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Influence of initial block packing on rock avalanche flow and emplacement mechanisms through FEM/DEM simulations

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ABSTRACT

The importance of the initial packing of a rock cumulus on its flow process and emplacement mechanism has been highlighted by several small-scale experiments (Manzella and Labiouse, 2009) where thousands of terracotta bricks were either randomly settled as a loose material or orderly piled one on top of the other before releasing them on an inclined slope. When bricks were piled, longer runout were observed compared to tests run with loose bricks. The reason of this difference has been highlighted using a 2D Finite Element-Discrete Element code by explicitly accounting for the shape of the blocks and the interactions between them. When bricks are piled, the mass has originally an ordinate structure that tends to be preserved during the downhill motion and only after the slope break it shatters, whereas in the case of bricks randomly settled into the box, the mass behaves as a loose material from the start and more energy is lost from the beginning through both friction and collisions at the base and within the granular mass. When the slope break is smoother, the relatively-coherent structure of the block cumulus is even less disaggregated, as a consequence less energy is dissipated at the toe, the mass can travel further and it preserves the inherited geometries. Simulations confirm the experimental results, giving a better insight on the understanding of the effect of the initial block packing on the longitudinal spreading and on the mechanisms underneath the process of rock avalanche propagation.

RÉSUMÉ

L'importance de la structure initial d'un cumulus de roche sur le processus de flux et le mécanisme de mise en place a été soulignée par plusieurs expériences à petite échelle (Manzella et Labiouse, 2009) où des milliers de briques de terre cuite ont été soit disposées comme un matériau en vrac ou empilées les unes au-dessus des autres de manière ordonnée avant de les relâcher sur une pente inclinée. Quand les briques ont été empilées, des plus longues distances parcourues ont été observées par rapport aux tests effectués avec des briques en vrac. La raison de cette différence a été mise en évidence en utilisant un code par éléments finis-éléments discrets combinés, en tenant compte explicitement de la forme des blocs et des interactions entre eux. Dans le cas de briques en vrac l'énergie est dissipée à la fois par friction et par collisions à la base et au sein de la masse granuleuse tout le long de la pente et du processus d'accumulation. Alors que quand les briques sont empilées, l'énergie est dissipée principalement par le frottement à la base sur le plan incliné, là où la masse reste relativement structurée. Puis, l'énergie est dissipée aussi par frottement et par des chocs à l'intérieur de la masse dans la zone d'accumulation. Lorsque le pied de pente est plus doux, cet effet est encore plus évident: la structure relativement cohérente du cumulus est moins désagrégée au pied, moins d'énergie est donc dissipée, la masse peut parcourir des plus longues distances et la structure des colonnes des briques empilées est préservée jusqu'à dans le dépôt final. Les simulations confirment les résultats expérimentaux, en donnant une meilleure compréhension de l'effet de la structure initial du cumulus de roche sur les mécanismes de propagation des avalanches rocheuses.

1 INTRODUCTION

Rock avalanches are a landslide hazard derived from a bedrock failure and characterized by a very high mobility resulting in runouts much greater than the one that could be predicted using frictional models (Hungr et al, 2001). Rock avalanche phenomena generally involve very large volumes (e.g., several millions of cubic metres) which are impossible to stabilize and, once destabilized, they destroy everything they encounter flowing downhill. Thus, the only possible mitigation and prevention method is to estimate the area at risk by forecasting the travel path, distance, and velocity of the avalanche and the resulting deposit profiles. To this end, a satisfactory comprehension of the mechanisms involved in propagation is needed; although several theories have

been put forward to explain rock avalanche high mobility, at the present time, no general agreement has been achieved and there are still many questions to be answered (Hungr, 2002).

