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First insights for a simplified state parameter-based model for cyclic liquefaction triggering

Primeros acercamientos para un modelo simplificado basado en parámetro de estado para el desencadenamiento de licuación cíclica

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ABSTRACT: Despite the constant emergence of new case histories of liquefaction from recent earthquakes, the incorporation of these data has not led to substantial changes in semi-empirical methodologies, generating considerable uncertainties of their performance on underrepresented data. Accordingly, the aim of this study is to explore liquefaction models that are less empirically derived. To accomplish this, correlations derived from calibration chambers and cyclic tests from the literature were re-visited to develop a new triggering model based on the CPT-based state parameter approach. When using a purely state-parameter-based approach, the cyclic resistances ($CRR_{7.5}$) in dense sands appear to be considerably lower than those predicted by most state-of-practice models. To validate the resulting model, empirical databases were used to compare its prediction performance, showing important improvements over most of state-of-practice models. These results highlight that accounting for the possible mixed triggering and manifestation effects on back-analyzing case histories databases is essential to improve the prediction performances of a simplified state parameter approach.

KEYWORDS: state parameter, critical state soil mechanics, liquefaction triggering, liquefaction case histories.

1 INTRODUCTION

Soil liquefaction is a major area of interest in geotechnical engineering due to its known damage potential and the substantial economic impact that liquefaction mitigation can have on large-scale projects. Previous research regarding liquefaction has established that the primary factors influencing the triggering of liquefaction in sand samples are initial confining stresses, the intensity of shaking, and soil state (Seed and Idriss, 1971; Ishihara, 1993). Also, several methods of liquefaction triggering have been developed, primarily based on the "simplified procedure" proposed by Seed and Idriss (1971) which has a semi-empirical basis using liquefaction case histories from sites with free field conditions subjected to past earthquakes (Cetin et al., 2004; Idriss and Boulanger, 2006; Youd et al., 2001), where surface manifestations were documented.

While several semi-empirical liquefaction triggering methodologies are currently available in the state-of-the-art, the evidence that backs up these methodologies is limited in the range of the available data. Since these data is not always representative of all local conditions, significant efforts have been made to overcome the problem of a lack of case histories to improve state-of-practice methodologies (Geyin et al., 2021; Montalva et al., 2022; Zimmaro et al., 2019).

However, even with databases containing a comparable number of case histories to Boulanger and Idriss (2014), the differences between model performances could be too low to ensure statistically significant improvements (Geyin et al., 2020), implying that it may take several decades before having a sufficiently heterogeneous database to accurately represent the liquefaction phenomena.

In this context, the state parameter proposed by Been and Jefferies (1986) is a parameter that accounts for the proximity to the critical state line of a soil, combining the influence of density and effective stress following the critical state soil mechanics theory. As "screening" procedures have been proposed to estimate the in-situ state parameter with similar input data to most simplified CPT-based liquefaction triggering models, we evaluate the use of state parameter as the state index for three simplified semi-empirical models, in lieu of the fines corrected cone tip resistance (q_{c1Ncs}).

The objective is to provide a comparison between semi empirical procedures (Moss et al., 2006; Boulanger and Idriss, 2014; Green et al., 2019) and state parameter approaches (Plewes et al., 1992; Been and Jefferies, 1992) in accounting for the cyclic resistance ratio. Furthermore, in order to minimize the empirical adjustments and subjectivities in the resulting state parameter-based model, we propose a new state parameter-based cyclic resistance ratio correlation, based on published laboratory cyclic tests. Finally, the model is tested against case histories databases to provide insights regarding the empiricism on semi-empirical state-of-practice procedures.

2 A SIMPLIFIED PROCEDURE FOR STATE PARAMETER APPROACH

2.1. *Simplified procedure by Seed and Idriss*

Current CPT-based liquefaction triggering procedures consist of calculating factors of safety or the probability of liquefaction using a simplified stress-based approach, originally developed by Seed and Idriss (1971). In this approach, factors of safety against liquefaction can be computed using Eq. 1, where $CSR_{7.5}$ denotes the cyclic stress ratio and is used to quantify the cyclic loading scaled to a 7.5 M_w earthquake, and $CRR_{7.5}$ is used for cyclic resistance ratio and represents the cyclic stress ratio of the soil at which liquefaction occurs after 15 uniform cycles or the equivalent of a 7.5 M_w earthquake.

$$FS = \frac{CRR_{7.5}}{CSR_{7.5}} \tag{1}$$

According to Seed and Idriss (1971), the cyclic stress ratio can be estimated by Eq. 2 following a rigid body behavior of the soil column. In this equation, a_{max} denotes maximum ground surface acceleration, σ_{vo} and σ'_{vo} are total and effective initial vertical stresses, respectively, r_d is a stress reduction coefficient that accounts for the deformability of the soil profile, and MSF a magnitude scaling factor that converts the cyclic loadings of different durations into a M_w 7.5 equivalent (or 15 cycles) cyclic stress ratio.

$$CSR_{7.5} = 0.65 * \left(\frac{a_{max}}{g}\right) * \left(\frac{\sigma_{vo}}{\sigma'_{vo}}\right) * \left(\frac{r_d}{MSF}\right) \tag{2}$$

Currently, there are several proposals to compute the stress reduction coefficient r_d (e.g. Cetin and Seed, 2004; Idriss, 1999; Lasley et al., 2016; Liao and Whitman, 1986) and magnitude scaling factor MSF (e.g. Boulanger and Idriss, 2015; Cetin and Bilge, 2012; Moss et al., 2006), that have been proposed to adjust the semi-empirical approach to liquefaction case history databases.

However, the stress reduction coefficient equations are commonly developed independently of case histories databases and performing comprehensive site response studies. For this study, three semi-empirical methods were used to compute $CSR_{7.5}$ from Eq. 1: Moss et al. (2006), Boulanger and Idriss (2014) and Green et al. (2019).

2.2. *State parameter as a CPT-based state index*

In the state-of-practice CPT-based liquefaction models, cyclic resistance is computed using a clean sand equivalent normalized tip resistance (q_{c1Ncs}) through well-known factors of overburden stress (C_N) and "fines content" correction. The "fines content" correction aims to incorporate the factors that have demonstrated to affect liquefiability of soils, such as fines content, soil plasticity, mineralogy, sensitivity, and stress history (Robertson and Wride, 1998).

