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Comparative analysis of P2PSand and NorSand models for static liquefaction assessment in tailings dams

Análisis comparativo de los modelos P2PSand y NorSand para la evaluación de licuación estática en presas de relaves.

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ABSTRACT: Tailings dams play a critical role in mining operations by serving as containment facilities for waste materials. Ensuring the stability of these structures is essential to prevent catastrophic failures. This study discusses the application of two constitutive models based on critical state soil mechanics available in FLAC3D, P2PSand and NorSand, for assessing static liquefaction susceptibility in a tailings dam following upstream construction.

The research conducts a comparative analysis in terms of parameters needed, calibration process and results, and provides a valuable insight into the performance of these models highlighting their applicability and effectiveness in predicting static liquefaction susceptibility. Understanding their strengths and limitations contributes to informed decision-making in tailings dam design and risk management.

KEYWORDS: static liquefaction, tailings dam, constitutive model, NorSand, P2PSand

1 INTRODUCTION.

Tailings dams are essential structures in mining operations, designed to contain waste materials securely. Ensuring the stability of these dams is crucial to prevent devastating failures. The Brumadinho tailings dam collapse in Brazil in 2019 highlighted the critical need for thorough assessment and continuous monitoring of tailings dam stability.

Numerical simulations are becoming increasingly popular among practicing engineers to evaluate soil liquefaction and its destructive effects on the build environment. However, the selection of soil constitutive models and their calibration remain challenging even at element level conditions.

Advanced constitutive models are increasingly used in engineering practice to numerically simulate the behavior of geotechnical structures on liquefiable soils. Examples include Sanisand by Dafalias & Manzari (2004); UBCSand by Beaty & Byrne (2011), PM4Sand by Boulanger et al. (2015), among others.

This study presents the application of two constitutive models based on Critical State Soil Mechanics, namely NorSand (Jefferies, 1993) and P2PSand (Cheng and Detournay, 2021) both available in FLAC3D (Itasca, 2019), for evaluating the susceptibility for static liquefaction in a tailings dam built using the upward construction method. The advantage of using CSSM is that they require as input parameters the critical state friction angle and the initial relative density (or state parameter). The formulation allows for a unique set of parameters to capture the contractive and dilative behavior of a material and, unlike simpler and more widely used models such as Mohr-Coulomb, CSSM do not need to be re-calibrated every time density or confining stress changes. The study examines the parameters needed, the calibration process, and the results, providing valuable insights into the performance of these models. The findings underscore their applicability and effectiveness in predicting static liquefaction occurrences, thereby aiding in informed decision-making for tailings dam design and risk management.

2 CONSTITUTIVE MODELS.

2.1 *P2PSand constitutive model*

P2PSand is a practice-oriented, two-surface plastic constitutive model, recent-developed,by (Cheng and Detournay, 2021). This model was developed to model sands for earthquake engineering applications such as the parameters are calibrated based on in-situ test data (e.g., relative density, standard penetration test results) within a reasonable calibration effort. This model is based on an existing critical state model developed by Dafalias and Popov (1975, 1976); therefore, it preserves the characteristic that a same set of parameters should capture soil behavior for different densities and confining stresses. The model retains the general three-dimensional formulation used in SANISAND and uses the Lode angle for the effect of the second principal stress. The model uses default parameters that are compatible to the cyclic resistance chart in the semi-empirical procedure but also allows for the user to assign customized parameters. P2PSand mainly requires ten material properties. Three of them are related to elasticity properties, four parameters depend upon the critical state, and the rest are the minimum and maximum void ratios and the reference pressure (usually taken as 100 kPa). The other parameters particular to the model can be calibrated against monotonic and cyclic tests or can be taken as the default values based on the relative density.

The elastic law is based on the formulation proposed by Dafalias and Manzari (2004). Therefore, the shear modulus is based on the current mean effective stress (p) and modified in terms of relative density as indicated in Equation 1.

$$G = G_r P_{atm} \left(\frac{p}{P_{atm}}\right)^{0.5} = g_0 (D_r + C_{Dr}) P_{atm} \left(\frac{p}{P_{atm}}\right)^{0.5} \quad (1)$$

where G_r is a density-dependent material parameter as a function of g_0 and C_{Dr}, with default values of 1240 and 0.01, respectively. A high correlation is observed for both laboratory tests and in-situ data for values of relative density, D_R, between 15 and 85%.

