Variable Cohesion Model for Soil Shear Strength Evaluation

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ABSTRACT
Soil cohesion is the result of the reaction of an extremely large and countless number of resisting elements on the failure surface, each considered by its own resistance value. As such, considering soil cohesion as a constant parameter is far to be accurate. This paper presents a new model for behavior of soils which is based on the variability of soil cohesion. By adopting this model and approach, a lower factor of safety may be selected to determine the bearing capacity of soil and its shear strength. In this paper, following a brief discussion on the nature of cohesion and the various factors which cause or affect soil cohesion, a simple equation is presented through which, based on the results of common soil mechanics tests, soil cohesion can be calculated and evaluated.

RÉSUMÉ
La cohésion du sol est le résultat de la réaction d'un nombre très grand d'éléments résistant sur la surface de rupture, considérées chacune par sa propre valeur de résistance. En tant que tel, la cohésion de sol, considéré comme un paramètre constant, est loin d'être précis. Cet article présente un nouveau modèle pour le comportement des sols qui est basée sur la variabilité de la cohésion de sol. En adoptant ce modèle et approche, un plus faible facteur de sécurité peut être utilisée pour déterminer la capacité portante de sol et sa résistance au cisaillement. Dans cet article, après une brève discussion sur la nature de la cohésion et les différents facteurs qui causent ou affectent la cohésion de sol, une équation simple est présentée à travers lequel, sur la base des résultats des essais communs de mécanique des sols, la cohésion du sol peut être calculée et évaluée.

1 INTRODUCTION

Soils are quite complex and their behaviour is still not well understood. In the traditional/classical soil mechanics, soils are considered to be elastic, frictional or plastic. However, they are neither truly elastic nor frictional in behaviour, nor are they completely plastic. Therefore, any such idealized treatment will produce differences between predictions and experimental results.

The Ultimate Limit State (ULS) designs as well as the stability problems (stability issues) are related to the conditions of ultimate failure of a mass of soil. Soil lateral earth pressure, bearing capacity and stability of slopes are considered in this category. In such condition, the most important subject is the determination of the loads which will cause failure of the soil mass. Solutions to these problems are generally obtained using the theory of perfect plasticity. In such cases, when the solution to the problem is defined, a factor of safety is applied to limit the stresses and consequently to safeguard the structure. In such condition, if the soil model is not selected appropriately, a larger factor of safety should be applied to adjust the obtained results with the reality and with the full scale test results.

In the present paper a variable cohesion model for soil shear strength evaluation is presented that, when applied, could reduce the required factor of safety in soil related engineering problems.

In order to obtain soil shear strength, frictional and cohesional resistances should be studied and considered concurrently. However, discussion on evaluation of soil frictional resistance is beyond the scope of the present paper.

2 NATURE OF SOIL COHESION

In soil mechanics, geotechnical and foundation engineering, understanding soil shear strength is essential, as most soil failures involve a shear-type failure. This is due to the nature of soil which is composed of individual soil particles that slide beside each other when soil is loaded. The shear strength of soil is required for many different types of engineering analyses, such as the bearing capacity of shallow and deep foundations, slope stability analyses and the design of retaining walls.

The basic definition of soil shear resistance, explained in the Mohr-Coulomb fundamental shear strength equation, is as follows (Eq. 1a):

$$\tau = C + \sigma' \tan \varphi \qquad [1a]$$

Where:
τ is soil shear resistance,
C: soil cohesion,
σ': the effective pressure normal to the surface of failure, and
φ is the soil angle of internal friction.
As maintained by Mohr-Coulomb equation, cohesion of a soil is defined as the shear strength at zero normal pressure on the surface of failure. Based on this definition, soil cohesion (C) is a constant parameter.

Classical soil mechanics is based on Mohr-Coulomb equation and they both imply that, friction and cohesion are properties available in a material (they are material fundamental/intrinsic properties). When we multiply the two sides of Mohr-Coulomb equation by the concerned area of soil shear resistance (Eq. 1b) we obtain soil shear force (shear resistance) which is the sum of soil cohesional and soil frictional forces (Eq. 1c).

$$\tau \,.\, A = C' \,.\, A + \sigma'_v \,.\, A \,.\, \tan \varphi' \qquad [1b]$$

$$F_R = C' \,.\, A + F_v \,.\, \tan \varphi'$$

$$F_R = F_c + F_f \qquad [1c]$$

Where:

A represents the concerned area for soil shear resistance (plane area);

F_v: Effective force normal to the shear failure surface (normal force on a plane);

F_R: Total shear resistance (shear force);

F_f: Total frictional resistance (frictional force);

F_c: Cohesional resistance (cohesional force);

Equation 1c also shows that soil cohesional resistance:
- is a constant value;
- is proportional to the plane area; but
- is independent of the force normal to the plane.