The "clean sand equivalent" cone tip resistance has turned into a reference and has been used on most CPT-based models. To date, different parameters have been used to quantify the "fines" correction factor in CPT-based procedures, including direct fines content measurements (Boulanger and Idriss 2014), and derived parameters as friction ratio (Moss et al., 2006) and soil behavior index (Juang et al., 2008; Robertson and Wride, 1998; Suzuki et al., 1997).

Alternatively, Jefferies and Been (2015) proposed the use of the state parameter as a state index for cyclic liquefaction evaluation. Accordingly, and similarly to NCEER (Youd et al., 2001), Jefferies and Been (2015) proposed an empirical equation (Eq. 3) for cyclic resistance ratio in terms of the state parameter estimated from CPT estimations using Plewes et al. (1992) method, after analyzing the case histories from Moss et al. (2003) liquefaction database.

$$CRR_{7.5} = 0.06 * \exp(-9\psi) \tag{3}$$

Regarding the state parameter estimations, a general inversion procedure was proposed by Been et al. (1986, 1987) to predict the in-situ state parameter from CPT, using data from six sands on large calibration chamber tests. Based on this procedure, methods from "screening" to advance constitutive modeling have been developed. The general inversion procedure is shown in Eq. 4, which involves coefficients k and m, where k is the normalized q_c value at ψ=0 and m is the slope of the normalized q_c-ψ relationship.

$$q^* = Q_p\left(1 - B_q\right) = k^* * \exp(-m * \psi) \tag{4}$$

Since the inversion coefficients are soil specific and require additional laboratory and field-testing data to be determined for each stratum (Ghafghazi and Shuttle, 2008; Shuttle and Jefferies, 1998), Plewes et al. (1992) and Been and Jefferies (1992) used Eq. 4 and Eq. 5 to correlate the inversion coefficients and the critical state soil parameters M_{tc} and λ_{10}.

$$\frac{k^*}{M_{tc}} = 3 + \frac{0.85}{\lambda_{10}} \tag{5}$$

$$m = 11.9 - 13.3 * \lambda_{10} \tag{6}$$

In addition, for estimating λ_{10} parameter directly from CPTu data, Been and Jefferies (1992) and Plewes et al. (1992) proposed the Eq. 7 and Eq. 8, respectively. Certainly, an advantage of the state parameter approach is that more advanced methods can be used in presence of more data, such as triaxial testing and field seismic investigations, however, the use of these correlations is primarily based on its simplicity and in their capabilities to represent overall trends for soil compressibility (e.g. Reid, 2015).

$$\lambda_{10} = \frac{1}{34 - 10\, I_{c,BJ}} \tag{7}$$

$$\lambda_{10} = \frac{F_r}{10} \tag{8}$$

Therefore, the cyclic resistance ratio in a simplified state parameter approach can be defined by the parameters M_{tc} and K_0, along with common CPT-derived data such as Q_p, B_q and F_r or $I_{c,BJ}$. Q_p is analogous to Q_t (Robertson and Wride, 1998), but using a three-dimensional stress normalization. B_q is the normalized excess pore water pressure, which uses the dynamic pore water pressure measured behind the shoulder of the cone (u_2). F_r is the friction ratio expressed as a percentage, and $I_{c,BJ}$ is the soil behavior type index as defined in Jefferies and Been (1992) and a function of Q_p, F_r and B_q.

As both Eq. 4 and Eq. 7 includes the expressions $Q_t(1\text{-}B_q)$ or $Q_p(1\text{-}B_q)$ on its formulation (Eq. 7 has it on $I_{c,BJ}$ equation), it is expected that the model will be less reliable in soft soils (soils with low Q_t and high B_q) because these expressions may take very small values, leading to inaccuracies when using standard commercial cones (i.e. subtraction-type).

2.3. *Semi-empirical vs. a laboratory based CRR*

To derive Eq. 3, several assumptions must be considered: (1) a unique value of cyclic resistance exists for each value of state parameter predicted, and (2) a liquefaction triggering can be adequately developed based on case histories classified by its surface manifestation. While it is known that the first assumption is inaccurate and the correlation between the state parameter and its cyclic resistance is soil specific (Jefferies and Been, 2015), it is of practical importance to consider it unique in absence of further soil specific data, analogously to most semi-empirical procedures (Boulanger and Idriss, 2014; Youd et al., 2001).

However, the second assumption has recently been addressed by Upadhyaya et al. (2022), who observed that the existing triggering curves of semi-empirical models could be "combined triggering and manifestation curves" due to the direct use of the classification scheme in published liquefaction databases.

Accordingly, the use of only specific "critical layers" to represent liquefaction triggering of the stratum based on its manifestation may cause bias on the resulting field cyclic resistance curves, by excluding the possibility of cyclic liquefaction to occur on medium to dense sands. Similar suggestions have been provided Dobry (1989), emphasizing that current triggering models may be failing to predict the triggering of cyclic liquefaction at greater densities.

Therefore, to avoid introducing bias on a mechanistic approach, an alternative cyclic resistance model was introduced independently of case histories data. With this aim, cyclic triaxial tests of 13 different sands based on the compilation of Jefferies and Been (2015) and additional data from TP Lisbon sand (Viana da Fonseca et al., 2023), Ticino sand (Fioravante and Giretti, 2016), Fraser River sand (Ghafghazi and Shuttle, 2010; Thomas, 1992), and West Kowloon sand (Shen and Lee, 1995) were used as a reference to constrain a representative cyclic resistance ratio curve ($CRR_{7.5}$), as shown in Figure 1.