The critical state is related to a physical condition where the soil reaches a steady state of deformation while shearing continues. It is given by:

$$\eta = q/p = \eta_c = M(\theta) \quad (2)$$

Where q is the deviatoric stress, η is the stress ratio with c denoting the critical state condition, and θ is the Lode angle.

For the current study the three-parameters equation was used for P2PSand:

$$D_{Rc} = D_{Rc0} + \lambda_r (P/P_a)^{\xi} \quad (3)$$

where D_{Rc0} is the critical relative density at the reference pressure Pa, λ_r is the slope of the critical state line in the D_R – p space and ξ is a constant parameter usually equal to 0.7 for sands.

The Lode Angle dependency of P2PSand adopts a generalized function (Van Eekelen, 1980) as:

$$g(\theta, c) = \left[\frac{1+c^{1/z}+(1-c^{1/z})cos3\theta}{2}\right]^{z} \quad (4)$$

where, c is the ratio of the triaxial compression strength to the extension strength and z = -0.25.

The state parameter in P2PSand is defined in terms of the relative density and is defined is the distance between the current state and the D_R at the critical state.

The bounding surface is defined from Lashkari (2009) as follows:

$$M^b = gM^c(1 + n^b I_r) \quad (5)$$

where $M^c = 6sin\phi_{cs}/(3 - sin\phi_{cs})$ and n^b is a material parameter.

The dilatancy surface has the form of:

$$M^d = gM^c(1 - n^d I_r) \quad (6)$$

where n^d is 4 to 6 times n^b.

The π-plane is shown in Figure x.

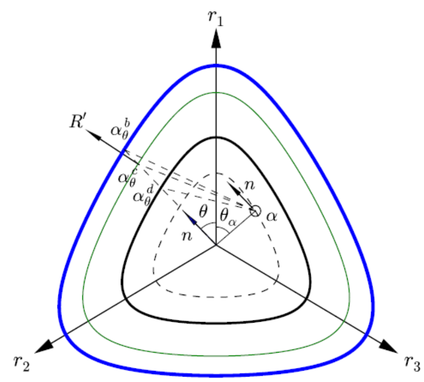

Figure 1. Schematic of surfaces of P2PSand in the π-plane sharing the same shape but with different sizes: bounding surface, critical state surface, dilatancy surface, and maximum back-stress ratio memory surface. The small circle on the maximum back-stress ratio surface is the yield surface (from Cheng and Detournay, 2021).

2.2 *NorSand constitutive model*

NorSand, first introduced by Jefferies (1993), is a generalized CSSM model for soil able to capture the softening and dilatant behavior of sands through the state parameter and by postulating an infinity of normal consolidation loci. It is a classic elasto-plastic model for soil, as such, and in common with other elasto-plastic models, it comprises four items: elasticity, a yield surface, a flow rule and a hardening law. The NorSand yield surface has a bullet-like shape with a limit that avoids having unrealistically large dilations of dense soils. The yield surface expands and contracts as required by the hardening law, depending on the current state parameter and the direction of loading. NorSand has evolved with time and the version used in this study corresponds to the one available in FLAC3D 7.0 (Itasca, 2019).

NorSand idealizes plastic work dissipation and hence, work conjugate stress and strain are needed. Accordingly, the shear strain increment is written as:

$$\dot{\varepsilon}_q = \frac{1}{3}\left[\begin{matrix}(sin\,\theta + \sqrt{3}\,cos\,\theta)\dot{\varepsilon}_1 - 2\,sin\,\theta\,\dot{\varepsilon}_2 \\ +(sin\,\theta - \sqrt{3}\,cos\,\theta)\dot{\varepsilon}_3\end{matrix}\right] \quad (7)$$

where θ is the Lode angle (for triaxial compression and extension $\theta = 30^o$ and $\theta = -30^o$, respectively), and $\dot{\varepsilon}_i$ i = 1,2,3 are the principal strain increments.

