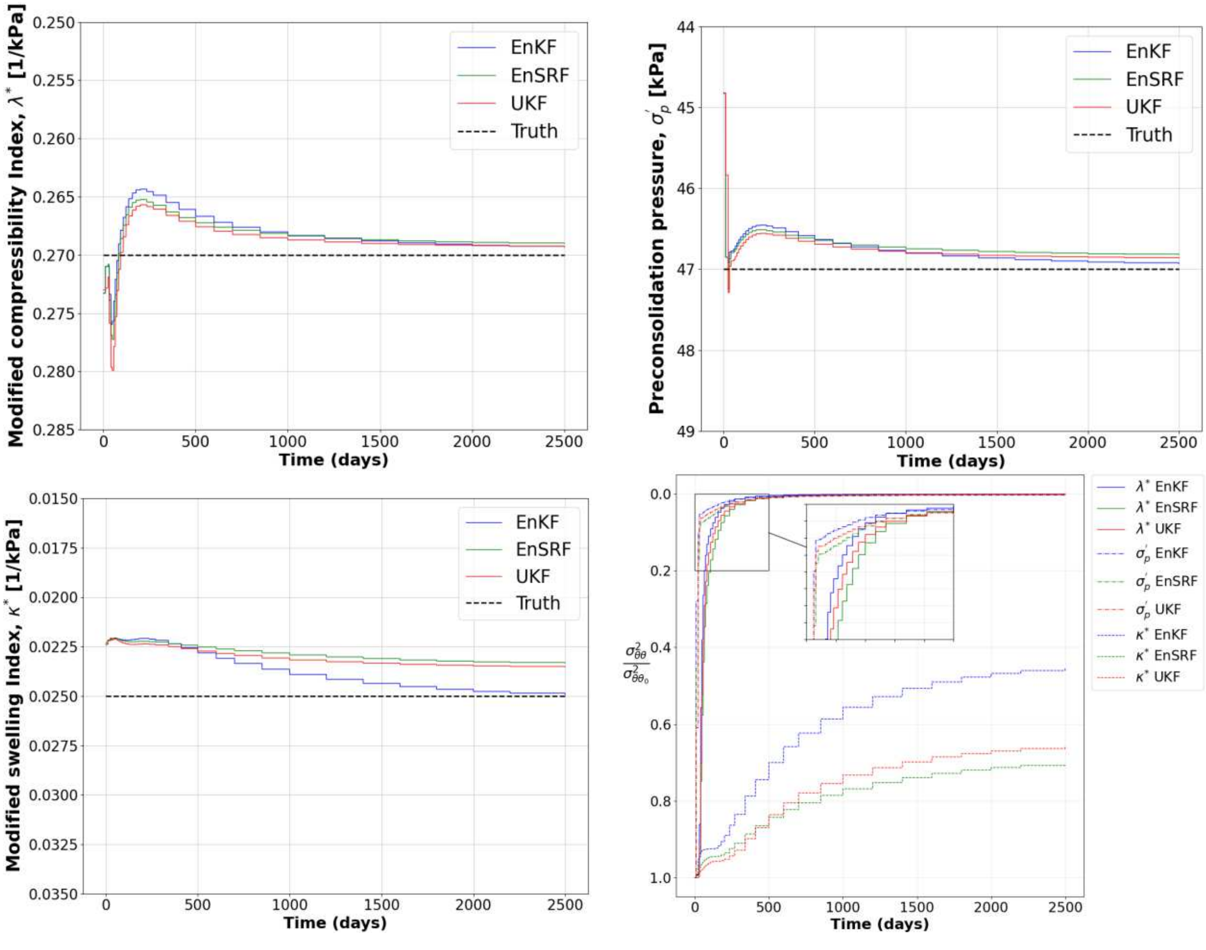


Figure 1: Convergence of parameters for elasto-plastic model using different DA procedures

