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Maximum Shear Modulus by In-situ Tests and its use for Site Characterization of a Tropical Soil

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SUMMARY: Site characterization is the process of identifying the homogeneous zones and defining index, stiffness and strength parameters for the soils within these zones. The maximum shear modulus (G_o) is an important geotechnical parameter to be used in dynamic problems (e.g. earthquakes and vibration problems) as well as in static deformation analysis such as excavations and foundation design. So, the demand for site investigation including the G_o profile is increasing. There are several in-situ seismic tests to determine G_o such as the cross-hole and the down-hole techniques, as well as hybrid tests (e.g. seismic cone - SCPT and seismic flat dilatometer - SDMT). This paper presents cross-hole, down-hole, SDMT and SCPT tests carried out in an unsaturated tropical soil profile from Brazil. Advantages and limitations regarding the test procedures and interpretation are briefly discussed. The differences observed between G_o profiles determined by these techniques are also discussed. The G_o can be used to identify unusual geomaterials, like the tropical soils. In addition, the applicability of the modulus degradation curve to predicted settlement of shallow foundations for the studied site is also presented and discussed.

KEYWORDS: Site Investigation, In-situ Tests, Tropical Soil, Maximum Shear Modulus, Modulus Degradation Curves.

1 INTRODUCTION

Geotechnical site characterization consists in determining the stratigraphical profile, the groundwater level and the estimative of geo-mechanical designs parameters required for each project.

For purposes of the dynamic analysis of soils, shear wave velocity (*Vs*) and, as a result, the maximum shear modulus (G_o), is widely used. For this reason, it is necessary to better understand the differences between several in-situ seismic techniques, as well as the way for their data interpretation.

The modulus degradation curve is an important soil characteristic and is crucial in geotechnical projects (Clayton, 2011).

G_o is the stiffening parameter that refers to the initial undisturbed state of the soil and allows assessing the stress-strain-strength response of soils for static, cyclic and dynamic loads, both for drained and undrained conditions. It can be calculated from shear wave velocity (*Vs*) by in-situ or laboratory tests (Woods, 1978).

The maximum shear modulus (G_o) is calculated by the equation:

$$G_o = \rho_T (V_s)^2 \qquad (1)$$

where $\rho_T = \gamma_T / \mathrm{g}$ = total mass density, γ_T = soil unit weight, and g = 9.8 m/s² = gravitational constant.

This paper presents the cross-hole, the down-hole, the seismic flat dilatometer test (SDMT) and the seismic cone penetration test (SCPT) tests carried out in an unsaturated tropical soil site located at Bauru, São Paulo state, Brazil. Advantages and limitations regarding the test procedures and interpretation are briefly discussed. Moreover, the differences observed between G_o profiles determined by these techniques are also discussed. G_o was used to identify unusual geomaterials, like the tropical soils. In addition, the applicability of the modulus degradation curve to predict settlement of shallow foundations in this soil was studied.

2 IN-SITU SEISMIC TESTS

2.1 Cross-hole tests

The cross-hole seismic test is one of the most effective techniques for the in-situ determination of G_o. The main objective of this technique is determining the compression (P) and/or shear (S) propagation velocities along depth, being regulated by ASTM (2007).

The test consists of generating seismic waves in a borehole and registering their arrivals in one or more adjacent boreholes. The spacing between the source borehole and the first receiver borehole have to be around 1.5 to 3.0 m and the distance between subsequent receiver boreholes have to be 3.0 to 6.0 m apart. A typical layout is illustrated in Figure 1. For two boreholes, spacing between the source borehole and the receiver borehole have to be 1.5 to 5.0 m. (ASTM, 2007). The source and the seismic receivers (geophones or accelerometers) are positioned at the same depth, and the *Vs* is typically determined every meter depth interval.

The interpretation of cross-hole test data to calculate G_o basically consists of identifying the first arrival of the shear waves (*S*). *S* waves are characterized by an increase in the amplitude of the signal as well as by the fact that it polarizes: inverting the direction of the blow, all phases corresponding to the shear waves appear inverted.

