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# Association of flow and equilibrium in geotechnical engineering: an application for earth dams

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## ABSTRACT

The phenomena relating to a variation in stresses and deformations of soils and water flow can occur simultaneously, so that a more realistic forecast requires a solution where combined modeling is used. Mathematical coupling is important but quite complex and very often its use in practical geotechnical engineering problems unfeasible. A simpler proposal that can bring good results would be to associate the two phenomena that would work consequentially with an association of events using numerical methods such as the Finite Element Method (FEM). The purpose of this article is to develop a numerical implementation with two types of software that use the finite element method: one that works with flow in porous media in stationary or transient conditions (SEEP/W) and the other that analyzes the stress-deformation behavior of saturated and unsaturated soils (UNSTRUCT). The UNSTRUCT program uses double density for collapsible or expansive soils and single density for inert soils. It is possible therefore in a very simplified and practical form to obtain the mechanical parameters of the materials, while the program considers the influence of seepage force in saturated soils from the data provided by the flow analysis. Of course, this form may consider various simplifying hypotheses, but it attends practical engineering proposals and can be used in any geotechnical work involving flow and equilibrium.

## RÉSUMÉ

Les phénomènes liés à une variation dans souligne et de déformations des sols et écoulement d'eau peut se produire simultanément, afin qu'une prévision réaliste exige une solution où combiné modélisation est utilisée. Mathématiques couplage est important, mais très complexe et très souvent son utilisation dans la pratique géotechnique problèmes irréalisable. Une simple proposition qui peut apporter de bons résultats serait d'associer les deux phénomènes qui fonctionnerait en conséquence avec une association d'événements utilisant des méthodes numériques tels que la méthode des éléments finis (MEF). Le but de cet article est de développer une implémentation numérique avec deux types de logiciels qui utilisent la méthode des éléments finis: celui qui travaille en collaboration avec écoulements en milieu poreux fixes ou des conditions transitoires (SEEP/W) et les autres que les analyses le stress-déformation comportement de saturés et insaturés sols (UNSTRUCT). Le programme utilise UNSTRUCT double densité pour pliants ou expansive sols unique et densité pour sols inerte. Il est donc possible dans un très simplifié et forme concrète d'obtenir les paramètres mécaniques des matériaux, tandis que le programme considère l'influence des infiltrations vigueur en sols saturés des données fournies par les flux analyse. Bien sûr, ce formulaire ne peuvent envisager diverses hypothèses simplificatrices, mais il s'occupe pratique ingénierie propositions et peut être utilisé dans les travaux géotechniques impliquant flux et l'équilibre.

## 1 INTRODUCTION

Dams are often used in the semi-arid region of Brazil for different purposes, such as, for example, human water supply, irrigation, flood control and so on. Dimensioning the reservoir to be formed by the dam is a function of the topographical characteristics and hydrological studies in the region, as well as its own water balance defined, among other factors, by the purpose of the reservoir. After dimensioning the reservoir, the type of dam to be built is then to be defined.

In Northeast Brazil, earth fill dams are very commonly used with material placed on the job site and compacted with adequate moisture and compaction energy. However, due to the irregular rainfall in this region, small

earth dams are very often built with material containing little moisture content and no compaction to create an embankment consisting of a metastable structure.

The metastable structure is produced by the little water and air in voids in the soil that allows the generation of much lower water pressures than the atmospheric pressure. This results in cohesion between the particles that hinders compaction, absorbing the impacts of the equipment, and the soil has, therefore, a very porous and momentarily stable structure. This porous structure is destroyed as soon as there is an increase in moisture content when the dam is first filled.

The collapse of unsaturated soils is triggered during the transient water flow that changes the stress state of the soil to cause deformations at various points in the

embankment that may completely destroy the dam. Miranda (1988), Pereira (1996) and Silva Filho (1998) presented models using the finite element method (FEM) to predict the behavior of small dams during the first filling.

This paper continues with the proposal by Miranda (1988) and modified by Silva Filho (1998) and uses a finite element program to calculate the deformations occurring as a result of the variation in suction (air pressure less water pressure). In this article two programs are used: the SEEP/W (Geo-slope, 2001) calculates on a transient basis the variation in suction and moisture content during the filling, while the UNSTRUCT program (Silva Filho, 1998) uses these data to predict the stress-deformation behavior. The association of these two programs was undertaken by Cerqueira (2004).