These statements are considered as beliefs in the classical soil mechanics and it is difficult to accept that they might not be correct. However, later on in the following sections, we will notice that soil cohesion is composed of diverse variable components and cannot be a constant value.

In order to define and explain the different factors that affect and shape soil cohesion, some simple laboratory tests have been imagined and/or carried out and the obtained results have been analyzed. In the following sections, these soil tests and their results are briefly described and the effects of selected parameters on the test results are discussed.

2.1 Soil Disturbance and Cohesion

In the classical soil mechanics it is believed that cohesion is the relation and inter-connection between soil particles due to water polar molecules and soil polar particles. To investigate that, the results of the following test may be looked upon. In a shear box apparatus, an undisturbed soil sample with a significant cohesion (shear strength at zero normal pressure on the surface of failure) is tested and soil cohesion is measured. After disturbing and remoulding the soil with a very low (near zero) applied stress (e.g. in a shear box mould), the new sample showed no significant cohesion.

Since water has been always present and there has been no change in the particles and particle size distribution, it could be concluded that the polar molecules of water and polar particles of soil have no major role in the creation of cohesion. They are not the true factors that affect soil cohesion.

2.2 Compaction[1] and Cohesion

A disturbed sample of a clayey soil is compacted / consolidated under a high effective stress. When a shear stress is applied on the sample (e.g. in a shear box apparatus), it was noticed that, at zero normal stress, the soil sample showed some cohesion (for the moment, cohesion is considered to be the shear strength at zero normal stress on the surface of failure). Increasing the pre-consolidation /pre-compaction stress increased the cohesion (Figure 2.2).

Therefore, one of the factors that influence and create cohesion is the energy of compaction. This energy, as a potential energy, accumulates and stores in soil and causes an increase in soil shear strength. The result of applied energy is compaction / densification of soil sample and cohesion is the result of compaction.

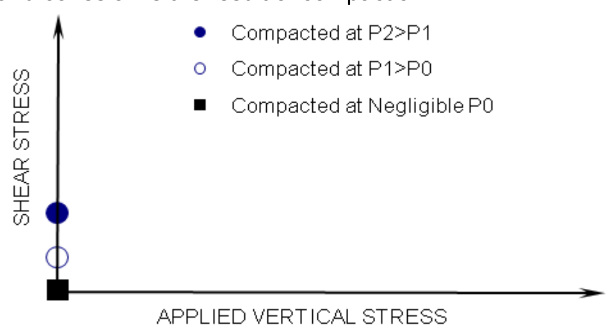

Figure 2.2 – Effect of Compaction on Cohesion

2.3 Other Components of Soil Cohesion

In addition to the above, there are other factors that can affect soil cohesion. These are briefly described below:

- Cohesion due to cementation (which exists, more or less in a large percentage of undisturbed native material),
- Cohesion due to thixotropy (Which is a reversible characteristic that exists in some highly plastic soils and is a result of interaction between soil polar particles),
- Cohesion due to negative capillary pressure (which is lost upon saturation),
- Cohesion due to negative pore pressure during undrained loading (which may be lost through time),
- Cohesion due to soil aging (which could be considered as a type of cementation),
- Cohesion due to osmotic pressure,
- Cohesion due to adhesion and interlocking of soil particles,

[1] Compaction is soil compression and densification under anisotropic stresses

- etc.

With the exception of compaction and cementation, other components of cohesion generally have less influence on generation of soil cohesion in engineering practice.

The type of interparticle bonds that make and affect soil cohesion may by classified in the three (3) following categories:

- Chemical bonds (in cementation and aging),
- Electrostatic and electromagnetic bonds (in consolidation / compaction, capillary stresses and surface tension in non-saturated soils, thixotropy)
- Mechanical bonds (in adherence and interlocking of soil particles)

Each one of cohesion components react independently, may or may not be present and participate in soil shear resistance and the effect of each component of cohesion in soils may be measured accurately by simple laboratory tests.

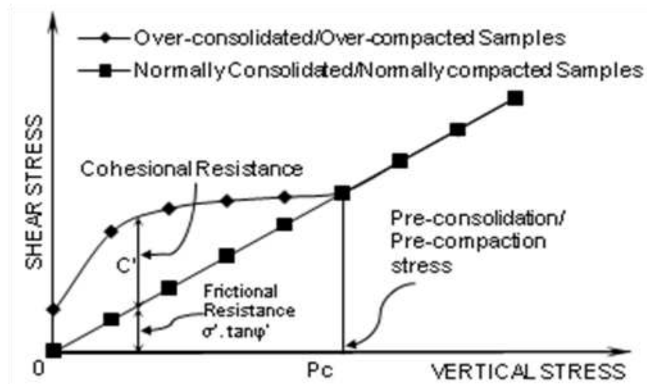

Figure 3.1a–Effect of Pre-consolidation/Pre-compaction on Soil Shear Strength

3.0 COHESION AND VARIABLE VERTICAL STRESS

3.1 Pre-Consolidation / Pre-Compaction

Some identical samples were prepared from a disturbed clayey soil by using the shear box testing mould. The samples had a very little degree of compaction. Then, on each sample a constant surcharge (effective vertical stress σ'_v) was applied and the shear resistance of each sample was measured using the shear box apparatus. The results are presented in Figure 3.1a. These soil samples have been normally consolidated / normally-compacted as at the time of failure, the level of stresses on them was higher than the pre-consolidation/pre-compaction stress.