Special attention should be given in opening and preparing the source and the receiver boreholes. The procedure suggested by ASTM (2007) consists of coating them with metallic or PVC (polyvinyl chloride) pipes and grouting the borehole by a small diameter grout tube insert to the bottom of the borehole, by means of using a cement mix.

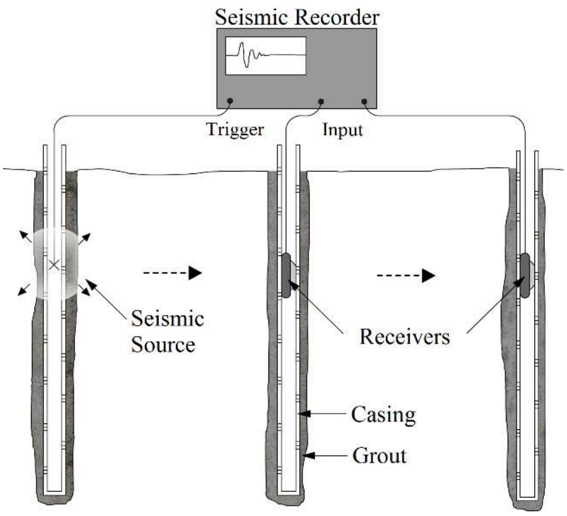


Figure 1. Cross-hole seismic test (ASTM 2007).

2.2 Down-hole tests

The down-hole seismic test (ASTM, 2008) is carried out using a single borehole. This test is performed inserting a seismic probe in a prepared borehole or into the soil mass and, in this case, there is no need for preparing the borehole.

The test consists in determining the arrival time of seismic waves generated on the ground surface and travelling down to an array of vertically installed seismic sensors positioned at different depths. The interpretation of the test data considers that the travel path between the source and the receiver follows a linear trajectory. In heterogeneous materials, this path is not a straight line and Snell's law of refraction can to be used.

The determination of shear wave propagation velocity can be done by three different methods: first arrival, cross-over and cross-correlation. According to Campanella and Stewart (1992), the cross-correlation method surpasses the others because it is less affected by signal distortions, leading to more consistent and reliable results.

2.3 Seismic Cone Penetration Test (SCPT)

In the mid-1980s, a system for the acquisition of seismic waves by down-hole technique was incorporated into the electrical cone penetration test (CPT), which was known as the seismic cone (SCPT). The cone resistance (q_c), sleeve friction (f_s) and Vs can be quickly determined, with accuracy and repeatability (Campanella et al., 1986).

The seismic cone presents the same characteristics of a standard electrical cone (Figure 2), with seismic receivers (geophone or accelerometer) incorporated into it.

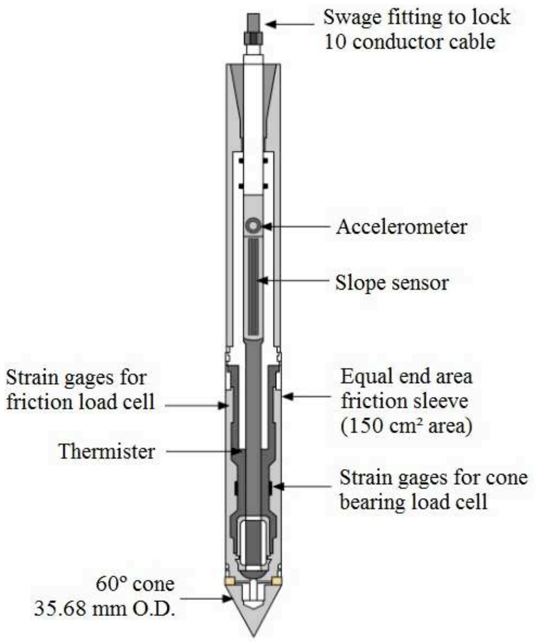


Figure 2. Seismic cone (SCPT) (adapted from Campanella et. al. 1986)

The procedure to push the seismic cone into the ground provides a firm mechanical contact between the soil mass and the seismic sensors, which allows an excellent signal reception. In addition, the sensor orientation and the depth definition can be accurately done.

The down-hole technique with the SCPT involves three steps: S-wave arrival time, S-wave velocity determination (Vs) at each test depth and calculation of the maximum shear modulus (G_o) for each of these depths.