## 2 MODELING OF UNSATURATED SOILS

### 2.1 UNSTRUCT Program

The UNSTRUCT program uses two procedures to analyze the effects of the variation of suction in the unsaturated soils: in the first, a thermal analogy is applied to the deformations of expansive soils, submitted to the variation in moisture content (contraction with an increase in suction and otherwise in swelling) and in the second, the variation in rigidity of the soil that collapses with reduced suction (Silva Filho, 1998).

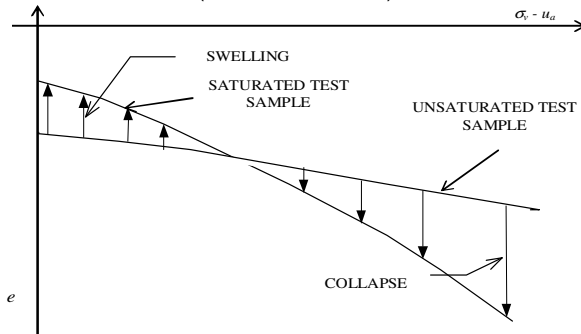


Figure 1. Double oedometer test described by Jennings & Knight (1957)

The soil parameters are obtained from the double oedometric test described by Jennings and Knight (1957). Figure 1 shows a typical result of this test presented in terms of vertical stresses and void index. Supposing linear behavior, the UNSTRUCT program uses the elasticity module in the initial conditions ( $E_0$ ) and the elasticity module of saturated soil ( $E_s$ ) to determine the unsaturated module interpolated from the extreme values, as can be seen in Figure 2 and equation 1. In Figure 2, the double density test results are expressed in a specific deformation ( $\epsilon$ ) versus total vertical stress ( $\sigma_v - u_a$ ) and restricted to the stress interval in which the soil can be considered linearly elastic.

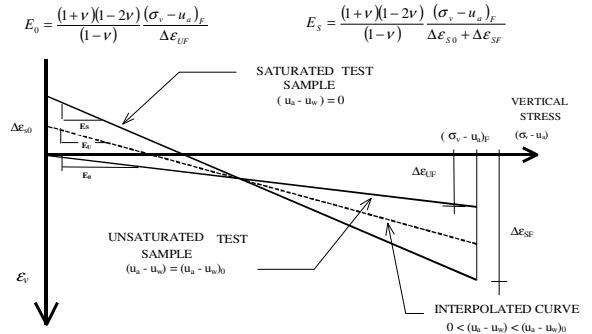


Figure 2. Calculation of an elasticity module  $E_0$ ,  $E_s$  and  $E_u$  (Silva Filho, 1998)

$$E_u = \frac{E_0}{\left(1 - \frac{E_0}{E_s}\right) \left[ \frac{(u_a - u_w)}{(u_a - u_w)_0} - 1 \right] + 1} \quad [1]$$

where:

$(u_a - u_w)_0$  - initial suction of the test sample used to define  $E_0$  (assumed constant)

$(u_a - u_w)$  - suction of the soil for which calculating  $E_u$  is desired.

An analogy is used to determine the stresses and deformations in the soil caused by the variation of suction in swelling soils, in which the vector of initial deformations enters Hooke's law in a similar way to the deformations produced by variations in temperature, which is why this procedure was called thermal analogy.

$$\underline{\sigma} = \underline{D}_e (\underline{\epsilon} - \underline{\epsilon}_0) + \underline{\sigma}_0 \quad [2]$$

where:

$\underline{\sigma}$  - vector of stresses;

$\underline{D}_e$  - stress-deformation matrix;

$\underline{\epsilon}$  - vector of deformations;

$\underline{\epsilon}_0$  - vector of self-deformations resulting from an increase in crystals, variations in temperature, collapse (Zienkiewicz, 1985) or swelling of the soils;

$\underline{\sigma}_0$  - vector of initial stresses.

When using the second procedure, the calculation of the states of initial stresses and the alterations due to various loads, such as surface forces applied to the soil, are adopted to determine collapse deformations. In other words, the soil accumulates deformations compatible with the loads and rigidity for the acting stress level. Increase in moisture content may cause reduced rigidity, with consequent reduction of the elasticity module and increase in the Poisson coefficient during saturation,

characterizing in terms of the proposed modeling the phenomenon known as collapse. The increase in the Poisson coefficient from the phenomenological viewpoint was also accepted by a number of researchers, such as, for example, Pereira and Fredlund (1997).

As a result of the alteration in the rigidity of the soil with the diminished suction, the initial structure does not remain stable and undergoes obligatory deformations, configuring the collapse of the soil. The formulation used in finite elements can be divided into two parts: initial equilibrium and collapse.