Due to the rareness of rock avalanche events, the number of well documented real cases is limited and therefore precious data such as unstable volume and coefficient of friction are difficult to accurately recover. Consequently, several authors have looked at laboratory tests and numerical simulations to improve the understanding of this kind of phenomenon, e.g. Hutter and co-workers, Davies and McSaveney (1999), Derlinger and Iverson (2001), McDougall and Hungr (2004), Banton et al (2009), Manzella and Labiouse (2009).

Manzella and Labiouse (2009) have carried out several small-scale granular flow experiments at the Rock

Mechanics Laboratory of the Ecole Polytechnique Fédérale de Lausanne where several parameters including avalanche volume, fall height, basal friction, shape of the blocks and their initial arrangement were varied. Although it is generally not possible to obtain long runout with laboratory experiments (Friedmann et al (2006), some of the experiments carried out by Manzella (2008) have given particularly long spreading. Two main factors have a strong influence on the propagation of granular flow, namely the regularity of the pathway and the regularity of the initial block packing and deserve to be investigated further with the contribution of numerical modeling.

Rock avalanches are mainly modelled using either (i) sled block models, or (ii) discrete element models, or (iii) continuum mechanics models. The models of Heim (1932), Hsü (1975), Davies (1982) and Van Gassen and Cruden (1989), based on frictional law and energy dissipation, belong to the first category. Whereas the ones of Bagnold (1954) and Drake (1990, 1991) which take into account the interaction between particles, belong to the second one. Continuum mechanics models are more frequently used for debris flows, mudflows, lava and snow avalanches as in the case of the models of Hutter et al. (1988, 1991), Hungr (1995), Ancey et al. (1997, 2004), Pouliquen et al. (1999, 2002), Derlinger and Iverson (2001). The latter approach is sometimes extended to dry granular flows, rockslides and rock avalanches (e.g. Manzella et al, 2007; Naaim et al, 1997; Pirulli and Mangeney, 2008) when these are considered to be dense one-phase flows.

In this context, a new two-dimensional hybrid Finite-Element Discrete Element (FEM/DEM) numerical method has been chosen to study the effect of the regularity of the pathway and of the initial block packing and gain some insights into the flow and interaction mechanisms that take place during rock avalanche propagation.

2 THE COMBINED FINITE-DISCRETE ELEMENT METHOD

The combined finite-discrete element method (FEM/DEM) is a numerical technique developed by Munjiza et al. (1995) for the dynamic simulation of multiple deformable and fracturable bodies. Discrete Element Method (DEM) algorithms are used to model the interaction between different solids, while Finite Element Method (FEM) principles are used to analyze their deformability. In particular, within the framework of FEM/DEM, each discrete element is meshed into finite elements. These meshes define the shape and boundaries of discrete elements and contact between them, and allow the discrete elements to deform. The key-processes in FEM/DEM therefore include: contact detection, interaction and friction between elements and element deformation. Furthermore, since an explicit time-marching scheme is used to integrate Newton's equation of motion, fully dynamic simulations are allowed.

In the context of rock avalanche modelling, a multitude of interacting distinct bodies are simulated and therefore proper treatment of contact interaction is of crucial importance. As soon as two discrete bodies are

detected in contact, the interaction algorithm is applied to compute the reaction forces between them. A penalty function method (Munjiza and Andrews, 2000), is used for the interaction algorithm. In normal direction, contacting couples tend to penetrate into each other, generating distributed contact forces, which, depend on the shape and size of the overlap between the two bodies. Body impenetrability condition is therefore only satisfied as a limit condition for normal penalty values that tend to infinity. Since a potential function is integrated to calculate interaction forces, principle of energy conservation is automatically satisfied. In tangential direction, a Coulomb type friction law is employed to calculate shear interaction forces based on the sliding distance between element edges (Mahabadi et al., 2010).

Although no fracturing was allowed to occur in the simulations carried out for this research, it is worth noting that FEM/DEM is also able to explicitly simulate material sudden loss of cohesion (i.e. brittle failure) by means of fracturing and fragmentation algorithms.