A digital oscilloscope and a filter are used to reduce ambient noise. A trigger system is also essential for accurate identification of wave arrival time, which must always be checked at each test start to ensure performance and repeatability. A schematic representation of down-hole test together the CPT is shown in Figure 3. Details on the execution and interpretation of SCPT can be obtained in Butcher *et al*. (2005).

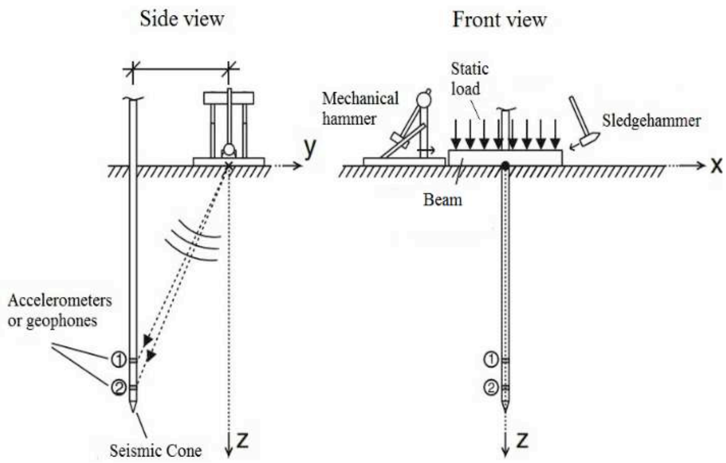


Figure 3. Schematic representation of the SCPT (Karl et al., 2006).

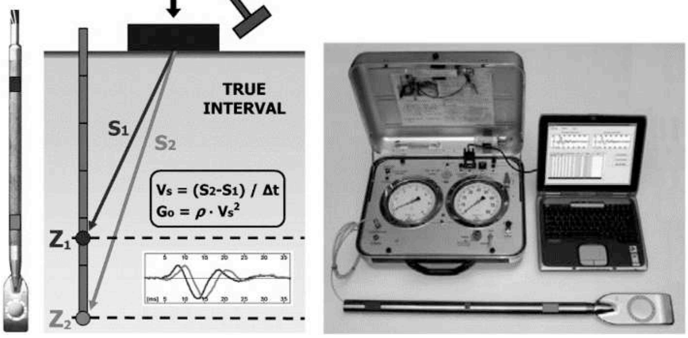
2.4 Seismic Flat Dilatometer Test (SDMT)

The Marchetti dilatometer test (DMT) consists of a stainless-steel blade with a thin flat circular expandable membrane on one site. It is pushed into the soil mass using a hydraulic system and a reaction structure. The blade penetrates vertically into the ground at a constant velocity (20 to 40 mm/sec). At 0.2m depth intervals, gas pressure is applied through the control unit, inflating the membrane.

Two readings are registered during the DMT (readings A and B). Reading A corresponds the pressures necessary for the membrane loses contact with the sensitive equipment. The pressure necessary for the membrane to move 1.1 mm is the reading B (Marchetti, 1980). Both readings are corrected for membrane stiffness to determine pressures p_o (corrected reading A) and p_1 (corrected reading B). The intermediate DMT parameters (Material Index - I_D, Horizontal Stress Index - K_D and Dilatometer Modulus - E_D) are calculated from p_o and p_1. They are used in the soil classification and estimative of geotechnical parameters.

The seismic dilatometer (SDMT) is a combination of the Marchetti dilatometer test (DMT) with a seismic probe for the determination of wave propagation velocities (*Vp* and *Vs*) (Marchetti et al., 2008). The test is conceptually equivalent to the seismic cone (SCPT).

Figure 4a shows the seismic probe consisting of a cylindrical element installed above the DMT blade, equipped with two 0.5 m spaced geophones. Figure 4b shows the schematic representation of the SDMT and Figure 4c the equipment for performing the seismic dilatometer test.



a) b) c)

Figure 4. a) DMT blade and seismic probe; b) Schematic layout of SDMT; c) SDMT equipment (Marchetti et al., 2008).