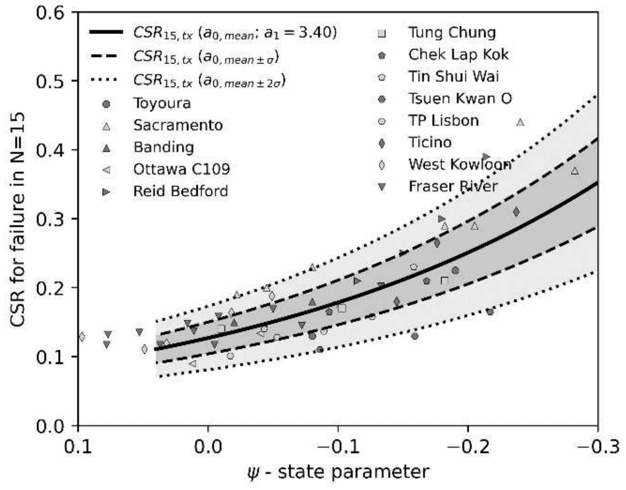


Figure 1. Cyclic stress ratio for failure at 15 cycles for 13 sands from cyclic triaxial tests.

The trend was obtained after minimizing the root mean square error (RMSE) for cyclic tests data in the functional form of Eq. 9, adopting a unique value for a_0 and soil-specific values for a_1. The equation corresponding to the mean value for a_0 and best fit for a_1 is shown in Eq. 10. The subscript for $CRR_{15,tx}$ correspond to the 15 loading cycles to trigger liquefaction in triaxial conditions.

$$CRR = a_0 * \exp\left(-a_1 * \psi\right) \tag{9}$$

$$CRR_{15,tx} = 0.127 * \exp\left(-3.40 * \psi\right) \tag{10}$$

As the above equations were developed from cyclic stress ratios measures in cyclic triaxial tests, a correction is applied to convert it into a cyclic stress ratio for simple shear conditions ($CRR_{15,ss}$) using Eq. 11 (Ishihara et al., 1977), more closely related to field conditions. Therefore, assuming K_0 equal to 0.60, the resulting cyclic resistance ratio model usable for field conditions is shown in Eq. 12. The proposed model along with the $CRR_{7.5}$ proposed by Jefferies and Been (2015) (Equation 3) is shown in Figure 2.

$$CRR_{15,ss} = \left(\frac{1+2K_0}{3}\right) * CRR_{15,tx} \tag{11}$$

$$CRR_{15,ss} = 0.093 * \exp\left(-3.40 * \psi\right) \tag{12}$$

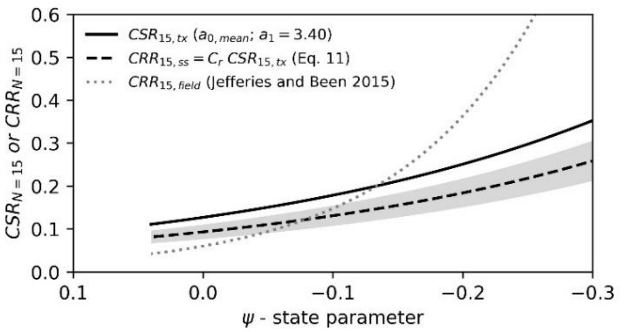


Figure 2. Proposed model for cyclic resistance for field conditions derived from laboratory tests.

Note that, as only fully saturated specimens were tested from the 13 sands of Figure 1, the generalized cyclic resistance ratio curve will only suffice for predicting liquefaction triggering on saturated sands. However, each sand on unsaturated or partially saturated conditions may have a whole different cyclic resistance curve, which cannot be assessed through the proposed model. Despite this, stablishing a state parameter approach for cyclic liquefaction may be a good starting point to include the complexities of CPT interpretations and liquefaction resistances of partially saturated soils.

3 COMPARISONS WITH EXISTING MODELS

3.1. *State parameter-based vs. state-of-practice models*

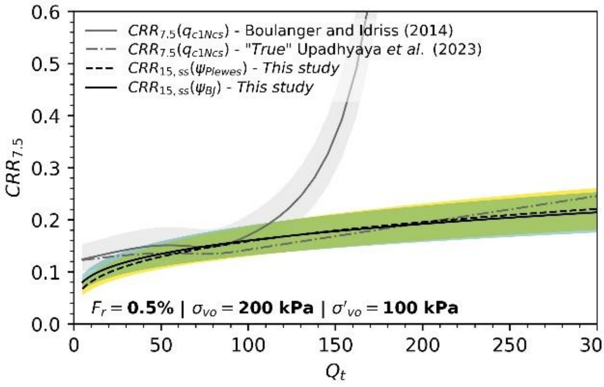
To provide further insight into the use of the proposed state parameter approach, a parametric analysis was performed in terms of tip resistance Q_t and friction ratio F_r. For developing this, values of F_r equal to 0.5 and 2.0 were assumed for plotting Q_t against the computed cyclic resistance ratio ($CRR_{7.5}$) in Figures 2.(a) and 2.(b), respectively. Both figures highlight the differences between the proposed state parameter approach and the state-of-practice model of Boulanger and Idriss (2014) and the recently published "true" triggering curve from Upadhyaya et al. (2023).

The "true" triggering curve proposed by Upadhyaya et al. (2023) is the result of adopting a more rational and consistent framework to handle liquefaction triggering and manifestation effects on case histories databases. By using this more consistent framework, the "true" triggering curve is allowed to be better guided by fundamental liquefaction mechanics according to its authors (Upadhyaya et al., 2023). The equation for computing $CRR_{7.5}$ according to the "true" triggering curve is shown in Equation 13.

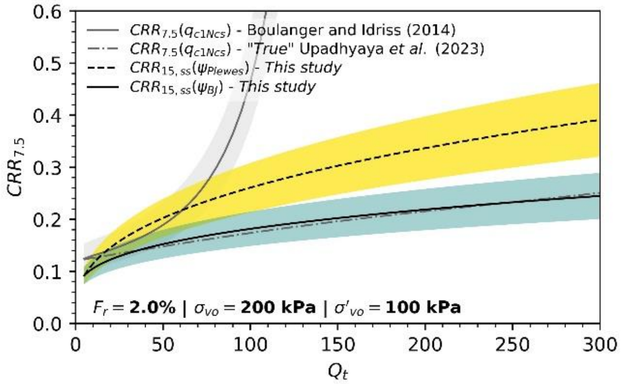
$$CRR = \exp\left[\ln\left(\frac{q_{c1Ncs}}{1919.2}+0.09\right)+0.243*\Phi^{-1}(P_L)\right] \qquad (13)$$

Since the "true" triggering curve was proposed in terms of q_{c1Ncs}, it is interesting to compare this "more mechanistic" perspective based on q_{c1Ncs} with the proposed state parameter approach. As shown in Figure 3.(a), for low values of friction ratios (F_r=0.5%), the cyclic resistance ratio ($CRR_{7.5}$) values resulting from the proposed state parameter approach are in close agreement with the "true" triggering curve, regardless of which method for predicting the state parameter from CPT data. Moreover, either the proposed state parameter approach or the "true" triggering curve shows an important decrease in cyclic resistance for greater values of Q_t.