- (a) Equilibrium – the initial stresses in the element can be assumed to be in equilibrium with the initial deformations caused by earlier loads.

$$\int_{V_e} B^T \sigma_0 d(vol) = \int_{V_e} B^T D_e \varepsilon_0 d(vol) \quad [3]$$

where:

- $\sigma_0$  - state of stresses compatible with the load applied to the soil;
- $\varepsilon_0$  - deformation also compatible with the applied load and rigidity of the soil;
- $D_e$  - initial soil rigidity (before collapse).

- (b) Collapse – with the diminished soil rigidity, further Deformations must occur in the element to obtain equilibrium again.

$$\int_{V_e} B^T \sigma d(vol) = \int_{V_e} B^T \overline{D}_e B d(vol) x a^e + \int_{V_e} B^T \overline{D}_e \varepsilon_0 d(vol) + \int_{V_e} B^T \sigma_0 d(vol) \quad [4]$$

where:

- $\sigma$  - new stress state after collapse of soil;
- $\overline{D}_e$  - final rigidity of the soil;
- $a^e$  - displacements of the element's nodes.

The new rigidity, in which the elasticity module ( $E_u$ ) and Poisson coefficient ( $\nu_u$ ) are associated, is calculated by interpolation between the unsaturated and saturated extreme values of the double density test (equations 1 and 5):

$$\nu_u = \nu_s - (\nu_s - \nu_0) \frac{(u_a - u_w)}{(u_a - u_w)_0} \quad [5]$$

where:

- $(u_a - u_w)_0$  - suction of test sample used to define  $E_0$ ;

$(u_a - u_w)$  - suction of the soil for the desired calculation of  $E_u$ ;

$E_0$  - elasticity module for the initial condition with  $(u_a - u_w)_0$ ;

$E_s$  - elasticity module for the saturated condition;

$\nu_0$  - Poisson coefficient for initial condition with  $(u_a - u_w)_0$ ;

$\nu_s$  - Poisson coefficient for the saturated condition.

Britto and Gunn (1987) use the technique to check the equilibrium between the stress and total load vectors (e.g. mass force vector, distributed loads, excavation, etc.) in the CRISP program. In this program the equilibrium is checked for initial stresses and each increased load in order to accompany the development of the analysis throughout its performance.

In the numerical procedure presented herein, the imbalance caused by the variation in suction is used to determine the stresses and deformations of the collapse, and the total load vector cannot be used to calculate the imbalance following the reduction in suction, since it does not have information on the material's rigidity. The solution is obtained by using the accumulated deformation vector to calculate the nodal loads. These nodal loads cause the collapse deformations of the soil's structure.

During the saturation process with the consequent collapse, the increase occurs in the suction, which in turn modifies the soil parameters and then imbalance occurs. This procedure does not imply reduction in the final collapse value with a higher number of increases, since the suction is divided rather than relaxing the stress. This explanation is important because Farias (1997) showed that stress relaxation, as proposed by Nobari and Duncan (1972), determines a reduction in volume (collapse) decreasing with the number of increases.

The proposal of Nobari and Duncan (1972) was used successfully in saturating rock fills in only one step (Pereira, 1996). But in this study the elastic parameters of the soil are functions of the suction and new values are calculated at each increase. Accordingly, the collapse deformations will always be accumulated and the imbalance must always exist as long as full deformation does not occur.

Adopting a linear interpolation when calculating the elastic parameters of the unsaturated soil can lead to significant differences between both the measured and predicted collapse values. Some test results presented in scientific literature (Jucá, 1993) indicate, for example, that the elasticity module does not accompany the variation in suction for very high values of this variable. It is therefore worth analyzing whether the range of variation in the suction that is being modeled is compatible with the hypothesis of the linear variation. In order to solve the problem it is suggested that when calculating the interpolation it is adopted for the unsaturated test sample with a suction of no more than that indicated below (Silva Filho, 1998):

Table 1 – Maximum suction values for the dry sample

Soil	$(U_a - U_w)$ (kPa)
Sands and silts with low plasticity	500
Clays with low plasticity	4000
Clays with high plasticity	8000

Bearing in mind the non-linear behaviour of the soil, the UNSTRUCT program uses the variable elasticity modules (linear by intervals). Miranda (1988), doing analyses of the first filling of small dams, adopted the stress-deformation behaviour restricted to the interval considered to be this linear ratio. On looking at a density test presented in a vertical stress curve  $\sigma_v$  (in logarithmic scale) and index of voids, two intervals are clearly identified: the first before stress and the second after the pre-density stress.