The performance of the Particle Filter is assessed in a manner similar to Shuku et al., (2012), where a separate assessment is conducted with hierarchical number of parameters. Due to the use of a resampling procedure (SIR), it is sufficient to generate 100 particles for single and dual parameter assimilation and 300 particles for all 3 parameters of the elasto-plastic model. The results are shown in Figure 2. It can be seen that the Particle Filter gives a stable and accurate response for estimating a single parameter. As soon as more parameters are included, the filter becomes increasingly unstable and when trying to assimilate all parameters, the solution is least accurate. This is also observed in Shuku et al., (2012) for a similar elasto-plastic model but for general stress path testing. The poor performance of the Particle Filter with increasing dimension is well-known (Bengtsson et al., 2008) and clearly shows that its application is not recommended for general geotechnical applications which often comprise a large number of uncertainties. Furthermore, given the computational demand (in this simple case the run time was around 600 s for assimilating all parameters), parallel computing methods are required for more advanced 2D and 3D numerical models which is not sustainable in everyday practice. However, the final assimilated settlement in all cases is similar to the synthetic truth. The fitted values in each of these cases does not pose a major issue, if the future behaviour is well predicted. This, however, is usually not the case and the effectiveness need to be measured on the proximity of the estimate to the true value which is believed to maintain consistency and provide a better understanding of the underlying physics.