The FEM/DEM computer code used for this study is based on the Y-Code of Munjiza (2004). The code has been the subject of ongoing research and development by the Geomechanics Group at the University of Toronto.

3 EXPERIMENTS AND SIMULATIONS

3.1 Description of the experiments

As explained in details in Manzella and Labiouse (2008 and 2009), the experimental set-up (see Figure 1) used by Manzella (2008) mainly consists of a slope (3 m×4 m) at 45° degrees, ending with a slope break and a horizontal accumulation zone. Different amounts of material were poured into a wooden container measuring 20 cm height×40 cm width×65 cm length and placed on the slope. By rapidly opening the downhill gate the material was let free to fall and therefore an unconstrained flow was triggered.

During the experiments, several parameters were varied including the type of released material (aquarium gravel and small terracotta bricks of 1.5 cm × 3.1 cm × 0.8 cm), the slope angle (37.5°, 45°), the fall height (1 m, 1.5 m), the material volume (20,000 cm³, 40,000 cm³); and the basal friction angle (two different types of basal covering - wood and smooth plastic – were used). In addition two arrangements of bricks before failure were used, i.e. poured in randomly in the container, called Random bricks (BrR) and piled orderly one on top of the other, called Piled bricks (BrP) as shown in Figure 2. Main goal of using the latter arrangement was to study how potential rock avalanche phenomena could develop in a rock mass characterized by the presence of three sets of fully persistent joints. The longest dimension of the bricks (3.1 cm length) was perpendicular to the dip slope direction, whereas the larger surface (1.5 cm × 3.1 cm) was positioned parallel to the slope plane. For the 40,000 cm³ tests, 9680 bricks were thus disposed in 22 elements height × 11 elements width × 40 elements length (in the reservoir of 20 cm height × 40 cm width × 65 cm length). To investigate the influence of the shape of the toe of the slope on the runout distance, another

series of tests were designed where, for the 45° slope sharp angle at the toe was replaced by a curved connection as shown in Figure 2. The radius of the arc of the curve constituting the smoother slope break was of approximately 0.5 m.

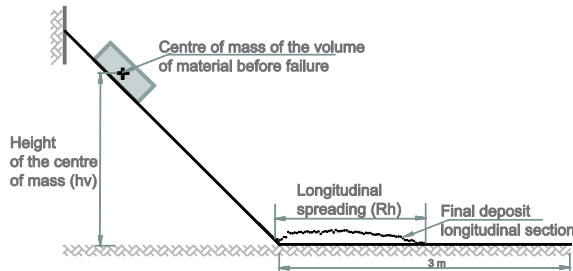


Figure 1. Schematic representation of the experimental set-up used by Manzella (2008) and of the measured parameters h_v and R_h

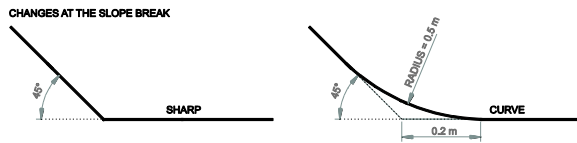


Figure 2. Changes at the slope break: sharp and curve connection at the toe of the slope (Manzella and Labiouse, 2011)

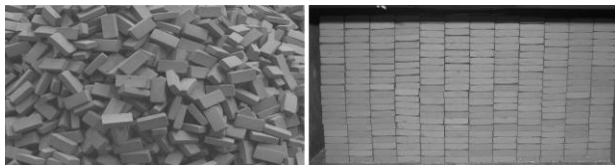


Figure 3. Brick arrangement before release: random on the left and piled on the right (Manzella and Labiouse, 2009)

According to Davies and McSaveney (1999), for each test two non-dimensional factors were computed, namely the normalized length (R_h/h^*) and the normalized vertical fall height (h_v/h^*) which are respectively the deposit length and the total height of the centre of mass before release (see Figure 1), both normalized with respect to the cube root of the volume, h^* . This allowed to compare the longitudinal spreading of experimental and field granular avalanches, scaling the longitudinal spreading with the size of the event. The experimental results have been compared to two real events which can be considered as unconstrained granular avalanches on an average slope of 45° as used in the experiments: the Elm (Switzerland, 1881) and Frank (Canada, 1903) rock avalanches, a 10 million and a 37 million cubic metres event, respectively.