Similarly, Figure 3.(b) shows the same models but now assuming a friction ratio of F_r=2.0%. In this scenario, the proposed state parameter-based model shown to be in close agreement with the "true" triggering curve only when the Been and Jefferies (1992) model was used to assess the state parameter.



a)



b)

Figure 3. Comparison between parametrical analyses of $CRR_{7.5}$ from triggering models by Boulanger and Idriss (2014), Upadhyaya et al. (2023), and the proposed ψ-based approach using BJ92 and PEA92 with: (a) F_r=0.5; (b) F_r=2.0.

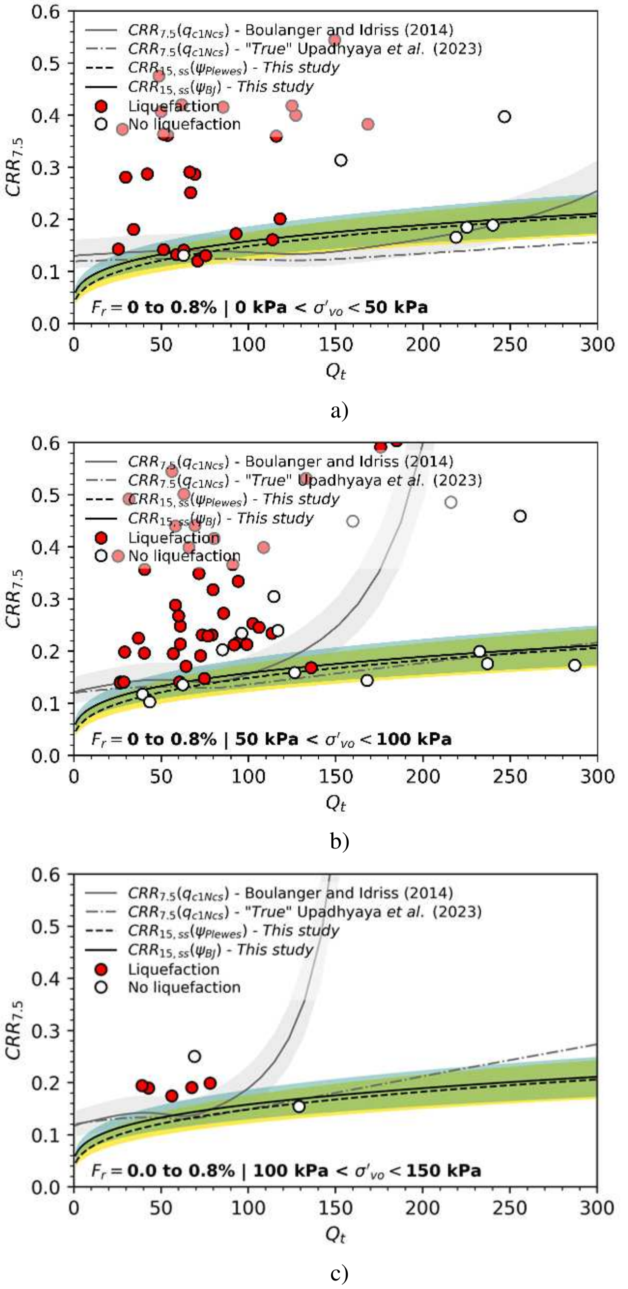
This first result is particularly important since it reflects the similarity between the results of two approaches better relied on mechanics (the state parameter approach and the "true" triggering curve). Moreover, a noticeable aspect of the proposed state parameter approach is that it was formed without using any case histories database. This aspect allows to use the case histories databases to test the prediction of the model avoiding the overfitting caused by using the same training and testing dataset, which can be present in many of the current state-of-practice models (Maurer and Sanger, 2023).

4 VALIDATION OF THE PROPOSED MODEL

While the differences between Boulanger and Idriss (2014) model and the proposed state parameter approach are remarkable for greater values of Q_t, its significance in how this impacts state-of-practice procedures is not trivial. Accordingly, it is interesting to know how this new cyclic resistance ratio equation modifies the prediction performances of models when tested against empirical data.

4.1. Case histories from Moss et al. (2003) database

The CPT case histories database compiled by Moss et al. (2003) was used to evaluate the modifications suggested by the state parameter approach. From the database, several selected critical layers of liquefiable soils were characterized by CPT parameters and then used as an input for liquefaction triggering model (Moss et al., 2006). To illustrate the differences between models in terms of case histories data, Figure 4 and 5 shows several bins of data divided by its friction ratio value (F_r) and its effective stress (σ').