Identical samples of the same soil were prepared by using the shear box testing mould, but have been initially compacted / consolidated under a pre-compaction/pre-consolidation vertical stress (σ'_p). Similar to the above series of tests, each sample was then tested under a constant value of surcharge (effective vertical stress σ'_v), lower or higher that the initial pre-compaction/pre-consolidation stress.

The results of soil shear resistance under each applied vertical stress are also presented in Figure 3.1a.

Regarding the induced cohesion in the over-consolidated/over-compacted soil samples the following statements are important:

1. Under a specified stress, soil cohesion due to over-compaction/over-consolidation is the difference between shear strength of an undisturbed soil and the shear strength of the same soil when it is completely disturbed and remoulded (with a comparatively negligible confining stress).
2. Soil cohesion is not a constant parameter. It is closely related to the intensity of applied stresses (past and present). In a pre-consolidated (pre-compacted) soil, the variation of shear resistance (τ) versus applied vertical stress (σ'_v) is far to be considered as a straight line and as a result presenting cohesion as a constant parameter is far to be accurate.
3. At low vertical stresses the minimum stress is negative and appearance of a tensile stress reduces the effect of over-consolidation/over-compaction.
4. At pre-consolidation/pre-compaction vertical stress and higher stresses, where the effect of pre-consolidation/pre-compaction is wiped out (and soil is considered as normally consolidated / normally-compacted), the curves representing two series of tests match together.
5. The highest effect of over-compaction/over-consolidation (the highest cohesion) does not show itself under the zero normal stress.
6. A normally consolidated soil has (practically) no cohesion (it may only have some cohesion due to its thixotropy). Cohesion decreases to around zero at vertical stresses higher than the pre-consolidation/pre-compaction pressure. At high normal stresses, when the sample is no more considered as pre consolidated, the pre-consolidation / pre-compaction (stress history) has no influence on the shear strength of soil and soil sample has no cohesion. Its strength is provided merely by friction.
7. As one of the factors that generate and cause cohesion is pre-consolidation/pre-compaction pressure, increasing the stresses on a soil sample leads to

decreasing the pre-consolidation/pre-compaction ratio and consequently decreasing soil cohesion.

8. Soil cohesion has a peak value. Increasing the applied stresses increases the shear resistance effects of friction (frictional resistance) but, following the peak, decreases cohesion (cohesional resistance).

9. Soil cohesion, density, modulus of elasticity, permeability and many other soil parameters are not soil intrinsic/fundamental properties. They are a function of applied stresses. However, Atterberg limits are the intrinsic/fundamental characteristics of a soil (They are not affected by the applied stresses or soil disturbance). Therefore, there could be no relation between cohesion and plasticity (Atterberg limits) of soils.

10. When cohesion of a soil is due to its over-compaction /over-consolidation, it expands at the failure point and if it is saturated, the pore pressure along the failure surface will be negative (This does not generally happen when soil cohesion is due to cementation. However, discussion on soil cementation is beyond the scope of the present paper).

11. If a soil is cohesive, it must be over-consolidated /over-compacted (or cemented) and depending upon its potential of absorbing the compacting energy, an over-consolidated / over-compacted soil is cohesive.

12. As a result of increasing the normal stress, the over-consolidation/over-compaction ratio and cohesion decrease to reach their minimum (one and zero respectively).

13. When the pre-consolidation/pre-compaction stress (Pc) is increased, the curve representing soil shear resistance moves upward; towards higher stresses and as such, soil cohesion and its shear resistance increase accordingly (Figure 3.1b).