3 SHEAR MODULUS DEGRADATION

Settlement prediction of shallow foundations is necessary in the design of structures. Numerical simulations via finite element method are the most appropriate approach for foundation settlement prediction. Such prediction can also be performed by using G_o as the input parameter (Archer and Heymann, 2015). The settlement prediction methods that use G_o require the knowledge of the deformations mobilized for a given applied load to the soil mass. The soil stiffness is relatively higher at low deformations and it tends to decrease with increasing loading (Atkinson, 2000).

The soil stiffness degradation is non-linear, and some models have been developed to represent it. Most of them were developed for dynamic loading using the resonant column test or improved triaxial tests (Seed and Idriss, 1970; Yamashita and Toki, 1994). After the 80's the static tests have also been used to evaluate the behavior of non-linear soil stiffness (Jardine et al., 1986; Jovicic and Coop, 1997). In general, models to satisfy this demand are based on routine testing and simple calculations (Oztoprak and Bolton, 2013).

Fahey and Carter (1993) incorporate the nonlinearity of soil stiffness based on the numerical solutions using nonlinear elastic, Mohr-Coulomb plastic soil model with an axisymmetric plane strain finite element software. The authors suggested a hyperbolic model, according to Equation 3.

$$\frac{G}{G_o} = 1 - f\left(\frac{\tau}{\tau_{max}}\right)^g \qquad (3)$$

where is G_o the small-strain stiffness, g and f are empirical fitting parameters that distort the shape of the hyperbolic function, τ_{max} is the shear strain at failure and τ is the shear strain.

3.1 G_o in the load-settlement prediction

The concept of modulus degradation can be used in the foundation design considering an equivalent elastic modulus compatible with soil deformation (Mayne et al., 1999). Some authors incorporate the hyperbolic models for the of the load-settlement prediction of foundations.

The displacement (s) of shallow foundations under axial compression loading (q) can be calculated using the equivalent elastic modulus (E_s) by Equation 4 (Mayne et al., 1999).

$$s = \frac{qI}{BE_s} \tag{4}$$

where s is the vertical deflection settlement, B is the foundation width, q is the applied axial loading, I is the displacement influence factor, and E_s is the equivalent elastic modulus

Mayne (2000) proposed a simple analytical solution based on the elasticity theory using the modified hyperbole suggested by Fahey and Carter (1993), according to Equation 5.

$$s = \frac{qI}{BE_o\left[1-f\left(\frac{q}{q_{ult}}\right)^g\right]} \tag{5}$$

where s is the vertical deflection settlement, B is the foundation width, q is the applied surface stress, I is the displacement influence factor, and E_o is equal to $2G_o(1+\nu)$, q_{ult} is the ultimate axial loading from bearing capacity theory, g and f are empirical fitting index that distort the shape of the hyperbolic function.

The f and g indexes are adjusted by using torsional, triaxial, and/or simple shear tests. The index value $f = 1$ and $g = 0.3$ provide reasonable approximations for non-aged and non-cemented sands and unstructured clays (Mayne, 1995; Burns and Mayne, 1996) and Mayne et al. (1999).

Elhakim (2005) also assessed the shallow foundation settlements using the soil small-strain stiffness. The author proposed an algorithm for generating non-linear load-displacement curves for footings using an equivalent elastic framework. The author considers that this approach turns the calculation of settlements in shallow foundations easier and compares reasonably well with other solutions (e.g. Harr, 1966; Carrier and Christian, 1973; Mayne and Poulos, 1999).

Originally, Puzrin and Burland (1996) proposes that the degree of non-linearity in the stress-strain relationship can be expressed by a normalized limiting strain x_L. This index is calculated by the ratio of the limiting strain and the reference strain. Alternatively, the value of x_L can be calculated by the ratio between of the small-strain stiffness (E_o or G_o) and secant modulus at failure (E_{min} or G_{min}) (Elhakim, 2005), according to Equation 6.

$$x_L = \frac{G_o}{G_{min}} \tag{6}$$

The G_o values can be calculated based on in-situ seismic test data, while G_{min} is the secant modulus at the point of maximum deviatoric stress determined from triaxial test. The x_L value is used to obtain the hyperbolic fitting parameter (g^*).