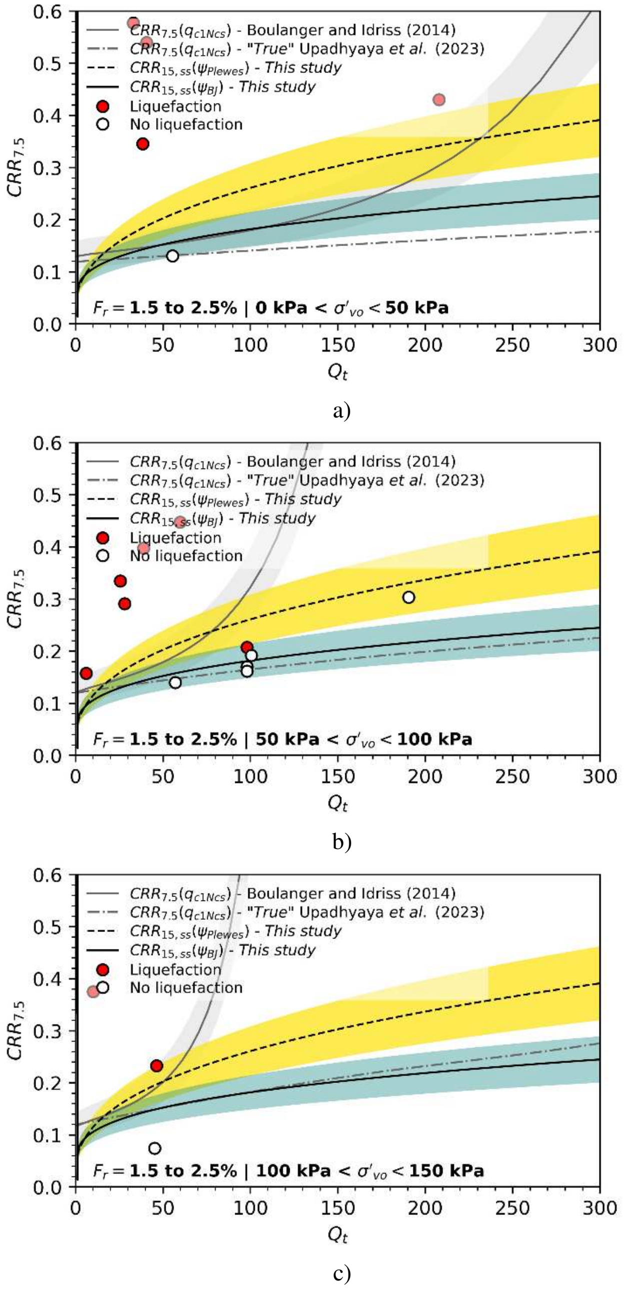
a)

b)

c)

Figure 4. Case histories data from Moss et al. (2003) database and 0.0%<F$_r$<0.8%, along with CRR$_{7.5}$ curves from Boulanger and Idriss (2014); Upadhyaya et al. (2023); and the proposed model binned for effective stresses: (a) 0<σ'$_{vo}$<50 kPa; (b) 50<σ'$_{vo}$<100 kPa; (c) 100<σ'$_{vo}$<150 kPa.

As the Figure 4 shows, all the "liquefaction" data lie above or in the range of the proposed models. Also, several "no liquefaction" data also groups in the range of the proposed model. While there is a significant amount of "no liquefaction" cases that lie above the curve, which according to current state-of-practice models would indicate an incorrect prediction, an alternative interpretation may be given based on Upadhyaya et al. (2023) and Dobry (1989).



a)

b)

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Figure 5. Case histories data from Moss et al. (2003) database and 1.5%<F$_r$<2.5%, along with CRR$_{7.5}$ curves from Boulanger and Idriss (2014); Upadhyaya et al. (2023); and the proposed model binned for effective stresses: (a) 0<σ'$_{vo}$<50 kPa; (b) 50<σ'$_{vo}$<100 kPa; (c) 100<σ'$_{vo}$<150 kPa.

Since the classification of data in empirical databases is guided primarily by its surficial manifestation, these cases could be considered as denser soils that may have experienced liquefaction (r_u=1) but with little to no consequences due to the dilative tendencies of the soil. A similar analysis is shown in Figure 5, which shows the same curves as Figure 4, but now for greater friction ratios (F_r).

The Figure 5 shows that for greater friction ratios (silty sands of case histories data) the proposed model shows a good agreement with the empirical databases. However, it is also observed that very few data points are present on each bin. This is conceivable since most of the knowledge of semi-empirical models comes from clean sands, and it is also known that greater friction ratios would tend to consider the soil non-liquefiable according to the $I_{c,RW}$ susceptibility criteria of Robertson and Wride (1998). As the main result extracted from Figures 4 and 5, it can be stated that while the proposed model shows significant differences at greater Qt values, this is not contrary to the current knowledge present in empirical databases.

4.2. *Canterbury Earthquake Database*

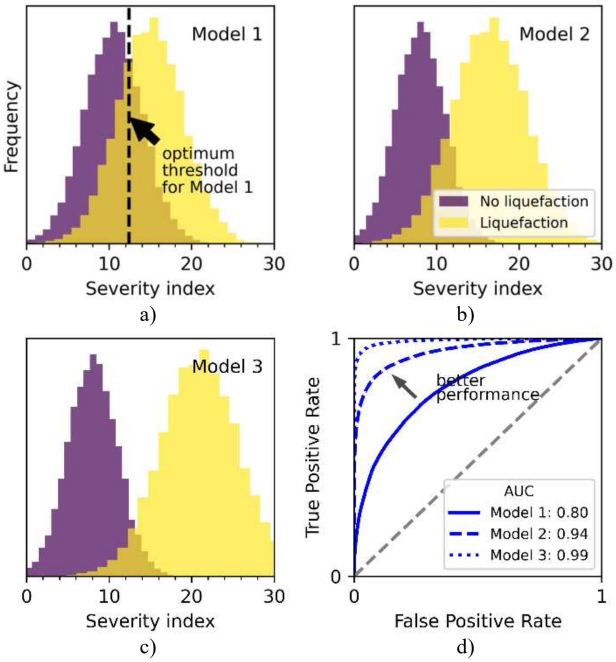
The Canterbury earthquake database (Geyin et al., 2021) is a comprehensive compilation of case histories of liquefaction, based on earthquake events of Christchurch 2010 M_w 7.1, Darfield 2011 6.2 M_w and Valentine's Day 2016 5.7 M_w. According to Geyin et al. (2021), most case histories of the database are originated from level-ground sites and are classified according to the severity of the manifestation.

This database, which contains 14788 CPT-based case histories, has the advantage that compiles entire CPT soundings rather than only critical layer values, allowing to perform more rigorous analysis by including the computation of manifestation models that are better suited to predict its surface manifestation. To evaluate the proposed model, the Canterbury dataset was filtered to consider only CPTu soundings that reached at least 10 meters of depth, resulting in a filtered dataset of 10435 case histories.