NorSand requires eight soil properties, of which 7 are obtainable from conventional triaxial compression tests and 1 is assumed (Poisson's ratio). The elastic shear modulus is ideally measured using bender elements and Poisson's ratio is usually assumed to be in a range between 0.1 and 0.3.

There are three parameters relating to the plastic behaviour: χ_{tc}, N and H. The property χ_{tc} is a constant for each sand and is defined as the slope of the maximum dilatancy (D_{min}) to the initial state parameter taking triaxial compression as a reference and scales the maximum dilatancy to the state parameter. N can be obtained by plotting the maximum stress ratio versus D_{min} for dense sands. The plastic hardening parameter, H, is obtained by fitting the results obtained by the drained triaxial tests and iterating to obtain the best match.

The critical state parameter in the stress-strain curve is defined by the critical state friction angle, ϕ_{cs}, for P2PSand and the critical friction ratio M_{tc} (the subscript 'tc' denoting the reference triaxial compression condition) as shown in eq. x. NorSand takes triaxial compression as the reference condition and leaves the variation of M with proportion of the intermediate stress (or Lode angle, θ) to the constitutive model, as described in equation x.

$$M_{\text{tc}} = \frac{6\sin\phi_{\text{cs,tx}}}{3-\sin\phi_{\text{cs,tx}}} \tag{8}$$

$$M(\theta) = M_{tc} - \frac{M_{tc}^2}{3+M_{tc}} cos(3\,\theta/2 + \pi/4) \tag{9}$$

For NorSand, the critical state line is defined as:

$$e_c = \Gamma - \lambda(P/P_a)^{\xi} \tag{10}$$

where Γ is the critical void ratio at the reference pressure Pa, λ is the slope of the critical state line in the e – p space and ξ is a constant parameter usually equal to 0.7 for sands.

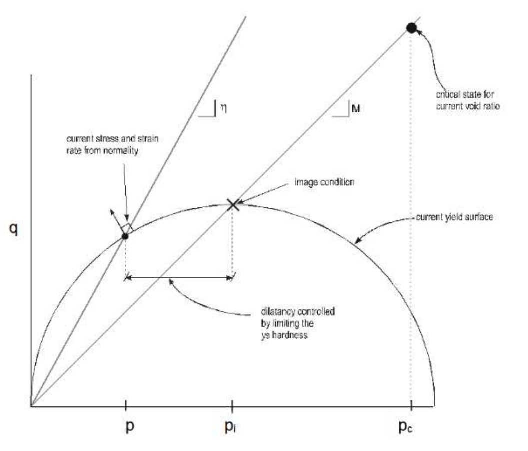

Figure 2. NorSand image condition on yield surface (ys) (from Jefferies and Shuttle, 2002)

Both models require the calibration of static and dynamic parameters however, as this study focuses on static liquefaction, only monotonic triaxial tests were used, and the parameters related to the undrained cyclic behavior of the material were considered as default values.

3 NUMERICAL MODEL.

3.1 *Model geometry*

The tailings dam under study was built using the upward construction method and consists of a 70 m high starter dam and a raise placed on top of a layer of tailings. Both the starter dam and the first raise were built with coarse material. The tailings were deposited in six different layers as shown in Figure 1. The mesh and pore water pressure distribution (in Pa) is shown in Figure 2.