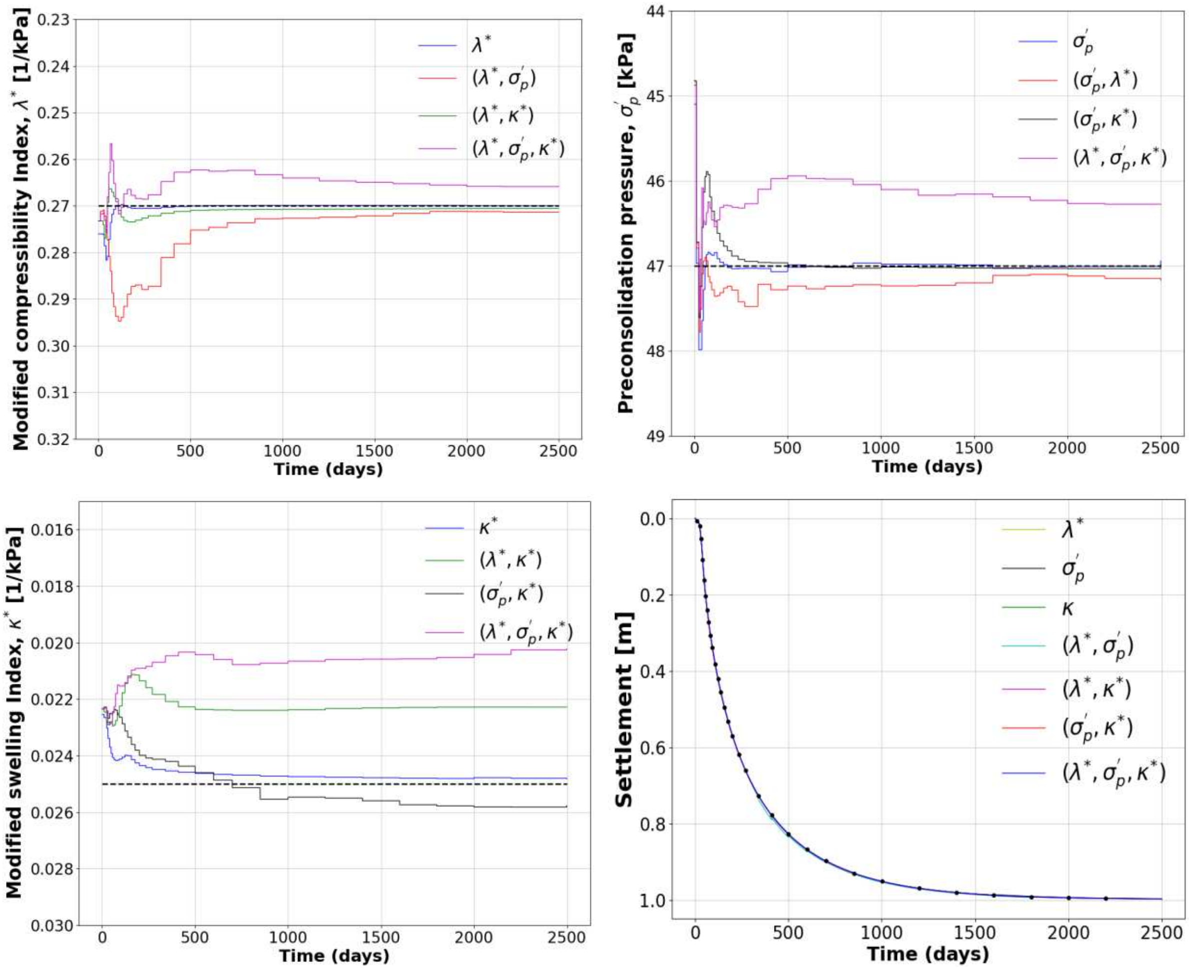


Figure 2: Convergence of parameters for elasto-plastic model using Particle Filter

3.2 Effect of sensor placement and measurement window

The performance of the Data Assimilation depends on the sensitivity of the parameters. In our specific case, the modified swelling index κ^* exhibits lower sensitivity than the other parameters. However, the sensitivity of the parameters is not constant with time. Due to the evolving effective stress level in the system, which in turn is a function of the ground water flow in the consolidation process, the sensitivity indices change in the spatial domain as well. Hence, the performance of the Data Assimilation procedure will also vary in the spatial-temporal domain. In view of this, the time and location of our measurement, which we include in our Data Assimilation scheme, dictates the convergence of the parameters. To illustrate this, consider the same case but with the drainage made impervious at the bottom of the model. Now we take the settlement measurement at 4 meters depth with high frequency (0.5 days) for the first 50 days and then the time intervals as used previously. Figure 3 shows the performance of EnKF to back estimate the elastoplastic model parameters and we can see that the modified swelling index (κ^*) converges relatively faster than the other parameters in the first 50 days (see Figure 3a). This is due to the effective stress level being still in the overconsolidated region. Now continuing the simulation with the aforementioned regular time intervals (shown until 300 days in Figure 3b), the other parameters reach convergence to their true value as now the stress level has reached the normally consolidated region. The time of stiffness transition from the elastic to elasto-plastic region at 4 meters depth is around 140 days and this corresponds well with Figure 3b where the assimilation of the modified compressibility index λ^* starts exactly around this time. Hence the groundwater flow conditions dictate the parameter sensitivity and thereby the Data Assimilation performance. This shows that the stability of the parameter estimation solution is related to the dependence of the parameters on the observations. The available observations should contain sufficient information (both spatial and temporal) to be able to determine the unknown parameters of interest. Hence, demonstrating that the parameter sensitivity is a function of observation strategy. Therefore, Data Assimilation, regardless of the methodology used, can only be expected to work efficiently in an optimally designed monitoring network i.e., optimal sensor placement and measurement frequency.