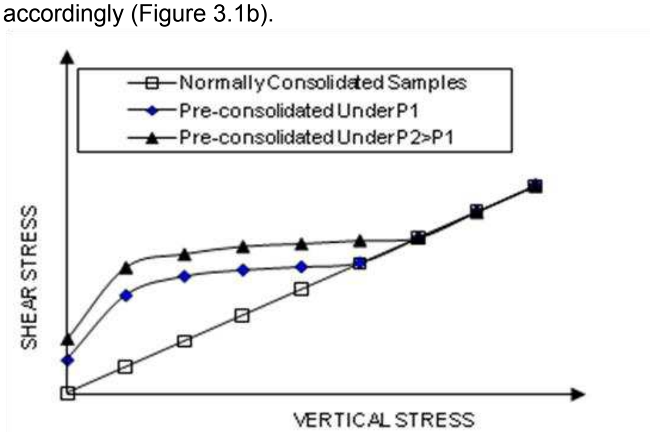

Figure 3.1b – Effect of Increase in Pre-consolidation/Pre-compaction Pressure on Soil Shear Resistance

3.2 Soil Cohesion in a Compression/Compaction Test

Soil cohesion can be studied through oedometer tests, plate load tests, zone tests, jacking or other similar tests.
When a pre-consolidation/pre-compaction surcharge acts on a soil sample, some small elastic deformations appear; soil compacts (Consolidates) under the pre-consolidation /pre-compaction stresses; its volume decreases plastically and the phenomenon of strain hardening caused by the applied stresses happens. Once this surcharge is removed, some negligible amounts of elastic / reversible strains appear and the induced plastic deformations (strains) generate cohesion.

If this soil is charged again and if the new applied stress (σ'_v) remains smaller than the pre-consolidation / pre-compaction stress (σ'_v<Pc), soil remains cohesive.

If the applied stress is larger than the pre-consolidation/pre-compaction stresses (σ'_v>Pc)

- Plastic strains will be added to those negligible elastic and reversible strains,
- Gradient of soil stress-strain curve will increase considerably,
- In the stress-strain space, there will be a breakage or knee at the pre-consolidation/pre-compaction point (Figure 3.2) and
- Soil is no more cohesive.

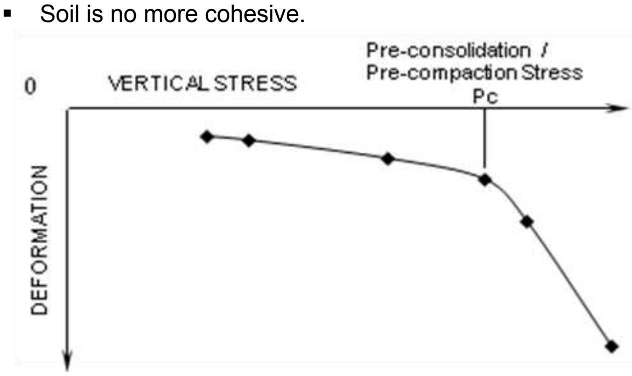

Figure 3.2 – Compaction/Consolidation Curve

Figure 3.2, is plotted in normal Cartesian coordinate system and is somewhat different from the consolidation curve of a clayey soil, which is generally presented in semi-logarithmic system.

A review of soil behaviour on compression/ consolidation reveals the following conclusions:

1 The applied stress (energy) stores in soil as plastic deformation. It increases soil cohesion and shear resistance. It is to be noted that, there should be no plastic deformation when the state of applied stresses is below the pre-consolidation/pre-compaction stress. In other words, plastic deformations appear when the applied stresses go beyond the pre-consolidation/pre-compaction stress. Similarly it can be said that, plastic deformations are generated when the state of the applied stresses runs on the virgin curve.

2 Both cohesion and pre-consolidation are generated by a common creating factor (the applied effective stresses), so they are related together.

3 A cohesive soil has to be necessarily over-consolidated / over-compacted (in the absence of the other cohesion components) and an over-consolidated/over-compacted soil is cohesive. A normally-consolidated/normally compacted soil has no cohesion.

3.3 Cohesion of Coarse-Grained Soils

The behaviour of a coarse-grained (granular) soil is also a function of their compaction (density) and as a result, a function of the maximum stresses that has been applied to the soil in the past (stress history). Therefore, regardless of their granulometry and gradation, soils can be grouped in two different categories:

- Normally consolidated / normally-compacted soils - This term describes the soils, which are compacted only under the actual state of applied stresses. It means that, the effect of previous state of stresses has been less than (or equal to) the effect of the stresses that are actually applied on the soil.
- Over-consolidated / over-compacted soils - This is a soil that, in comparison with the actual state of stresses, has been compacted under a higher state of stresses in the past.

The responses of these two types of soils to the applied stresses are completely different. Some of these differences are briefly explained in the following paragraphs.

Study of the behaviour of granular material (coarse grained silty sand, sandy silt, etc.) shows that, under a constant confining pressure, the shear resistance of an over-compacted granular soil is higher than the one of a normally compacted soil. Therefore, the behaviour of over-compacted coarse-grained (granular) soils is similar to the behaviour of over-consolidated fine-grained (silt and clay) soils (Figures 3.3a).