To quantify the predictive capabilities of the state parameter approach, the area-under-the-curve (AUC) from receiver-operating characteristic (ROC) curves was employed as the performance measure. ROC curves are constructed by evaluating classifications at various deterministic thresholds. These analyses have been very common in liquefaction problems, and several variables have been evaluated as potential thresholds to predict whether surface manifestation will or will not occur (e.g. Oommen et al., 2010; Schmidt and Moss, 2021). An example on how AUC operate on different models can be illustrated with the Figure 6.

An AUC of 1.0 means that the model can separate classes completely in an optimum threshold, while an AUC of 0.5 would result from a random binary classification. For this study, we used as variables two severity indices: the Ishihara-inspired liquefaction potential index (LPI_{ISH}) by Maurer et al. (2015) and Liquefaction Severity Number (LSN) by van Ballegooy et al. (2013). Also, to provide a comparison between the performance of the proposed model and other state-of-practice models, the performance of simplified CPT-based models of Moss et al. (2006) and Boulanger and Idriss (2014) were also evaluated in the same manner.

The evaluation in terms of AUC of the proposed cyclic resistance ratio along with the three state-of-practice models is shown in Figures 7, 8 and 9; corresponding to Moss et al. (2006), Boulanger and Idriss (2014), and Green et al. (2019) models, respectively.



a) b) c) d)

Figure 6. Performance evaluation of liquefaction triggering models with severity indices through AUC in the ROC space.

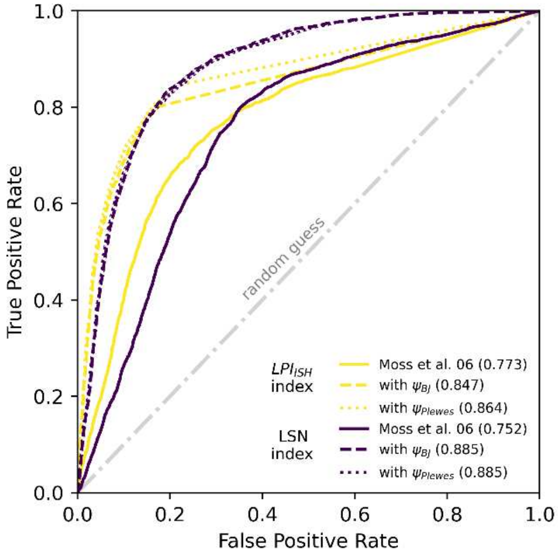


Figure 7. ROC curves for LPI_{ISH} and LSN manifestation models in Canterbury database with Moss et al. (2006) and ψ-approach by Plewes et al. (1992) and Been and Jefferies (1992).

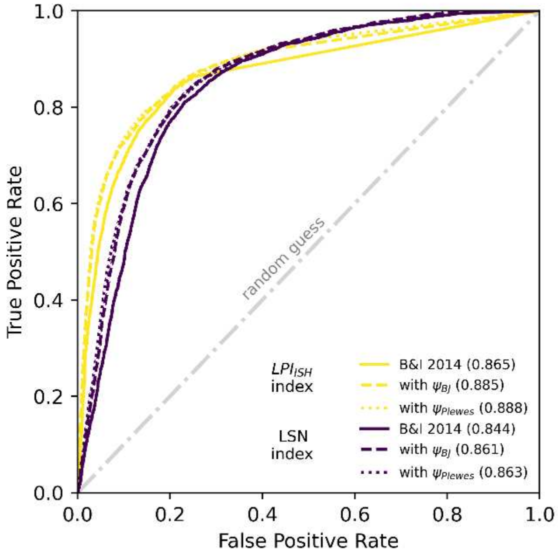


Figure 8. ROC curves for LPI_{ISH} and LSN manifestation models in Canterbury database with Boulanger and Idriss (2014) and ψ-approach by Plewes et al. (1992) and Been and Jefferies (1992).

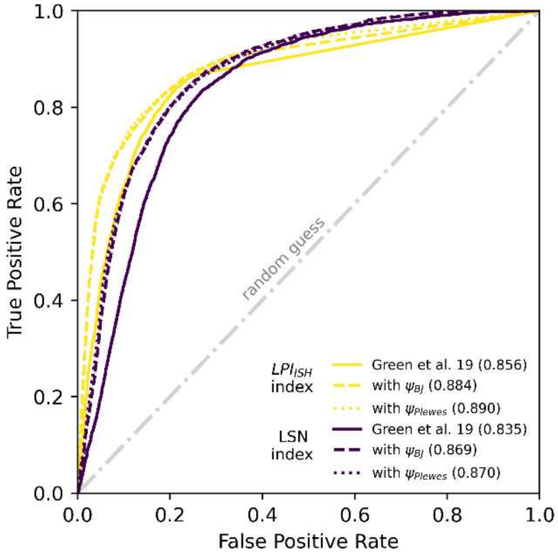


Figure 9. ROC curves for LPI_{ISH} and LSN manifestation models in Canterbury database with Green et al. (2019) and ψ-approach by Plewes et al. (1992) and Been and Jefferies (1992).

From the Figures 7, 8 and 9, it is shown that all state-of-practice models showed an improvement of AUC values on its state parameter version. This result was obtained regardless of which severity index was used, and regardless of which screening method was used to estimate the state parameter. In general, the model that showed to have the greatest improvements in terms of AUC was Moss et al. (2006), while the better overall predictive performances were obtained with the state parameter versions of LSN + Moss et al. (2006), LPI_{ISH} + Boulanger and Idriss (2014), and LPI_{ISH} + Green et al. (2019) models.

As the proposed model showed improvements on a comprehensive database of 10435 case histories from Canterbury Earthquake database, and the analysis of critical layers from previous empirical databases (Moss et al., 2003) showed little discrepancies against the proposed cyclic resistance ratio curve ($\text{CRR}_{7.5}$), we emphasize that the state parameter approach is a great alternative to assess cyclic liquefaction when proposed in a consistent manner with its fundamental mechanics rather than through empirical fitting.

5 CONCLUSIONS

The present study evaluated the use of a CPT-derived state parameter on liquefaction triggering models, in an attempt to develop a “less empirical” alternative for simplified semi-empirical models. With this aim, a generalized cyclic resistance ratio curve was proposed using cyclic triaxial tests data from 13 well characterized sands.