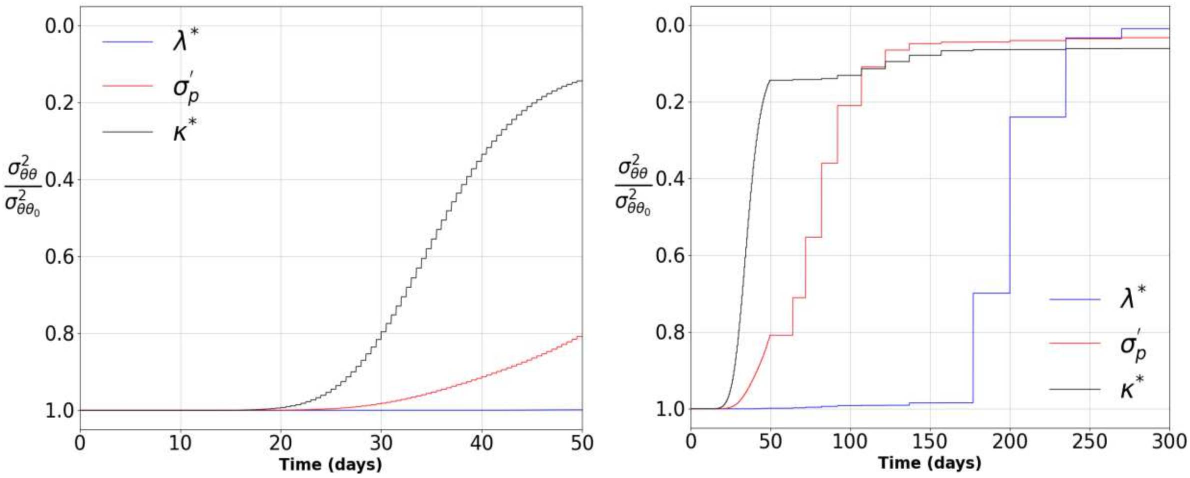
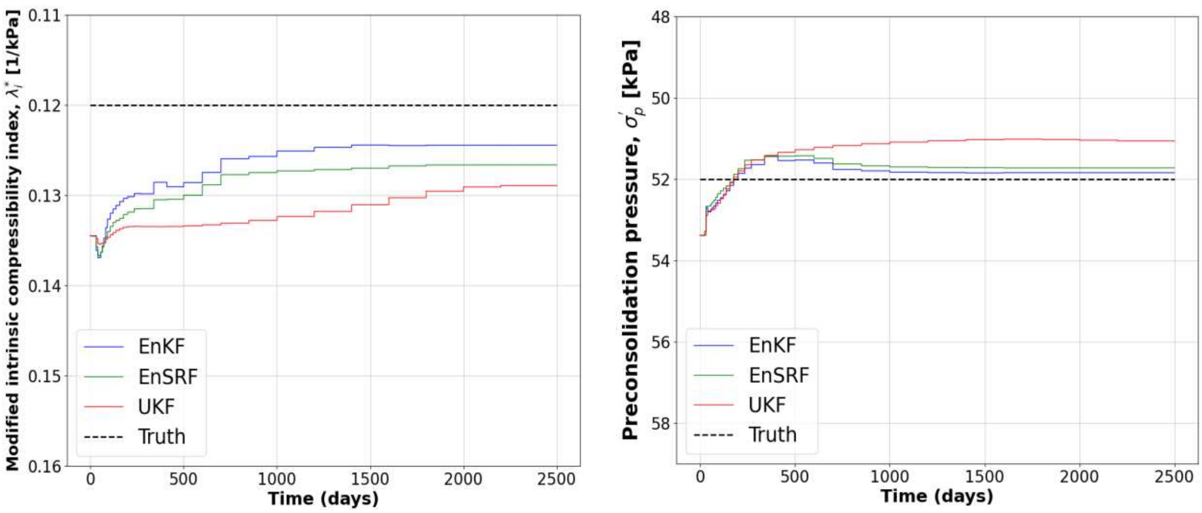


Figure 3: Normalised variance reduction plot for all elasto-plastic parameters (a) 0-50 days (b) 0-300 days

3.5 Performance with advanced constitutive model

To further evaluate the efficiency of the DA algorithms, their performance is assessed with advanced constitutive model i.e., an elasto-viscoplastic model that have an inherently large parameter set. The initial feasibility space for the parameters of the elasto-viscoplastic model are provided in *Table* 1. The reference time is taken as 1 day representing 24-hour duration of the load step in the incremental loading oedometer test commonly used to derive these parameters. The convergence of all the model parameters is shown in Figure 4. Clearly, the EnKF performs relatively better than the other filters that are evaluated. The initial prior belief of parameter feasibility space affects the DA performance. Especially for advanced constitutive models this is the case, due to their large parameter set which comprise parameters that might be insensitive for the loading path used for DA. A large uncertainty bound for sensitive parameters does not pose any issues in the performance of the DA. In contrast, for insensitive parameters the initial uncertainty bound need to be made strict which is similar to that observed in Wang et al (2008).