As shown in Figure 4, some of the experiments by Manzella (2008) showed particularly high values of R_h/h^* which are comparable to those of the two events considered. These experiments (marked by a star in Figure 4) were carried out with piled bricks and a smoother slope break.

In this framework, two-dimensional FEM/DEM simulations are used in the present paper to gain a deeper insight into the effect of these two parameters and to highlight the flow mechanisms by explicitly accounting for block-block and block-slope interaction.

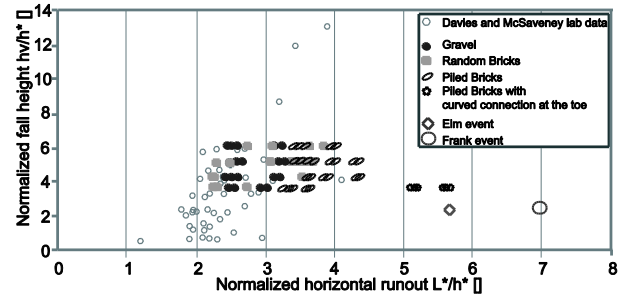


Figure 4. Normalized length (R_h/h^*) plotted against normalized vertical fall height (h_v/h^*) of: the experiments by Davies and McSaveney (1999) (empty hexagons) and Manzella (2008) (filled spheres, squares, ellipsoids and stars) and of two real events Elm (filled rhomb) and Frank (filled triangle).

3.2 Numerical set-up

Three different FEM/DEM models were built corresponding to the case of Piled bricks (Figure 5a) and Random bricks (Figure 5b) with a sharp slope break and Piled bricks with a smooth slope break (Figure 5c) from a realising height of 1 metre. Each brick was discretized with four triangular finite elements while the slope was meshed with 3 elements for the two cases with a sharp slope break and with 5280 elements for the smooth slope break case, using the software CUBIT to create this more complex mesh. Material properties and boundary conditions were assigned by using the Y-GUI program (Mahabadi 2010). The slope was assumed to be rigid while the bricks were assigned a Young's modulus and a Poisson's ratio equal to 10 GPa and 0.25, respectively. No brick fracturing was allowed to occur. Each simulation was run for a duration of 0.5 s corresponding to 10^8 timesteps with a time step size of 5×10^{-6} ms. For the case of BrR in order to distribute the element randomly before opening the gate a first run has been done to let the elements fall from a height of about 1 metre above the gate and distribute against it under the effect of gravity (see Figure 5b).

3.3 Numerical results and comparison with experimental ones

Figure 6 compares experimental and numerical final deposit distribution. By comparing the absolute values of the experiments and the simulations, it can be noticed that there is not a good match between the runout values of the experiments and of the simulations. This is mainly due to the fact that the two-dimensional nature of the FEM/DEM code does not take into account the energy loss in the lateral spreading and, in present experiments, we haven't considered the energy dissipation taking place during normal impact-style contact, both factors inducing shorter runout in the experiments.