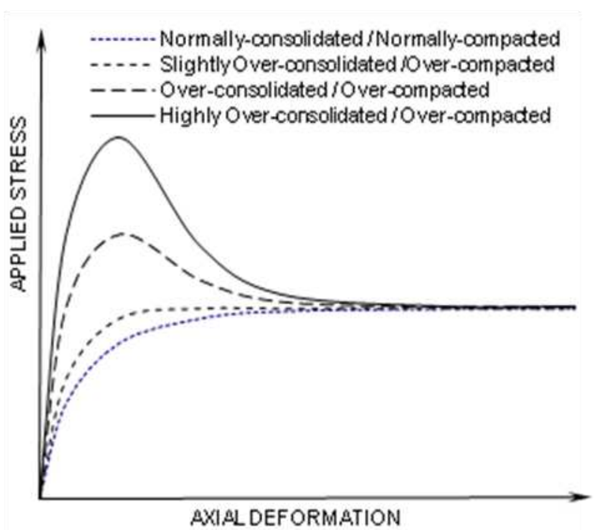

Figure 3.3a – Behaviour of normally-consolidated / normally-compacted and over- consolidated / over-compacted Soil

When a soil shear resistance is plotted in a $\tau - \sigma$ space, it could be noticed that at a similar vertical stress, the shear strength of an over-compacted soil is higher that the shear strength of the same soil when it is in normally-compaction condition. This is the effect of induced compaction / densification / plastic deformation / cohesion.

The stress-strain curve in over-compacted soils shows a peak, followed by a stress softening behaviour. No similar behaviour exists in normally compacted soils. The normally compacted soils have no peak and have always a strain hardening behaviour.

When a soil is over-compacted, its stress-strain modulus (modulus of deformation) is increased. Therefore, an over-compacted soil has a higher stress-deformation modulus. This is, of course, similar to the behaviour of fine-grained (silty clay to clayey silt) soils (Figure 3.3a).

A slightly over-compacted soil is a soil, which is considered as over-compacted at the beginning of a shear test. However, during the loading and prior to soil shear failure, as a result of increase in the applied loads or stresses, soil condition turns to a normally compacted soil.

There is a similarity between the behaviour of coarse-grained (granular) and fine-grained (silty clay/clayey silt) soils in an axial stress-volumic deformation space (Figure 3.3b).

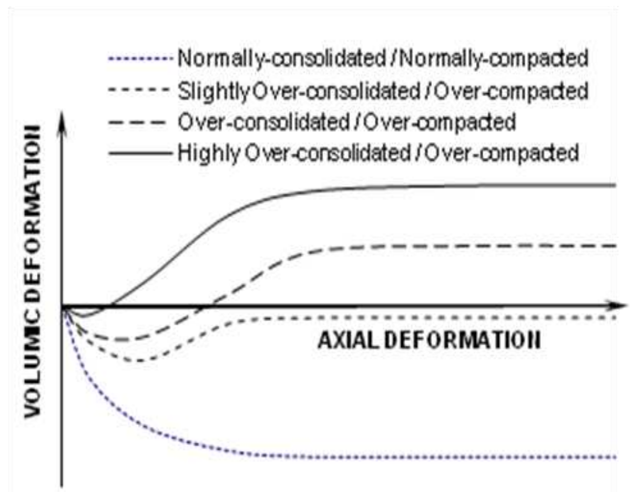

Figure 3.3b – Different Soil Types Volumic Deformation vs. Axial Deformation

Since under a constant vertical stress, the shear resistances of an over-compacted granular soil and a normally compacted granular soil are different, the followings could be concluded for all type of soil:

- When the stress-deformation curve has a peak, soil sample is over-compacted / over-consolidated and therefore, cohesive.
- When the stress–volumic deformation curve (in a consolidation test, plate load test, triaxial test, etc.), demonstrates a breaking point or a knee at a specific stress Pc (which is the representative of pre-consolidation / pre-compaction of the material), at any

applied stress below Pc, soil sample is over-compacted/over-consolidated and therefore cohesive.

4.0 SOIL COHESION MODELLING

Based on the above noted foundation, a variable cohesion model for evaluation of soil shear strength is defined and presented in the following sections.

Soil frictional and cohesional resistances should be studied concurrently in order to obtain soil shear strength. However, discussion on evaluation of soil frictional resistance is beyond the scope of the present paper.

4.1 Cohesion in Soil Micro-Elements

In a soil shearing process, When shear stress is increased and shear strain is imposed to the failure surface, the more rigid bonds on the failure surface yield gradually but the less rigid (more flexible) elements resist and provide /contribute to soil global resistance. As such, during the shearing process, it is the state of shear strain that governs soil resistance and not the state of applied / induced shear stress.

In stress-deformation system of coordinates (σ-ε space) under low or no confining pressure, the curve representing soil behaviour ascends to reach a peak. At the peak, the more rigid elements on the soil failure surface are actually failed but the less rigid and the more flexible elements are still resisting the applied shear stress. After the peak, the remaining softer and more flexible elements yield gradually and the curve descends progressively to reach a point where no more rigid bonds are present on the failure surface (Fig. 3.3.a).