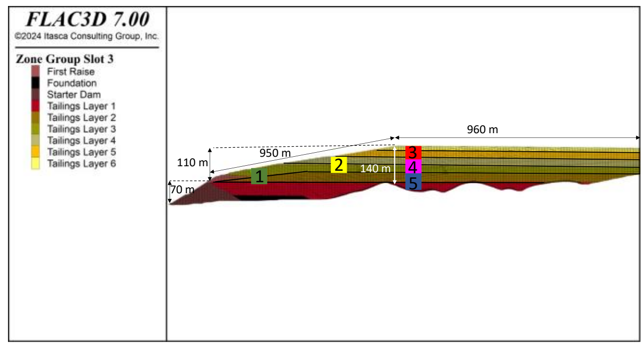

Figure 3. Dam model geometry in FLAC3D

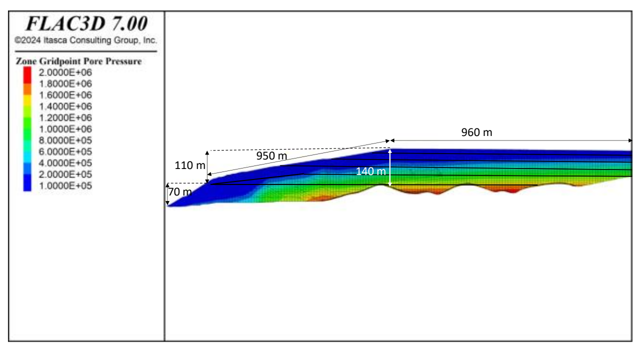

Figure 4. Pore water pressure distribution (in Pa)

3.2 *Constitutive models calibration*

The geotechnical characterization of the tailings was performed by means of undrained triaxial compression tests at effective confining stress of 100, 200 and 300 kPa. These tests were used for the calibration of the P2PSand and NorSand constitutive models. The critical state line in the DR -p' space is shown in Figure 5, and the stress-strain and stress path are shown in Figure 6 for the dilative samples. Figure 7 shows numerical simulations of contractive samples. While both models compare well for the dilative behavior and are able to capture the principal characteristics of the undrained behavior of the tailings in terms of strength and pore water pressure, the contractive behavior shows important differences in terms of pore water pressure development. Tailings parameters obtained from the calibration against dilative samples are shown in Table 1.

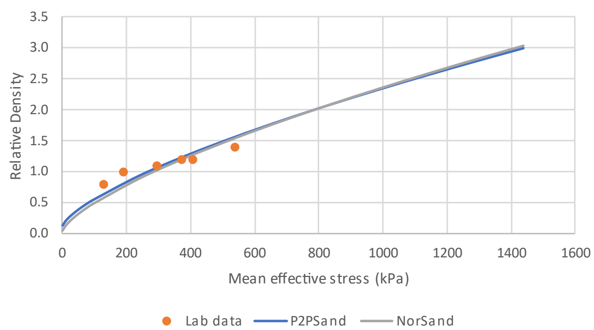

Figure 5. Critical state line calibration

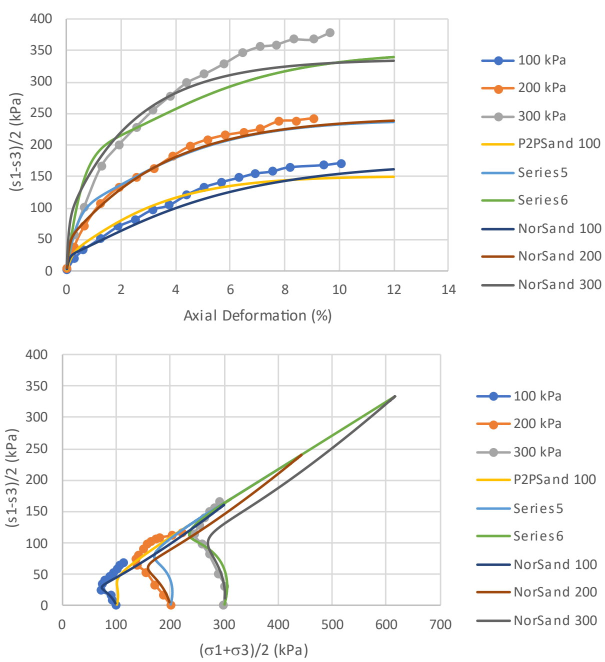

Figure 6. Triaxial calibration to NorSand and P2PSand for dilative samples for confining pressures of 100, 200 and 300 kPa