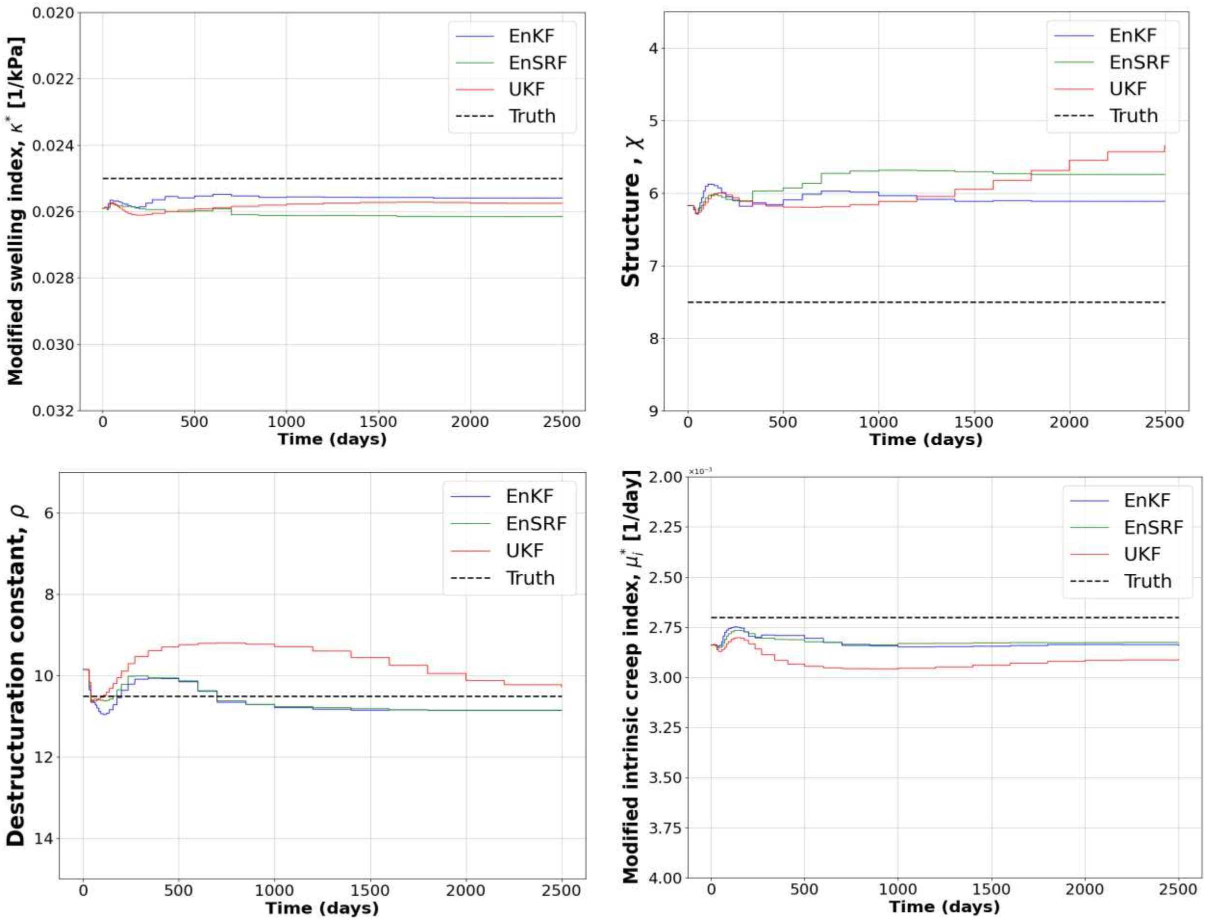


Figure 4: Convergence of parameters for elasto-viscoplastic model for different DA procedures

Table 1: Parameters for the models to generate synthetic truth.