Figure 5 shows the data for the adjusted hyperbolic functions from drained loading simulations for different friction angle values. In this case, f^* assumes values equal to 0.99 or 1.00. The value of g^* decreases with increasing x_L. Elhakim (2005) also provides solutions to other loading conditions.

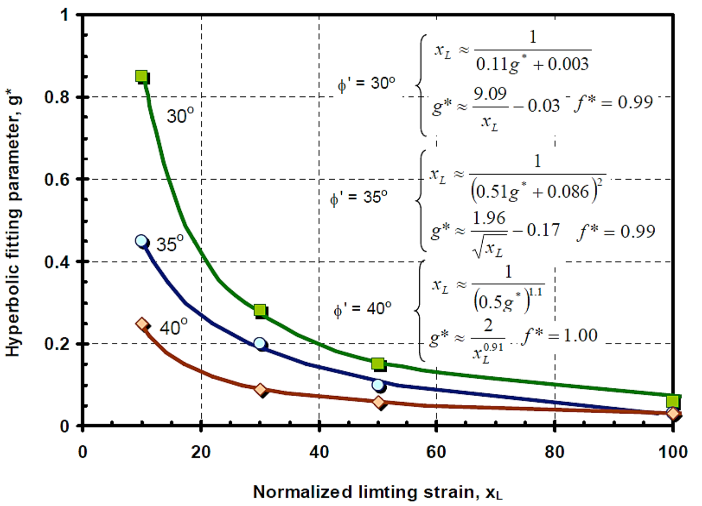


Figure 5. Variation of the hyperbolic fitting index g* for circular footings under drained loading conditions (Elhakim, 2005).

4 STUDIED SITE

The studied site is geologically characterized by sandstone rocks from Bauru Group (Upper Cretaceous), which recovers the volcanic rocks from Serra Geral Formation. Sedimentary rocks from Marilia Formation are predominant at the site, which are experiencing weathering processes over tropical conditions. De Mio (2005) emphasizes that these soils exhibit characteristics from the parent rocks, like sequence of layering and modifications on these geological materials by pedogenetic and morphogenetic processes.

The studied site has an unsaturated porous sandy soil profile with a high saturated hydraulic conductivity. The top 13 m has lateritic soil behavior (LA') (horizon B) followed by a saprolitic soil (horizon C) with non-lateritic behavior (NA') (Nogami and Villibor, 1981). The groundwater level was not found up to 20 m depth.

One cross-hole (CH1), two down-holes (DH1 and DH2), one seismic cone (SCPT 1) and two seismic dilatometers (SDMT 1 and SDMT 2) tests were carried out at this site. The cross-hole test was performed up to 14 m depth, while the other tests were performed up to 20 m depth. Figure 6 shows the location of the seismic tests.

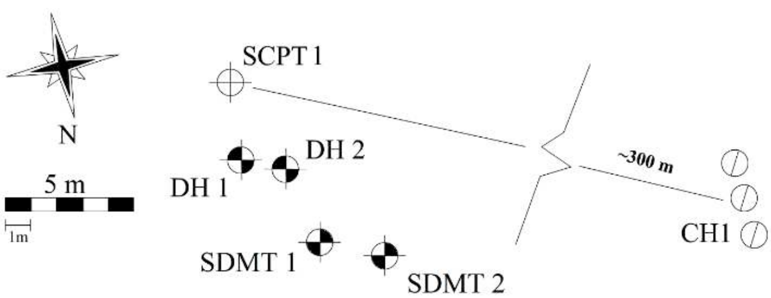


Figure 6. Test locations at the studied site.

5 TEST DATA AND DISCUSSION

5.1 G_o profiles

Figure 7 shows the seismic test data carried out at the studied site, showing the *Vs* and G_o values with depth.

The Vs profiles determined by the different techniques are presented in Figure 7a. Figure 7b shows G_o profiles calculated by Equation 1. Soil unit weight (γ_T) was determined from undisturbed soil samples collected in sample pits excavated at the site (ABNT, 1986).

There is reasonable agreement between the Vs and G_o profiles determined by the different seismic techniques. It can be observed in this figure a progressive increase in Vs and G_o values with depth up to approximately 12 m, followed by the tendency of the stabilization after this depth.