Although the characterization of the two intervals is important, the presentation of the double density results (Jennings & Knight, 1957) is made with the vertical stress  $\sigma_v$  on a natural scale versus the specific vertical deformation. When examining the test results, the non-linear stress-deformation behaviour of the test sample is found. The rigidity of the unsaturated soil is the same as saturated soil for high stresses, which is evident in the double density test by the convergence of the dry and saturated curves.

The version developed by Silva Filho (1998) presumes a general non-linear behaviour where the stress-deformation curve has a linear behaviour per interval. The results of the double test are supplied for the program by a set of points defined by the vertical stress applied to the test sample, and the corresponding specific vertical deformation. Between these points the stress-deformation ratio is represented by straight-line segments, which change slope at every stress interval. The number of linear intervals depending on the soil's general behaviour is defined by the user. It is important not to have wide variations in the slope of successive straight intervals in order not to have problems of convergence in the interactive process. With this the UNSTRUCT program keeps the single use characteristic, which is its major advantage for use in practical cases.

The process of calculating the stresses and deformations is done interactively until it reaches a maximum error between the stress values, adopted by the user, or when a maximum number of interactions is reached, also predetermined. In the first interaction, the values of the soil parameters will be those corresponding to the first interval of stresses.

When examining the double density test curves it is possible to see that there are variable potential deformations of collapse of a soil. The difference between the rates of voids of the two curves increases with the increase in compression stress until they reach a maximum value. From this maximum value the potential collapse deformations gradually decrease as the compression stress increases.

This behaviour is explained by the break in connections between the unsaturated soil particles, with

their consequent rearrangement, when the compression stress reaches higher values even if the matrix suction is kept constant, which is not real. While the saturated soil showing high compressibility for the lower stresses has its porosity reduced immediately at the start of the load it later gradually increases its confined compressive strength.

Theoretically for high loads, the two curves could eventually meet, showing the same void rate, when the collapse deformation then becomes practically null.

The calculation procedure for the collapse introduced in the UNSTRUCT program is also able to model this variation in collapse deformations. This is possible because the elastic parameters used by the program are directly removed from the two curves of a double test: one of unsaturated and the other of saturated soil.

## 2.2 SEEP/W Program

Since this is a commercial program and widely used in the geotechnical community, it will be addressed herein as restricted to the information required to understand the association process. The b SEEP/W program models the saturated and unsaturated flow and it is therefore possible to analyze the seepage process in function of time, such as, surface seepage caused by rainfall, filling reservoirs and so on.

The types of analyses that can be done in this study involve flow conditions in stationary or transient conditions. It may be confined or unconfined and the conditions can be axisymmetric. The contour conditions imposed may be constant hydraulic load and flow or functions of time.

As properties of the soils required for the analysis, a function of the hydraulic conductivity must be provided in relation to negative pore pressure of the water in the unsaturated zone of the embankment. The data can be provided for the program from discrete data or using the models proposed in scientific literature, curves being created as a result of the interpolation of the provided data. The following equation was proposed by Fredlund and Xing (1994) and correlates suction data with the volumetric moisture content.

$$\theta = \theta_s \left[ 1 - \frac{\ln \left( 1 + \frac{\psi}{\psi_r} \right)}{\ln \left( 1 + \frac{1000000}{\psi_r} \right)} \right] \left\{ \frac{1}{\ln \left[ e + \left( \frac{\psi}{a} \right)^n \right]} \right\}^m \quad [6]$$

where:

- $\theta_s$  - volumetric moisture content of saturation;
- $\psi$  - suction (kPa) of the soil;
- $\psi_r$  - total suction (kPa) corresponding to residual volumetric moisture content ( $\theta_r$ );
- a - parameter of the soil that is related the value of incoming air (kPa);

- $e$  - Neperian number (2.718282...);
- $n$  - soil parameter that controls the slope from the inflection point of the characteristic curve;
- $m$  - parameter relating to the residual moisture content of the soil.

As a result, the SEEP/W program provides data of the head at the elements' integration points; and the volumetric moisture content that will be useful to update the mass forces of the filling calculated in the UNSTRUCT program. In the association process, a file is produced with data of the suction and volumetric moisture content at each integration point and will be read by UNSTRUCT.

### 3 ANALYSIS OF THE FIRST FILLING OF AN EARTH DAM

This item presents an application of the association of the two programs SEEP/W and UNSTRUCT. The application is similar to that presented by Miranda (1988) who analyzed the behaviour of the filling of a small homogeneous earth dam 10m in height. The dam had slopes of 1 : 2.5 (V : H) and the crest breadth was 6m. Figure 3 shows the finite element grid used by both programs.