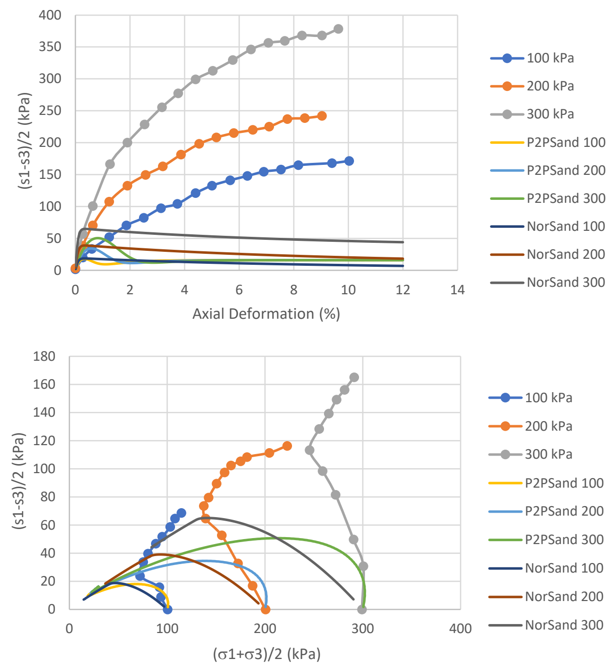

Figure 7. Triaxial simulations for NorSand and P2PSand for contractive samples for confining pressures of 100, 200 and 300 kPa

Table 1. Input parameters for NorSand and P2PSand for tailings (parameters not shown are set as default values)

Input parameters	*P2PSand*	*NorSand*
Pressure Reference (Pa)	101400	101400
Critical state friction angle (º) – Critical stress ratio	33	1.33
Poisson's ratio	0.32	0.32
Maximum void ratio	0.99	0.99
Minimum void ratio	0.42	0.42
Critical state parameter 1	0.115	0.92
Critical state parameter 2	0.5	0.098
Critical state parameter 3	0.7	0.55
Volumetric coupling factor	-	0.3
Plastic hardening modulus - 0	-	100
Dilatancy factor	-	4.5
Rate plastic shear	2.0	-
Dilatancy ratio minimum	0.8	-

Properties for the coarse material of the starter dam and the first raise are a friction angle of 40°, Poisson's ratio of 0.26 and Young Modulus of 2500 MPa, and it was modelled as a Mohr-Coulomb material.

To have a proper distribution of stresses in the tailings, the model was constructed in 9 stages under drained conditions, starting with the Foundation, the Starting Dam, Layers 1 and 2, First Raise, and Layers 3 to 6. At the beginning of each stage, vertical stresses in the materials deposited are initialized for gravity and density. Horizontal stresses are initialized following the recommendations of Jaky (1944). Boundary conditions at the bottom are fixed in both directions as to simulate a competent bedrock. To consider the densification of the tailings with depth, an initial relative density of 82% was used for Layer 1, 54% for Layers 2 and 3, and 35% for Layer 4 to 6, for the P2PSand constitutive model, corresponding to an initial state parameter of 0.1, 0.14, and 0.1 for the NorSand model. Static liquefaction was conducted by triggering undrained conditions at the end of construction by applying a water bulk modulus of 2Gpa and running the model for 10.000 steps. In this study the phreatic surface represents a design condition, therefore it was not part of the scope performing a flow analysis.

3 RESULTS.

The contours of total displacements along with displacement vectors are shown in Figure 8. Displacements go from the top of the tailings towards the starter dam. Figure 9 shows vertical displacements contours. In both cases, vertical displacements decrease with depth. For P2PSand larger downward vertical displacements are observed on top of the tailings (around 0.2 m) and upward vertical displacements close to the starter dam of 0.1 m. The largest downward vertical displacements for NorSand are observed at the middle of the downstream slope of the tailings (0.18 m approximately). Point histories of vertical displacements (see Figure 1) are shown in Figure 10. Vertical displacements on top of the tailings are very similar (red line - around 0.1 cm) for both models, and larger displacements re observed for the NorSand constitutive model in the middle, bottom and downstream slope of the tailings. Larger displacements are observed close to the starter dam for the P2PSand model.