The SCPT and SDMT were interesting tools for determining the Vs and G_o profiles. Moreover, its present a lower cost when compared to the cross-hole tests, since there is no need to prepare the boreholes, as previously mentioned.

5.2 Unusual soil behavior

The maximum shear modulus (G_o) has been used in the site characterization for geotechnical earthquake engineering, vibration problems, as well as in static deformation analysis. Recently, the G_o values have also been used to identify unusual soils, since pore pressure (u) measured by the piezocone tests sometimes cannot be considered useful to ensure proper soil characterization.

Schnaid *et al.* (2004) and Cruz (2010) have demonstrated that the use of the maximum shear modulus (G_o) together with another geotechnical parameter (q_c and E_D) is an interesting approach for identifying soils with unusual behavior (e.g. sensitivity, aging or cementation).

Schnaid et al. (2004) proposed a chart and limits to correlate G_o/q_c versus q_{c1}, a dimensionless normalized parameter defined as:

$$q_{c1} = \left(\frac{q_c}{p_a}\right)\sqrt{\frac{p_a}{\sigma'_{vo}}} \qquad (2)$$

where p_a = atmospheric pressure and σ'_{vo} = vertical effective stress.

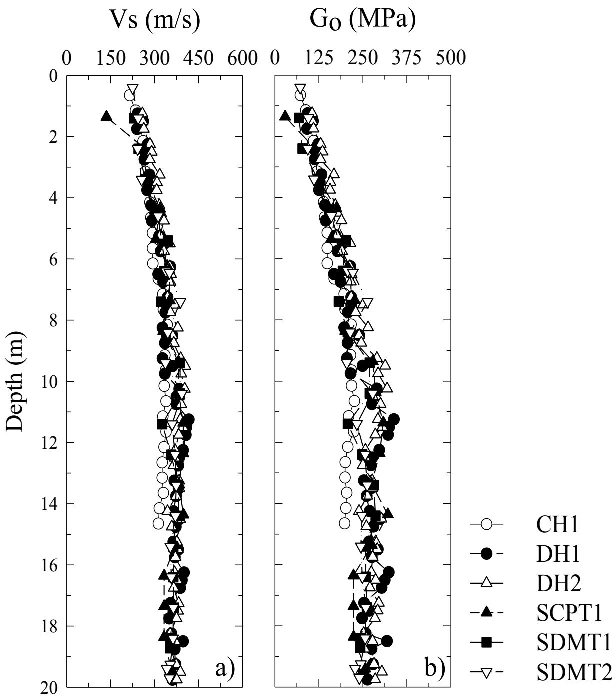


Figure 7. Seismic test data.

Cruz (2010) suggested another chart (G_o/E_D versus I_D) and boundaries based on the dilatometer test data similarly to what was suggested by Schnaid et al. (2004).

Figure 8 presents SCPT (q_c and f_s) and SDMT (I_D, E_D and K_D) data, in the Schnaid *et al*. (2004) and Cruz (2010) charts. The measured cone resistance (q_c) was considered equal to corrected cone resistance (q_t), since the pore water pressure cannot be measured since the studied site was unsaturated. Figure 8a and Figure 8b show that q_c and f_s tend to increase with depth. Figure 8c, Figure 8d and Figure 8e show the DMT data.

Figure 9 presents q_c and G_o values determined at the same depth from SCPT 1. It shows that unsaturated condition and cementation in tropical soils provide G_o/q_c ratios that are higher than those determined in sedimentary soils. Moreover, it is interesting to note that lateritic soils presented higher value of G_o/q_c than the saprolitic soils.