In a microscopic scale and in a stress-strain space (σ-ε space), as the shear stress is increased and shear strain is imposed to an element, it resists the applied stresses. So, its stress-strain curve ascends to reach a peak. At the peak point, the element fails. Its behaviour curve descends, reaches a steady state and from this time forth, there will be no cohesional bond between soil particles and the applied shear stress will be resisted only by the frictional resistance (the residual resistance) which is governed by the applied vertical stress on the failure surface (σ_v). From now on, the state of applied stresses (σ_1, σ_2, σ_3) and as a result the volumic stress $\sigma_m = (\sigma_1+\sigma_2+\sigma_3)/3$ remain constant.

During the shear process, before and after failure and up to the soil residual resistance, the cohesional resistance is transformed gradually to frictional resistance.

Cohesional resistance (which is the reaction of the inter-particle bonds) could be destroyed and dissipated through a combination of the extension strain, shear strain or normal (compressive) strain values which are applied on the failure surface.

4.2 Shear Failure Process

In a shear stress-vertical stress system of coordinates (τ-σ space), soil shear strength curve may be modelled and represented by some straight lines. As such four (4) different segments or stages (a, b c and d) may be distinguished on soil shear failure line (Figure 4.2).

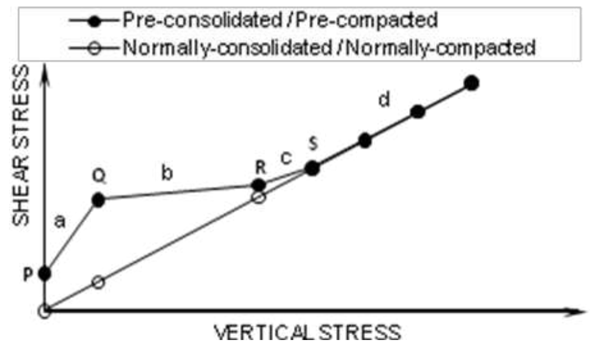

Figure 4.2 – Modeling Soil Shear Resistance

Stage a - At the first stage of loading (Segment PQ), the volumic stress that can generate friction is relatively low and the cohesional inter-particle bonds govern soil shear resistance. Soil frictional resistance (a failure line with a φ slope) is small and the inter-particle bonds (cohesional resistance) govern soil failure. As a result, the slope of the failure envelope (curve representing variation of soil shear resistance with applied vertical stress) is more than the soil angle of internal friction (φ). Around the end of this stage of loading (Point Q) soil cohesion has its maximum value. This segment is thus associated with mobilization of soil cohesional resistance.

Stage b - Following the peak of soil cohesion, the inter-particle bonds continue to fail and soil cohesional resistance decreases sharply (Segment QR). However as the applied vertical stress on the failure surface is rising, soil frictional resistance increases and as a result the general soil shear strength increases, though with a gentler slope in comparison with the frictional resistance (as more inter-particle bonds fail).

Stage c – At the end of Stage b, there are still some active inter-particle bonds and therefore, soil sample has some amount of cohesion on the failure surface. From now on, the rate of dissipation of soil cohesion and destruction of inter-particle bonds reduces considerably and cohesive soil approaches a granular soil is a decelerating mode.

Stage d - At point S all the inter-particle bonds are destroyed. No more cohesion is available on the failure surface. At this point and beyond it soil sample behaves exactly as a normally consolidated / normally compacted granular deposit whose behaviour is only governed by friction.

4.3 Soil Cohesional Resistance Mechanism

In order to simplify soil cohesional resistance mechanism at the failure surface, it may be assumed that at each stage of loading, all of the existing elements participate in generating soil shear strength. At the beginning stages of loading, when the stress level on the failure surface increases gradually, deformation is imposed to the failure surface. Consequently, some rigid bonds/links that cannot tolerate deformation resist deformation. As a result, the applied shear stress will be increasingly concentrated; converged and applied to these more rigid bonds/links to cause them yield. Therefore, at the failure surface, there are not the weaker or stronger bonds that collapse first, but the more rigid one. Typical behaviour of different elements that participate in soil shear resistance is shown on Figure 4.3 where elements that fail gradually are enumerated respectively.