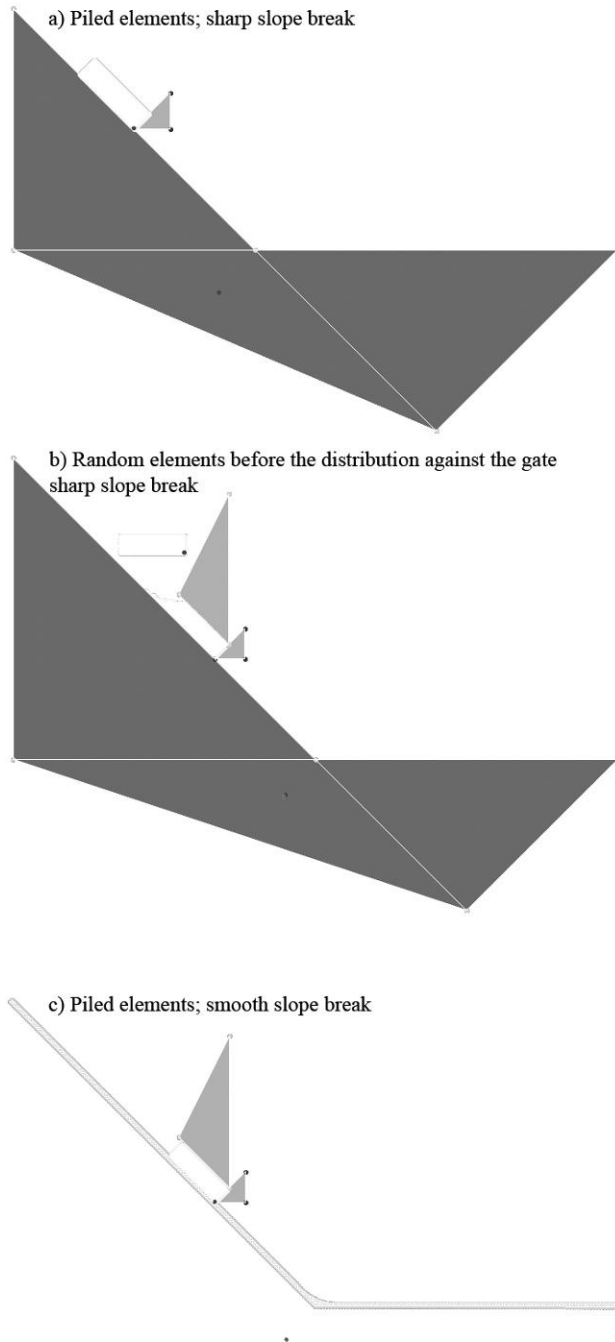


Figure 5. Numerical set-up: a) piled elements, sharp slope break; b) random elements before and after the distribution against the gate, sharp slope break; piled elements, smooth slope break

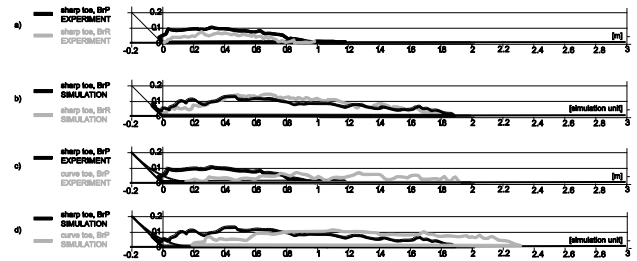


Figure 6 Comparison of experimental and simulation profiles: a) experiments; 40,000 cm³, 1 metre fall height, sharp toe, piled bricks (black line) compared to 40,000 cm³, 1 metre fall height, sharp toe, random bricks (grey line); b) simulations; sharp toe, piled elements (black line) compared to sharp toe, random elements (grey line); c) experiments; 40 litres, 1 metre fall height, curve toe, piled bricks (black line) compared to 40,000 cm³, 1 metre fall height, curve toe, random bricks (grey line); d) simulations; curve toe, piled elements (black line) compared to curve toe, random elements (grey line)