The proposed model was compared with other semi-empirical models, and the following results were observed:

(1) The proposed state parameter approach supports the idea that liquefaction triggering curves should include the triggering of denser sands, similarly as what the recently published “true” triggering curve by Upadhyaya et al. (2023) suggest.

(2) The use of the CPT-based state parameter approach instead of the normalized fines corrected cone tip resistance (q_{c1Ncs}) as a state index in state-of-practice models, showed to significantly improve the predictive performances of the original models when tested against the comprehensive Canterbury Earthquake database.

(3) Accordingly, the use of a mechanistic perspective based on critical state soil mechanics instead of semi-empirical fitting showed to be a good alternative to consider in the developments of future models.

(4) The results presented in this paper also highlights that accounting for the possible mixed triggering and manifestation effects on empirical databases and during model development is essential to improve the prediction performances of a state parameter-based model.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

Been, K., Crooks, J. H. A., Becker, D. E., & Jefferies, M. G. (1986). The cone penetration test in sands: Part I, state parameter interpretation.

Geotechnique, 36(2), 239–249. https://doi.org/10.1680/geot.1986.36.2.239

Been, K., Crooks, J. H. A., Jefferies, M. G., & Rothenburg, L. (1987). The cone penetration test in sands: Part II, general inference of state. Geotechnique, 37(3), 285–299. https://doi.org/10.1680/geot.1987.37.3.285

Been, K., & Jefferies, M. G. (1986). A state parameter for sands. Geotechnique, 36(1), 123–132. https://doi.org/10.1680/geot.1986.36.1.123

Been, K., & Jefferies, M. G. (1992). Towards systematic CPT interpretation. Predictive Soil Mechanics: Proceedings of the Wroth Memorial Symposium, 121–134.

Boulanger, R. W., & Idriss, I. M. (2014). CPT and SPT based liquefaction triggering procedures (Issue UCD/CGM-14/01).

Boulanger, R. W., & Idriss, I. M. (2015). Magnitude scaling factors in liquefaction triggering procedures. Soil Dynamics and Earthquake Engineering, 79, 296–303. https://doi.org/10.1016/j.soildyn.2015.01.004

Brandenberg, S. J., Zimmaro, P., Stewart, J. P., Kwak, D. Y., Franke, K. W., Moss, R., Cetin, K., Can, G., Ilgac, M., Stamatakos, J., Weaver, T., & Kramer, S. L. (2020). Next-generation liquefaction database. Earthquake Spectra. https://doi.org/10.1177/8755293020902477

Cetin, K. O., & Bilge, H. T. (2012). Performance-based assessment of magnitude (Duration) scaling factors. Journal of Geotechnical and Geoenvironmental Engineering, 138(3), 324–334. https://doi.org/10.1061/(ASCE)GT.1943-5606.0000596

Cetin, K. O., & Seed, R. B. (2004). Nonlinear shear mass participation factor (rd) for cyclic shear stress ratio evaluation. Soil Dynamics and Earthquake Engineering, 24(2), 103–113. https://doi.org/10.1016/j.soildyn.2003.10.008

Cetin, K. O., Seed, R. B., Der Kiureghian, A., Tokimatsu, K., Harder, L. F., Kayen, R. E., & Moss, R. E. S. (2004). Standard Penetration Test-Based Probabilistic and Deterministic Assessment of Seismic Soil Liquefaction Potential. Journal of Geotechnical and Geoenvironmental Engineering, 130(12), 1314–1340. https://doi.org/10.1061/(asce)1090-0241(2004)130:12(1314)

Dobry, R. (1989). Some basic aspects of soil liquefaction during earthquakes. In K. H. Jacob & C. J. Turkstra (Eds.), Earthquake hazards and the design of constructed facilities in the eastern United States (Issue 86, pp. 172–182). Annals of the New York Academy of Sciences.

Fioravante, V., & Giretti, D. (2016). Unidirectional cyclic resistance of Ticino and Toyoura sands from centrifuge cone penetration tests. Acta Geotechnica, 11(4), 953–968. https://doi.org/10.1007/s11440-015-0419-3

Geyin, M., Baird, A. J., & Maurer, B. W. (2020). Field assessment of liquefaction prediction models based on geotechnical versus geospatial data, with lessons for each. Earthquake Spectra, 36(3), 1386–1411. https://doi.org/10.1177/8755293019899951

Geyin, M., Maurer, B. W., Bradley, B. A., Green, R. A., & van Ballegooy, S. (2021). CPT-based liquefaction case histories compiled from three earthquakes in Canterbury, New Zealand. Earthquake Spectra, 37(4), 2920–2945. https://doi.org/10.1177/8755293021996367

Ghafghazi, M., & Shuttle, D. A. (2008). Interpretation of sand state from cone penetration resistance. Géotechnique, 58(8), 623–634. https://doi.org/10.1680/geot.2008.58.8.623

Ghafghazi, M., & Shuttle, D. A. (2010). Interpretation of the in situ density from seismic CPT in Fraser River sand. 2nd International Symposium on Cone Penetration Testing, 2(May), 8.

Green, R. A., Bommer, J. J., Rodriguez-Marek, A., Maurer, B. W., Stafford, P. J., Edwards, B., Kruiver, P. P., de Lange, G., & van Elk, J. (2019). Addressing limitations in existing 'simplified' liquefaction triggering evaluation procedures: application to induced seismicity in the Groningen gas field. Bulletin of Earthquake Engineering, 17(8), 4539–4557. https://doi.org/10.1007/s10518-018-0489-3

Idriss, I. M. (1999). An update to the Seed-Idriss simplified procedure for evaluating liquefaction potential. Proceedings, TRB Workshop on New Approaches to Liquefaction.