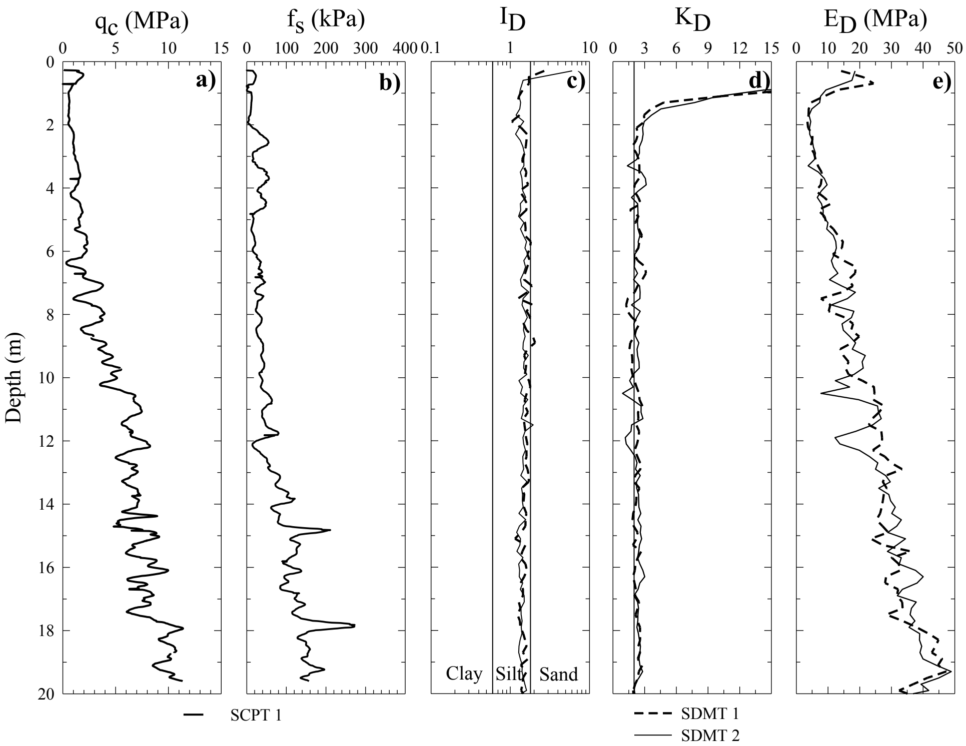


Figure 8. SCPT and SDMT *data*

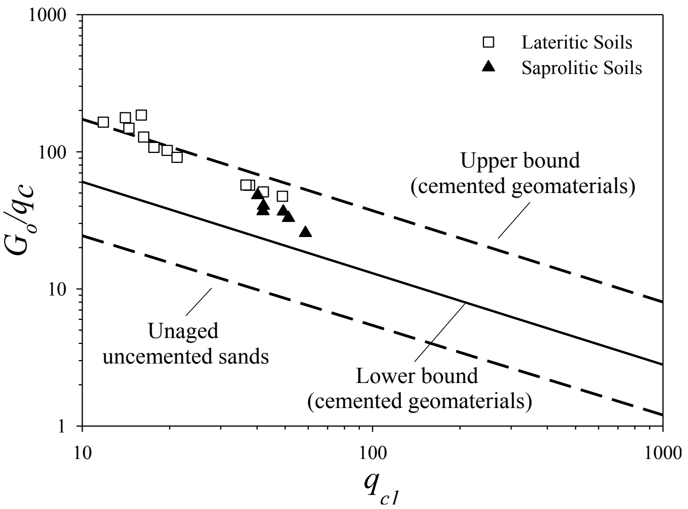


Figure 9. G_o/q_c for the studied site in the Schnaid *et al*. (2004) chart.

Figure 10 presents the average values of the G_o/E_D ratio determined at the same depth from SDMT 1 and SDMT 2 in the Cruz (2010) chart.

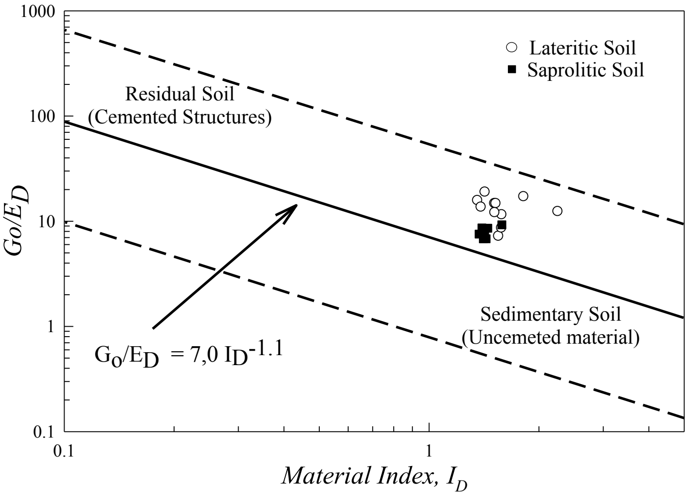


Figure 10. G_o/E_D for the study site in the Cruz (2010) chart.