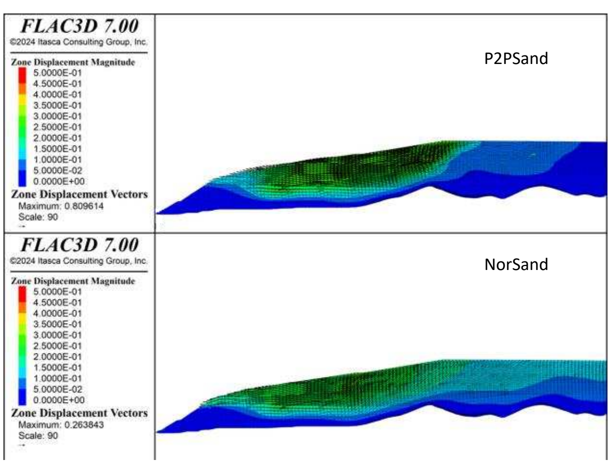

Figure 8. Total displacements contours and displacements vectors

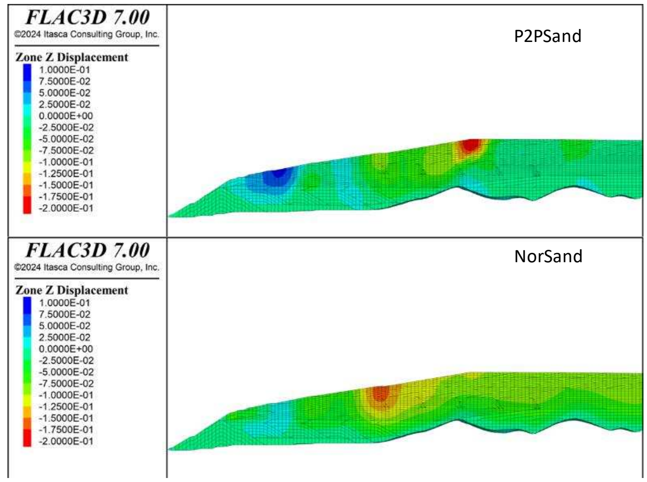

Figure 9. Vertical displacements contours

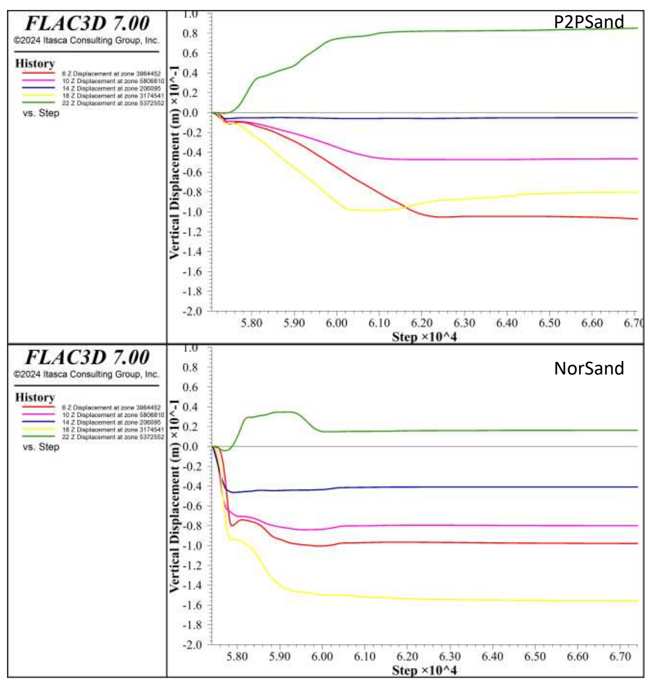

Figure 10. History points vertical displacements

Horizontal displacement contours are shown in Figure 11. Similar contours are observed, however P2PSand results in slightly larger values. History points of horizontal displacement (Figure 12) show that for P2PSand displacements on top of the tailings and close to the starter dam are very similar (around 0.18 m) and the largest displacement occur in the middle of the downstream slope of the tailings (approximately 0.25 m). For NorSand, the horizontal displacements on top and middle of the tailings are very similar (around 0.12 cm) and the largest displacement (close to 0.2 m) are obtained in the middle of the downstream slope of the tailings and close to the starter dam.