However, comparing the effect of the factors considered, i.e. the initial block packing and the shape of the slope break, it is possible to detect a similar influence on both experimental and numerical runout: when elements, i.e. bricks or rectangles, are piled, runouts are slightly bigger than when elements are poured into the releasing container. Major differences are detected when there is a smoother slope break; in this case runouts are much larger than when a sharper slope break is used. According to Manzella and Labiouse (2009), this is due to the fact that when bricks are piled orderly one on top of the other this gives a dense and structured initial packing to the mass, that remains packed together on the inclined slope where energy dissipation takes place mainly through friction at the base. After the impact with the horizontal panel, the mass shatters and energy is then mainly dissipated through friction/collisions between the bricks. Having “spared” a part of the energy in the first part of the sliding, the mass enters the accumulation zone with a higher velocity and can consequently travel further on the horizontal panel. When the slope break is smoother, less energy is dissipated at the impact with the horizontal and the mass can travel even further. Also, as shown in Figure 7, in the final deposit of experiments with a smooth slope break, piles of bricks that preserved their initial structure were detected; this as a confirmation that less shattering takes place at the toe and as a consequence less energy is dissipated.

These intuitive assessments have been confirmed by numerical simulations. Thanks to the simulation snapshots it is indeed possible to have an insight into the final deposit structure and to follow the flow and the interactions between blocks as they travel downhill. Similarly to what was observed in the experiments, when the rectangular elements are piled and the slope break is smooth, the mass has maintained most of its structure and a lot of rectangles are still piled one on top of the other even if they had gone under some shear (see Figure 8). In addition, observing the snapshots it is possible to detect how the mass behaves before and after the slope break, i.e. in the case of randomly poured elements at start and a sharp toe (Figure 9a), the mass

behaves as a loose material before and after the slope break; in the case of piled elements and a sharp slope break (Figure 9b), the initial packing is preserved only till the slope break, then it shatters and its structure is lost because of the impact with the sharp toe; on the other hand, in the case of piled elements and a curved slope break (Figure 9c), most of the piles of elements are preserved before and after the slope break, they slide on the curved smooth toe, the shattering of the mass at the impact with the horizontal is reduced and as a consequence less energy is dissipated at this stage of the downhill flowing process.



Figure 7 Evidence of the initial structure in a final deposit of a test of 40,000 cm³ of piled bricks with a smooth slope break (Manzella and Labiouse, 2011)



Figure 8 Disposition of the rectangular elements in the final deposit of the simulations with a sharp (a) and smooth (b) slope break. The black empty rectangles in (b) put in evidence how the structure is preserved in the final deposit when the slope break is smooth

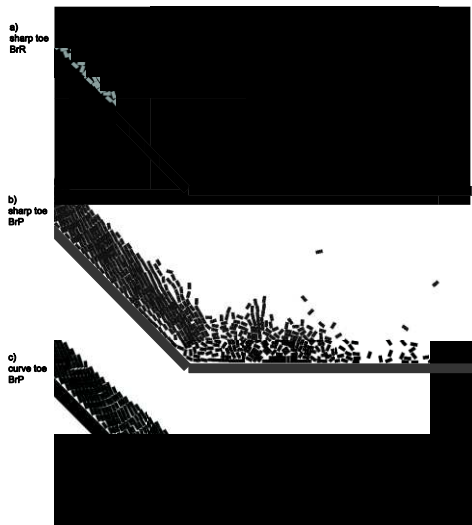


Figure 9 Difference in shattering with a) loose elements (BrR) at a sharp toe; b) piled elements (BrP) at a sharp toe; c) piled elements (BrP) at a smooth curve toe. It is possible to notice how in the first case the material is de-structured before and after the slope break, how the structures of the piled elements are broken in the second

case and how they simply slide on the curve in the last one.

4 DISCUSSION

The importance of the topography of the slope on rock avalanche runout has been already pointed out by Heim (1932), on the other side Davies (1982), Hewitt (1988) and Davies and McSaveney (1999) took into account the initial packing in their theory of the spreading of a coherent mass, i.e. the long runouts are due to the shattering and spreading of the failed, relatively-coherent block cumulus. These two aspects, however, had not been investigated in depth and deserve a greater attention.

Friedmann et al. (2006) carried out experiments with a smooth transition at the slope break and they concluded that this type of slope transition enhances the runout since less energy dissipation takes place at the toe of the slope and confirmed the importance of the changes in the flow geometry in the propagation. However, since the material used was sand, the initial block packing was obviously not considered.