It is observed that all the SDMT data are above the boundary that separates sedimentary to residual soils (cemented structures). The bonded structure of the studied tropical soil also produced G_o/E_D systematically higher than those measured in sedimentary soils. The G_o/E_D ratio allow identifying soil horizons similarly to the G_o/q_c ratio.

5.3 Modulus degradation curve to predict settlement of shallow foundations

The applicability of the suggested methods was assessed to predict in-situ plate load tests carried out at the studied site. The in-situ plate load was performed at 1.0 m depth by Agnelli (1997).

Considering the average seismic test data performed up to three meters depth, a G_o value equal to 90 MPa was determined. In addition, triaxial test data from undisturbed soil samples collected at 1.5 meter depth from Fagundes (2014) were also used.

The stress-strain curves were determined from the triaxial tests and the friction angle was assumed equal to 30º. They were used to calculate the bearing capacity by Vésic (1975) and to apply the Elhakim (2005) approach. These data allowed calculating the x_L equal to 202 and defining $g^* = 0.015$.

Figure 11 shows the plate load test data performed by Agnelli (1997) and the load-settlement curves predicted by assuming $f = 1$ and $g = 0.3$ and $f = 0.99$ and $g^* = 0.015$. It can be seen in this figure that in both cases the predicted settlements were smaller than those from the plate load test because a lower degradation was assumed for the soil from the studied site. So, this approach has limitation to predict foundation settlements in unusual soils.

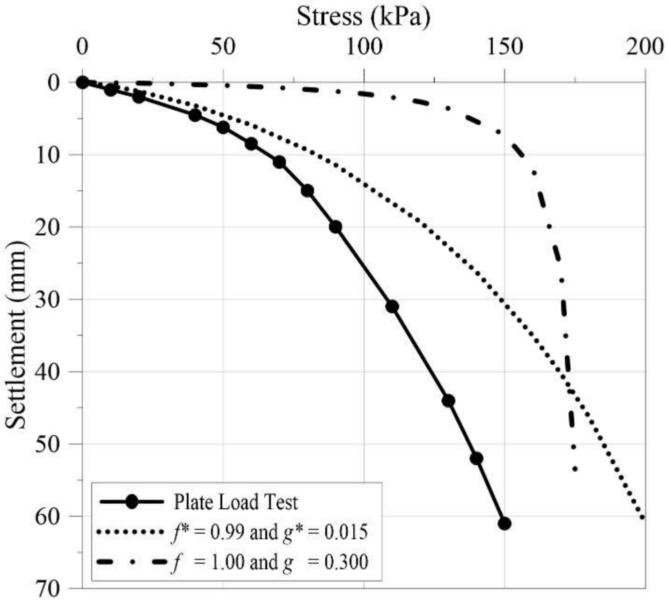


Figure 11. Plate load test and its prediction for the studied site.

5 CONCLUSIONS

The maximum shear modulus (G_o) is a very important geotechnical parameter for dynamic problems. There are several different seismic tests (cross-hole, down-hole, SCPT and SDMT) which allow determining G_o.

Vs and G_o profiles determined by different techniques presented reasonable agreement in the studied site. So, any of these seismic techniques can be used to determine G_o profiles.

G_o/qc and G_o/E_D ratios can be used to identify unusual soil behavior. The unsaturated condition and the cemented structures of the tropical soils produces G_o/qc and G_o/E_D that are higher than those obtained for sedimentary soils.

The load-settlement prediction based on the small-strain stiffness of the soil and its degradation were very different for the studied site.

The seismic tests are excellent to identify unusual soil behavior, but the G_0 and its degradation has limitation to predict foundation settlements in unusual soil.

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