Parameters	EP model	EVP model	EnKF	EnSRF	UKF	Unit
Modified compressibility index (λ*)	0.27 (0.190-0.360)	-	0.2693 (99.741, 4.56e-4)	0.2690 (99.630, 8.4e-4)	0.2692 (99.704, 7.97e-4)	1/kPa
Preconsolidation pressure (σ'$_p$) – EP	47.0 (37.0-53.0)	-	46.92 (99.829, 1.93e-3)	46.81 (99.596, 3.19e-3)	46.85 (99.681, 2.93e-3)	kPa
Preconsolidation pressure (σ'$_p$) - EVP	-	52.0 (48.0-59.0)	51.84 (99.692, 0.0564)	51.72 (99.463, 0.101)	51.06 (99.208, 0.122)	kPa
Modified swelling index (κ*) [EP]	0.025 (0.015-0.030)	-	0.0249 (99.6, 0.454)	0.0233 (93.426, 0.703)	0.0235 (94.176, 0.658)	1/kPa
Modified swelling index (κ*) [EVP]	-	0.025 (0.020-0.032)	0.0256 (97.63, 0.729)	0.0261 (95.51, 0.827)	0.0257 (97.24, 0.857)	1/kPa
Modified intrinsic compressibility index (λi*)	-	0.120 (0.110-0.160)	0.1244 (96.39, 0.172)	0.1266 (94.65, 0.249)	0.1289 (92.85, 0.528)	1/kPa
Destructuration constant (ρ)	-	10.5 (5.0-15.0)	10.852 (96.71, 0.050)	10.857 (96.66, 0.065)	10.282 (97.95, 0.063)	-
Structural parameter (χ)	-	7.5 (3.5-9.0)	6.11 (83.08, 0.497)	5.74 (79.08, 0.637)	5.34 (74.98, 0.484)	-
Modified intrinsic creep index (μi*)	-	0.0027 (0.002-0.004)	0.00284 (94.94, 0.734)	0.00282 (95.41, 0.8204)	0.00291 (92.52, 0.827)	1/day
Reference time (τ)	-	1	1	1	1	day

Notes:

- *values in blue are the feasibility space to generate the samples for the DA*
- *values in red are accuracy and indication of precision from each DA method*
- *Run time for Elasto-plastic model: EnKF (t=89.73 s), EnSRF (t=89.15 s), UKF (t=21.51 s)*
- *Run time for Elasto-viscoplastic model: EnKF (t=208.65 s), EnSRF (t=203.92 s), UKF (t=50.78 s)*

4. Conclusions

This study demonstrates that the statistical based Data Assimilation is promising to address state and parameter estimation for time-dependent geotechnical problem when combined with a deterministic geotechnical forecasting model. Combined they provide a powerful tool, although caution is required when choosing the appropriate method and observation data. Due to the variation of parameter sensitivity in the spatio-temporal domain, the location of the sensors along with the time interval of measurements dictate the performance of the Data Assimilation procedure. Among the Data Assimilation techniques tested, the Unscented Kalman Filter is the most computationally efficient and can be recommended for simpler geotechnical models but for advanced numerical models that require a large parameter set the Ensemble Kalman Filter is superior overall other DA methods. The performance of the Particle Filter is less suitable for geotechnics due to its less effectiveness in higher dimensions and requires a relatively large number of particles. The latter demanding higher computational cost and hence is not recommended for general geotechnical engineering practice. The EnKF and its variants counteract this problem by seeking an approximation to map the prior to the posterior by shifting rather than reweighting in the update step. The latter allows the algorithm to remain stable making it a powerful tool for inference in high-dimensional problems with far less computational demand. EnKF is efficient both in terms of accuracy and precision. Further evaluation is required with regards to the effect of quantity and quality of sensors on the DA performance, system uncertainty and a validation case with real in-situ measurement. Furthermore, other advanced DA techniques such as the hybrid methods combining the merits of EnKF and PF shall also be investigated.

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