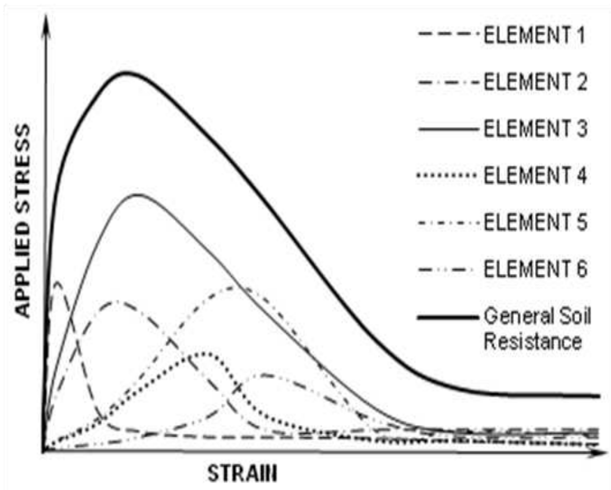

Figure 4.3– Typical behaviour of different elements that participate in soil shear resistance

If all the inter-particle bonds had the same rigidity or stiffness (Young's Modulus), then the bonds would fail together (before failure they could deform, but not slide on each other). Therefore, there would be no friction mobilization and soil behaviour would be completely elastic and ductile. However, gradual failure of different bonds between soil particles; accompanied by the applied stresses (volumic and shear stresses); generate soil deformation and consequently gradual mobilization of soil internal friction forces. Inter-particle bonds fail/yield gradually, their stress (δσ) is applied on the other particles. If on the failure surface the general effective stress is increased, some compaction would be imposed and this will increase soil friction angle (φ) and mobilize/activate additional soil shear resistance.

4.4 Soil Cohesion in σ-ε Space

The global cohesion on the failure surface is the result of the reaction of infinite and unlimited number of resisting elements that, each has a different behaviour and resists the applied shear stress. In mathematics and physics, the curve representing the infinite and unlimited number of resisting elements follows a Gaussian (Normal) distribution.

As cohesion is the result of inter-particle bonds, cohesional resistance on the failure surface (e.g. cohesional resistance under a constant normal stress in a shear box test) has to follow a Gaussian (normal) distribution curve (Figure 4.4). In other words cohesional resistance, as the sum of the resistance of the inter-particle bonds, is governed by the shear strain values applied to the failure surface.

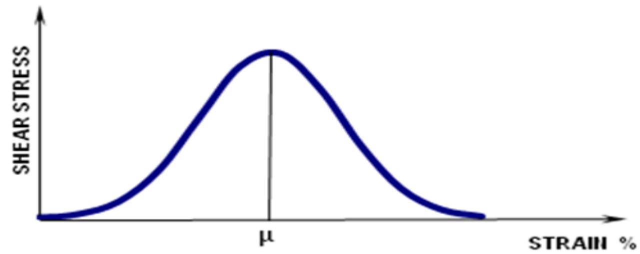

Figure 4.4 – Soil Cohesional Resistance at a Constant Normal Stress

In many natural processes, random variation conforms to a particular probability distribution known as the normal distribution, which is the most commonly observed probability distribution.

The normal distribution curve is described by the following probability density function:

$$F(x) = \frac{1}{\psi\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\psi}\right)^2} \quad [4.4]$$

In which:

μ: is the mean, and
ψ: is the standard deviation.

The normal distribution curve can be completely specified by two parameters:
- Mean (*μ*) and;
- Standard deviation (*ψ*).

As such, if the mean and standard deviation are known, then, one essentially knows as much as if one had access to every point in the data set.

In normal distribution, the empirical rule is a handy quick estimate of the spread of the data given by the mean and standard deviation of a data set that follows the normal distribution. It states that for a normal distribution:
- 68% of the data (68% of the area of the curve) will fall within 1 standard deviation of the mean,
- 95% of the data (95% of the area of the curve) will fall within 2 standard deviation of the mean, and
- Almost all (99.7%) of the data (99.7% of the area of the curve) will fall within 3 standard deviation of the mean.

The normal distribution has also the following characteristics:
- It is symmetric with reference to the mean value,
- It is unimodal and has only one peak (In mathematics, a function f(x) between two ordered sets is unimodal if for some value μ (the mode), it is monotonically increasing for x ≤ μ and monotonically decreasing for x ≥ μ. In that case,

the maximum value of f(x) is f(μ) and there are no other local maxima),

- It extends to +/- infinity,
- The area under the curve =1.

4.5 Formulation of Soil Cohesion in τ- σ Space

The same behaviour noted above could be used to describe dissipation of soil cohesional resistance and destruction of interparticle bonds in a τ-σ space (system of coordinates).

The variation of soil cohesional resistance in the space of shear stress (τ) versus applied normal stress (σ) should also follow a Gaussian distribution / normal distribution curve. Along the failure line in a τ-σ space, the shear and volumic strains increase. Considering:

F(x): soil shear strength, and
μ: the maximum of soil cohesional strength (depends on soil properties and pre-compaction stress),
ψ: the standard deviation (depends on soil properties, type of cohesion and the type of pre-compaction stresses. It is larger if soil is compacted in an isotropic manner).

In normal distribution curve, the area under the curve equals to unity. Therefore, in order to adjust the curve to soil strength, an adjusting coefficient "a" should be applied. A conformal transformation as X=x + b is also required to be applied to shift the curve and show the amount of cohesion for the negative values of stresses.