Idriss, I. M., & Boulanger, R. W. (2006). Semi-empirical procedures for evaluating liquefaction potential during earthquakes. Soil Dynamics and Earthquake Engineering, 26(2-4 SPEC. ISS.), 115–130. https://doi.org/10.1016/j.soildyn.2004.11.023

Ishihara, K. (1993). Liquefaction and flow failure during earthquakes. Geotechnique, 43(3), 351–451. https://doi.org/10.1680/geot.1993.43.3.351

Ishihara, K., Iwamoto, S., Yasuda, S., & Takatsu, H. (1977). Liquefaction of anisotropically consolidated sand. 9th International Conference on Soil Mechanics and Foundation Engineering (Tokyo), 4(16), 261–264. https://doi.org/10.1016/0266-1144(84)90012-8

Jefferies, M. G., & Been, K. (2015). Soil liquefaction – a critical state approach (2nd ed.). CRC Press. https://doi.org/10.1680/geot.2008.b.002

Lasley, S. J., Green, R. A., & Rodriguez-Marek, A. (2016). New stress reduction coefficient relationship for liquefaction triggering analyses. Journal of Geotechnical and Geoenvironmental Engineering, 142(11), 1–8. https://doi.org/10.1061/(ASCE)GT.1943-5606.0001530

Liao, S. S. C., & Whitman, R. V. (1986). Catalouge of Liquefaction and Non-Liquefaction Occurrences during Earthquakes.

Maurer, B. W., Green, R. A., & Taylor, O. S. (2015). Moving towards an improved index for assessing liquefaction hazard: Lessons from historical data. Soils and Foundations, 55(4), 778–787. https://doi.org/10.1016/j.sandf.2015.06.010

Maurer, B. W., & Sanger, M. D. (2023). Why "AI" models for predicting soil liquefaction have been ignored, plus some that shouldn't be. Earthquake Spectra, 39(3), 1883–1910. https://doi.org/10.1177/87552930231173711

Montalva, G., Ruz, F., Escribano, D., Bastías, N., Espinoza, D., & Paredes, F. (2022). Chilean liquefaction case history database. Earthquake Spectra, 38(3), 2260–2280. https://doi.org/10.1177/87552930211070313

Moss, R. E. S., Cetin, K. O., Seed, R. B., & Der Kiureghian, A. (2003). Field case histories for CPT-based in situ Liquefaction potential evaluation.

Moss, R., Seed, R. B., Kayen, R., Stewart, J. P., Der Kiureghian, A., & Cetin, K. O. (2006). CPT-based probabilistic and deterministic assessment of in situ seismic soil liquefaction potential. Journal of Geotechnical and Geoenvironmental Engineering, 132(8), 1032–1051. https://doi.org/10.1061/(ASCE)1090-0241(2006)132:8(1032)

Oommen, T., Baise, L. G., & Vogel, R. (2010). Validation and Application of Empirical Liquefaction Models. Journal of Geotechnical and Geoenvironmental Engineering, 136(12), 1618–1633. https://doi.org/10.1061/(asce)gt.1943-5606.0000395

Plewes, H. D., Davies, M. P., & Jefferies, M. G. (1992). CPT based screening procedure for evaluating liquefaction susceptibility. Proceedings of the 45th Canadian Geotechnical Conference, 41–49.

Robertson, P. K., & Wride, C. E. (1998). Evaluating cyclic liquefaction potential using the cone penetration test. Canadian Geotechnical Journal, 37(1), 442–459. https://doi.org/10.1139/t99-102

Schmidt, J., & Moss, R. (2021). Bayesian hierarchical and measurement uncertainty model building for liquefaction triggering assessment. Computers and Geotechnics, 132, 103963. https://doi.org/10.1016/j.compgeo.2020.103963

Seed, H. B., & Idriss, I. M. (1971). Simplified procedure for evaluating soil liquefaction potential. Journal of the Soil Mechanics and Foundations Division1, 97(9), 1249–1273.

Shen, C. K., & Lee, K. M. (1995). Hydraulic fill performance in Hong Kong. GEO Report 40.

Shuttle, D. A., & Jefferies, M. G. (1998). Dimensionless and unbiased CPT interpretation in sand. International Journal for Numerical and Analytical Methods in Geomechanics, 22(5), 351–391. https://doi.org/10.1002/(SICI)1096-9853(199805)22:5<351::AID-NAG921>3.0.CO;2-8

Suzuki, Y., Koyamada, K., & Tokimatsu, K. (1997). Prediction of liquefaction resistance based on CPT tip resistance and sleeve friction. 14th International Conference on Soil Mechanics and Foundation Engineering, 603–606. https://doi.org/10.1007/978-3-319-73568-9_174

Thomas, J. (1992). Static, cyclic and post liquefaction undrained behavior of Fraser River sand (Issue October) [PhD Dissertation]. University of British Columbia.

Upadhyaya, S., Green, R. A., Maurer, B. W., Rodriguez-Marek, A., & van Ballegooy, S. (2022). Limitations of Surface Liquefaction Manifestation Severity Index Models Used in Conjunction with Simplified Stress-Based Triggering Models. Journal of Geotechnical and Geoenvironmental Engineering, 148(3). https://doi.org/10.1061/(ASCE)GT.1943-5606.0002725

Upadhyaya, S., Green, R. A., Rodriguez-Marek, A., & Maurer, B. W. (2023). True Liquefaction Triggering Curve. Journal of Geotechnical and Geoenvironmental Engineering, 149(3). https://doi.org/10.1061/JGGEFK.GTENG-11126

van Ballegooy, S., Lacrosse, V., Jacka, M., & Malan, P. (2013). LSN – a new methodology for characterising the effects of liquefaction in terms of relative land damage severity. Proceedings of the 19th NZGS Geotechnical Symposium.

Viana da Fonseca, A., Molina-Gómez, F., & Ferreira, C. (2023). Liquefaction resistance of TP-Lisbon sand: a critical state interpretation using in situ and laboratory testing. Bulletin of Earthquake Engineering, 21(2), 767–790. https://doi.org/10.1007/s10518-022-01577-8

Youd, T. L., Idriss, I. M., Andrus, R. D., Arango, I., Castro, G., Christian, J. T., Dobry, R., Finn, W. D. L., Harder, L. F., Hynes, M. E., Ishihara, K., Koester, J. P., Liao, S. S. C., Marcuson, W. F., Martin, G. R., Mitchell, J. K., Moriwaki, Y., Power, M. S., Robertson, P. K., … Stokoe II, K. H. (2001). Liquefaction resistance of soils: Summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils. Journal of Geotechnical and Geoenvironmental Engineering, 127(4), 297–313. https://doi.org/10.1061/(ASCE)1090-0241(2001)127:4(297)