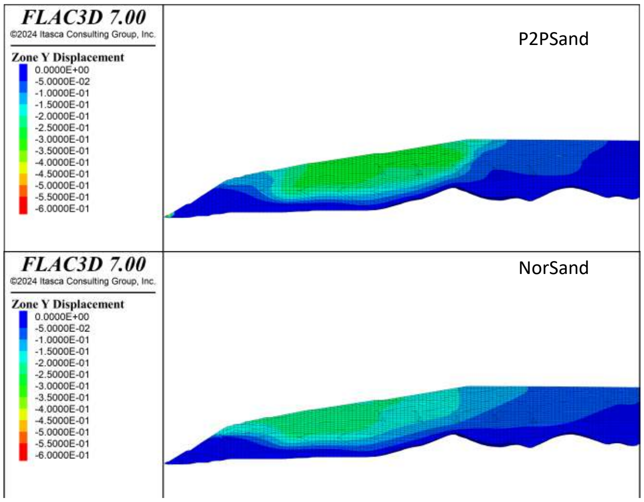

Figure 11. Horizontal displacements contours

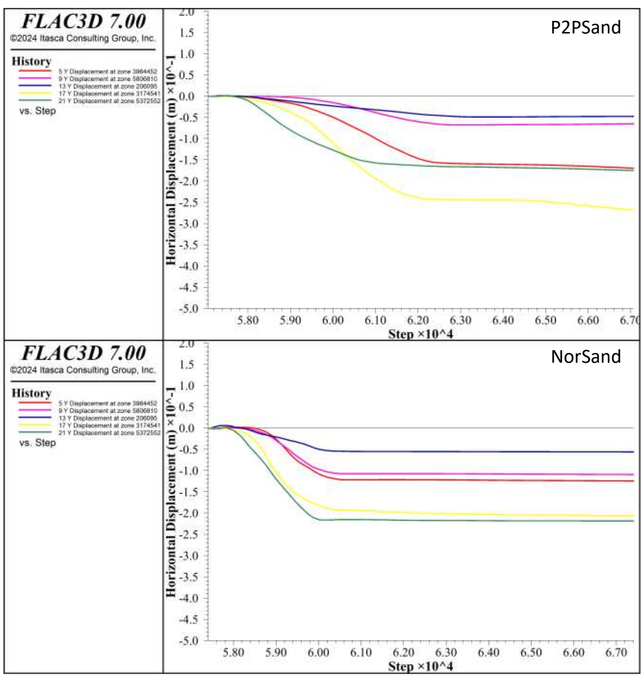

Figure 12. History points horizontal displacements

4 CONCLUSIONS

This study presents the comparison of two constitutive models based on Critical State Soil Mechanics, namely P2PSand and NorSand for assessing static liquefaction in a tailings dam.

For the calibration process, only triaxial monotonic tests were considered and the parameters related to the cyclic behavior of the material were considered as default values. For the P2PSand model, the parameters needed as inputs are material properties such as friction angle at the critical state, critical state parameters in the D_R-p space, elastic parameters, and extreme void ratios. It is important to notice that default parameters for P2PSand are based on in-situ testing and therefore, might not be representative of the triaxial tests used for the calibration process however, they should have little influence in the results since they describe the cyclic behavior of the material. For the NorSand model, 4 additional parameters are needed to represent the monotonic behavior rendering the calibration process more demanding.

In terms of displacement, both models result in general with the same order of magnitude; less than 20 cm for vertical displacements and less than 30 cm for horizontal displacements.

Vertical displacements show a more circular type of surface with P2PSand with its maximum downward value at the top of the tailings and the maximum upward value close to the starter dam. For NorSand, vertical displacements are more local and the maximum downward value is in the middle of the downstream slope of the tailings.

Horizontal displacements for P2PSand also show a more circular type of surface with maximum values at the top of the tailings, whereas maximum values for NorSand start in the middle of the downstream slope of the tailings.

Based on the calibration process and the response of the deposit, the P2PSand constitutive model appears to be a good candidate to be used in a static liquefaction analysis.

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