Therefore, the equation representing soil cohesion will be as noted on Equation 4.5.1.

$$\tau = a \frac{1}{\psi\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\sigma_V + b - \mu}{\psi}\right)^2} \quad [4.5.1]$$

A representation of soil cohesional resistance is shown on Figure 4.5.

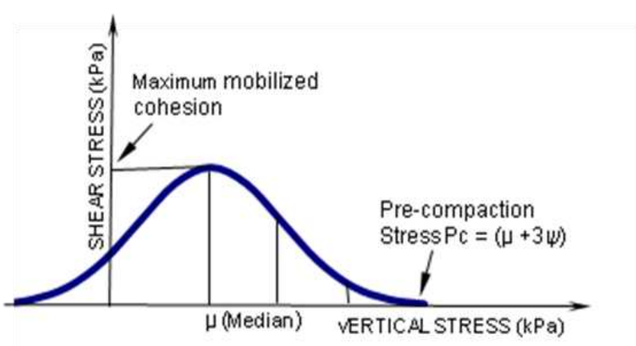

Figure 4.5 – Soil Cohesional Resistance in τ-σ space

As in normal distribution, almost all (99.7%) of the data (99.7% of the area of the curve) will fall within 3 standard deviation of the mean.

$$Pc = \mu + 3\psi \quad [4.5.2]$$

thus,

$$\psi = \frac{1}{3}(Pc - \mu) \quad [4.5.3]$$

4.5.1 Conclusions

One of the advantages of the above explained approach is that it is now possible to attribute formulas to each one of soil strength parameters to solve the geotechnical problems theoretically and with more accuracy. As stated earlier, cohesion may be analyzed into different components to which a mathematical formula may be attributed. This approach opens the doors to study soil behaviour in a more accurate manner. The following conclusions are also important to notice:

- In granular soils cohesion under zero vertical loads on the failure surface is zero. This means that soil cohesional resistance passes by the origin of coordinates. As such, based on the characteristics of normal distribution, the peak of the cohesional strength curve is at a vertical stress, which is equal to half of the pre-compaction stress.
- In over-consolidated /over-compacted clayey soils that tolerate negative stresses, soil can resist some shear stress at zero normal stress. Therefore, the peak of the cohesional strength curve is at a vertical stress, which is smaller than the half of the pre-compaction / pre-consolidation pressure.
- A normal distribution and Gaussian curve (bell like curve) representation for soil cohesional shear strength shows the real values of soil cohesional shear strength. However, as a result of the actual methods of soil sampling, this curve may not be obtained accurately in a laboratory soil testing. During the soil sampling procedure, as a result of unloading the existing surcharges, some non-reversible disturbance happens within the soil sample and as such, some interparticle bands will be destroyed and as such, the concave part of the beginning of soil cohesional shear strength turns to be convex.
- The normal distribution configuration could govern soil cohesion when only one of the major components of cohesion (compaction and cementation) is present within the soil media. In more complex cases where both compaction and cementation are present soil cohesional behaviour should be modelled in a more complex manner which is beyond the scope of the present paper. However, even in such condition, the normal distribution configuration could be used with a good accuracy.
- The intercept value for cohesion line with effective vertical stress axis has to be at pre-consolidation /pre-compaction stress (as maximum tolerable pressure)

and also at maximum extension (traction) stress. Cohesion line has also an intercept with shear stress axis at zero vertical stress (traditional cohesion).

- Soil cohesional shear resistance should be demonstrated as a three dimensional surface in the three-dimensional stresses space and in Mohr-Coulomb representation (τ-σ space), this volume is cut by a diagonal plane that passes through the diagonal axis (Isotropic axis).
- The values (parameters), necessary to define soil cohesion, may be obtained from the results of triaxial tests, shear box, plate loading, consolidation tests, etc. A multi-stage shear box test (carried out with multiple different vertical stresses) may provide the necessary information to calculate or plot the curve.
- Based on the above noted rationale, it is evident that the general soil shear resistance line at its intercept with the shear stress line (zero volumic/vertical stress) has two different slopes. In fact at negative values of volumic stress, friction does not exist and the slope of general soil shear resistance is the same as the slope of cohesion line. However, at positive volumic stresses the frictional effects of soil resistance will be added to the slope of cohesion line.
- Soil failure envelope line is not always ascending. If cohesion is brittle, the failure envelope curve may have also a descending section. In fact if the gradient of cohesion dissipation curve is more than the soil friction angle, the failure envelop that represents the sum of these two values will have a negative slope.
- The pre-consolidation / pre-compaction stress should be defined as an isotropic stress that has the same effect (on soil compaction and density) as the applied anisotropic stresses.
- The subjects discussed in the present paper are a part of a larger discussion on soil shear strength, stiffness and behaviour. Other topics related to soil cohesion have been discarded and not discussed on purpose due to volume restrictions.

5.0 References

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