As a matter of fact these two factors, i.e. smoother slope break and more structured initial block packing, combined together gave particularly long runout among several small-scale experiments carried out by Manzella (2008). Thousands of terracotta bricks were either randomly settled as a loose material or orderly piled one on top of the other into a releasing container. This was then placed at one meter height on a 45° slope ending with a slope break and a horizontal accumulation zone. When bricks were piled, longer runout were observed compared to those tests run with bricks disorderly thrown into the container before failure. This difference was even more evident when a smoother slope break was placed at the toe.

Only one other record by Okura et al (2000) has been found in literature where tests have been carried out with blocks instead of sand in the study of rock avalanche propagation; this is almost certainly due to the practical difficulty of performing these experiments. Nonetheless, small bricks better represent the dimensions of the blocks relative to small topographic irregularities in real events and they allow to take into account both packing and structure of the block cumulus before failure. Therefore they contributed to analyze phenomena which cannot be observed for gravel or sand flow at laboratory scale and consequently they could improve the understanding of the mechanisms involved in propagation (Manzella and Labiouse, 2009).

Even if some intuitive assessments were made to justify the long runout of these particular block flows, direct observation of flow processes and interaction between blocks was not possible within the experimental framework.

The use of a combined finite element-discrete element method has allowed us to observe the internal mechanisms and interactions between the blocks and topography. Because of the two-dimensional nature of the code, the absolute values could not be reproduced. However, although two-dimensional results are not

directly applicable to three-dimensional granular flows, they are useful from a theoretical point of view since, when the effects of interstitial fluid are negligible, the design preserves the essential physics of the interactions between particles in flows (Drake, 1991). We can thus confirm observations by Manzella and Labiouse (2009): when elements are piled, the original ordinate structure of the mass tends to be maintained during the downhill motion. Sliding energy is consequently mainly dissipated through friction at the base, and only after the impact with the horizontal panel, the mass shatters and energy is also dissipated through friction/collisions between the elements in the whole mass. In the case of rectangles randomly settled into the box, the mass behaves as a loose material from the start and energy is lost from the beginning through both friction and collision at the base and within the mass. Having dissipated more energy in the first part of the sliding, the mass enters the accumulation zone (flat base) with a lower velocity and consequently travels a shorter distance than the mass constituted of piled elements. When the slope break is smoother the relatively-coherent structure of the block cumulus is less disaggregated, less energy is dissipated at the toe, the mass can travel further and it preserves the inherited geometries, i.e. columns of piled elements undergo some shear but their sequence is preserved in the final deposit. This last feature of the experimental and numerical simulations is very important since the preservation of the stratigraphic order is one of the main characteristics of rock avalanche deposit as pointed out by Erisman (1979). He called it the coherence problem, i.e. the displaced mass, although sometimes disintegrated into small fragments, shows a surprising congruence of its sequential order before and after the event. According to the suggestion given by the observation of the simulations, this characteristic could be due to the fact that when the volume reaches a certain size, the dimensions of the packing structure become too large to feel the effect of medium to small changes in the topography of the slope which is felt like the smooth slope break described in the simulations. As a consequence the mass, even if it undergoes shear at the base, it preserves its structure, dissipating less energy in collisions between the blocks and travelling long distances.

5 CONCLUSIONS

This paper illustrates the use of a new numerical tool, called the Finite Element-Discrete Element Method (FEM/DEM), to evaluate the influence of the initial packing of a rock cumulus on its flow process and emplacement mechanism at the bottom of the slope. The FEM/DEM code has been able to give some important highlights on the internal mechanisms of a mass of blocks flowing down a slope, by explicitly accounting for the interactions between blocks and with the topography and it has proved to be a useful tool in the understandings of the mechanics underneath the process of the propagation of rock avalanches.

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