In order to model the transient flow the SEEP/W program must be fed with data of the soil retention curve used to build the dam. In the analysis the curve shown in Figure 4 was used.

The double density test data, also presented by Miranda (1988), were used to calculate the stresses and deformations calculated by the UNSTRUCT program. The earth dam fill using compacted material with low moisture content and no compaction was calculated and characterized as a metastable structure. The later collapse over time affected the upstream zone after a few days and after 130 days generalized the entire embankment, the water level staying at 8m (Figures 3 to 8). The deformed mesh shows a tenfold increase in the displacements.

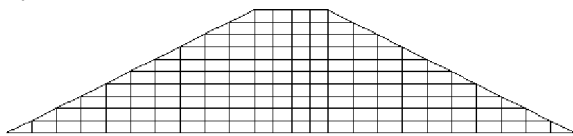


Figure 3 Mesh of finite elements used by the programs

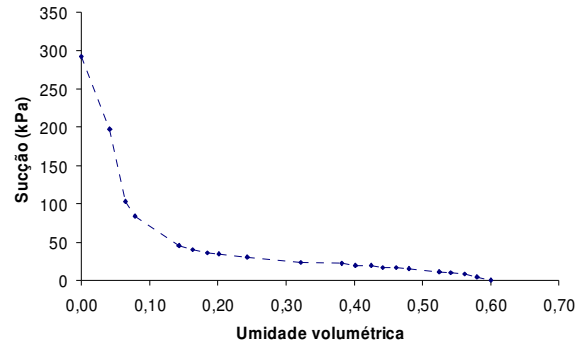


Figure 4 Characteristic curve of the soil used in the dam construction

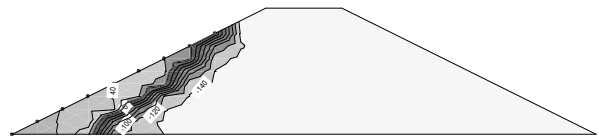


Figure 5 Advance of the saturation front three days later

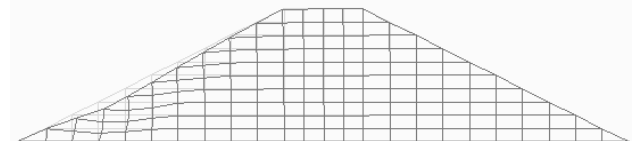


Figure 6 Deformed mesh three days later

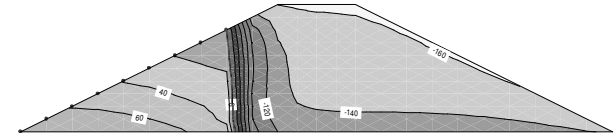


Figure 7 Advance of saturation front 16 days later

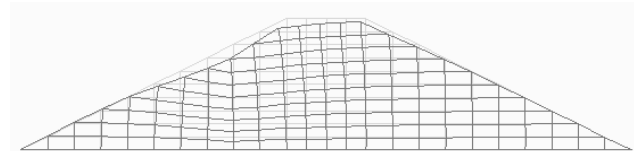


Figure 8 Deformed mesh 16 days later

### 4 CONCLUSIONS

This paper presented an application of association of two finite element programs with a view to analyzing the behavior of a small dam undergoing its first filling. During the dam construction stage it was assumed that the factors influencing the soil compaction, namely moisture content and energy, were applied with quite low values for the optimum condition of the Proctor test. In this situation the embankment has a metastable structure caused by the capillary stresses from the low moisture content in the voids in the soil and by high porosity.

During the filling stage the stress state changes considerably and transiently throughout the body of the dam. This variation causes localized collapses and sometimes causes the complete failure of the dam. To analyze this behavior it is fundamental to use an association or coupling of the two phenomena: flow and equilibrium. This article addressed the association of SEEP/W (Geo-slope, 2001) and UNSTRUCT (Silva Filho, 1998) programs. The SEEP/W program models the transient flow and provides suction and volumetric moisture content data for the UNSTRUCT program to calculate the new stresses and deformations.

There is an option in the UNSTRUCT program to supply only suction values and internally in the program calculate the volumetric moisture content using the different models proposed in literature, such as that of Fredlund and Xing (1994). The results obtained showed considerable coherence with what was expected, and it only remains, for a later study, to build a small dam in these conditions with instrumentation required for measurements throughout the filling process of the dam and check the numerical models. It should be mentioned that Muniz de Farias and Cordão Neto (2010) presented a proposal of coupling that brings good results but without the same practical connotation as this study.

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