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Constitutive Models for Municipal Solid Waste in Landfills

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Abstract. The effectiveness and safe operation of a municipal solid waste (MSW) landfill is determined by the hydraulic, mechanical, biological and thermal processes within the waste mass in landfill. The understanding of the behavior of MSW is further exacerbated by the simultaneous coupled interactions between the aforementioned processes. These complex interactions stand as a unique challenge in reliably predicting the overall performance of a landfill system. One of the most intricate issues in numerical modeling of MSW behavior in landfills is the prediction of mechanical response (strength and settlement) of the MSW under different stress states. Realistic representation and accurate prediction of the strength and deformation aspects of MSW is critical for analyzing the instability and integrity within the landfill and its components. In this regard, several researchers have formulated constitutive models to predict the stress-strain behavior of MSW. But these studies do make assumptions that omit some of the important characteristics that are unique to MSW, thus lacking the realistic representation of the MSW behavior in their mathematical formulation. This study presents a critical review of some of the most prominent constitutive models developed for MSW till today. The current research status and a practical approach to simulating the MSW mechanical behavior based on the review of the findings from the published literature is discussed. Finally, the key challenges that need to be addressed in accurate representation of the mechanics of MSW are highlighted.

Keywords. Municipal solid waste, landfills, constitutive model, settlement, shear strength.

1. Introduction

Municipal solid waste landfills unlike open dumpsites are engineered waste containment systems consisting of several engineered components designed to meet certain regulatory requirements. If one were to think of landfills as an engineered structure, then MSW is essentially the main construction material and the load bearing entity of a landfill. Generally, the MSW in landfills is placed in thick lifts and, based on the landfill capacity, the height of the landfill can reach over 100 m. The MSW in landfills is therefore subjected to enormous amount of load which induces high stresses causing the waste to compress and settle. Sometimes such high stresses may even cause instability within the MSW leading to failure/collapse of the engineered landfill. The

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stability of a landfill and thereby its performance is highly dependent on the deformation and strength characteristics of MSW. Moreover, the prediction of settlement of MSW is essential to ensure stability and integrity of landfill components (e.g., leachate and gas collection systems, cover system), maximizing the use of landfill airspace, and to aid in appropriate planning of the end-use of the site. Further, with the recent advent of the leachate recirculating landfills (bioreactor landfills) wherein the MSW undergoes enhanced rate of biodegradation, the accurate prediction of the stability and settlements in both short and long-term is of prime importance.

Unlike the traditional soils, the deformation characteristics of MSW are more involved, in that the settlement of MSW is not only dependent on load induced stresses but also on time-dependent viscous creep and the biological decomposition of the MSW. However, a major portion of the settlement of MSW is typically due to biodegradation of MSW in landfills. Simulating the effect of biochemical reactions within the MSW on the contemporaneous changes in its physical characteristics is quite complex thus resulting in a fundamentally different and unique constitutive behavior for MSW. Over the years, several researchers have studied the mechanical behavior of MSW with varying complexity mostly with an aim of accurate prediction of settlement of MSW in landfills. In the early studies, the settlement of MSW was based on simple soil mechanics models with the secondary compression coefficient (C_α) being indicative of the long-term time dependent biodegradation and creep settlement. Thereafter, many empirical time dependent and rheological models with limited general applicability were developed. A detailed review of these models is presented in [1] and [2]. This study presents a critical review of some of the recent and advanced constitutive models developed for numerical modeling of the mechanical behavior of MSW in landfills. The mathematical formulations of each of the models are briefly described and their shortcomings are discussed. An approach that is appropriate for the development of a realistic and practically useful model based on the findings from the published models is presented and the key challenges in this regard are highlighted.

2. Constitutive models for MSW

2.1. Babu, Reddy & Chouksey (2010)

The constitutive model proposed by Babu, Reddy & Chouksey [3] combines the settlement induced by load, creep and biodegradation of MSW in landfills. The total volumetric strain of MSW expressed in the increments of volumetric strain due elastic (*E*), plastic (*P*), creep (*C*) and biodegradation (*B*) effects is shown in Eq. (1). The elastic and plastic strain were based on the effective stress formulation of Modified Cam Clay (MCC) model. The increment in the volumetric strain induced by creep and biodegradation of MSW were adopted from empirical model proposed by Gibson and Lo (1961) and Park and Lee (1997), respectively.

$$d\varepsilon_v = \underbrace{\frac{\kappa}{1+e_0}\frac{dp'}{p'}}_{E} + \underbrace{\left(\frac{\lambda-\kappa}{1+e_0}\right)\left[\frac{dp'}{p'}+\frac{2\eta d\eta}{M^2+\eta^2}\right]}_{P} + \underbrace{cb\Delta p'^{e^{-ct}}dt}_{C} + \underbrace{E_{dg}de^{-dt}dt}_{B} \quad (1)$$

where, p' is the effective mean stress, $\eta = q'/p'$ is the ratio of deviatoric and mean effective stresses, c is the rate constant for mechanical creep, b is the coefficient of mechanical creep, $\Delta p'$ is the change in mean effective stress, E_{dg} is the total amount of strain that can occur due to biological decomposition of organic solids in the waste, d is the rate constant for biological decomposition, λ and κ are the compression and recompression index obtained from 1D compression test. Upon integrating Eq. (1), the modified expression for the deviatoric stress for MCC model is given by Eq. (2).

$$q = Mp' \sqrt{\left\{ \left[\left(\frac{p'_o}{p'} \right)^{\lambda} exp \left\{ \left[\frac{(e_o - e)}{1 + e_o} + b\Delta p' e^{-ct} + E_{dg} e^{-dt} \right] (1 + e) \right\} \right]^{\frac{1}{(\lambda - \kappa)}} - 1 \right\}} \quad (2)$$

2.2. Machado, Carvalho & Vilar (2002)

The model proposed by Machado, Carvalho & Vilar [4] assumed that the MSW essentially comprises of two components namely, the fibrous materials of the waste (composed mostly of plastic) and the MSW paste (composed of wood, food, rubber, glass etc.). The fibers were modeled with a linear elastic perfectly plastic model using the von Mises yield criterion. The MSW paste behavior was modeled by critical state framework with a non-associated flow rule. The increment in shear elastic strain of the fibers ($d\varepsilon^e_{sf}$) was calculated as a function of the increment in shear elastic strain of the paste ($d\varepsilon^e_{sp}$) (see Eq. (3)), through a mobilization function (f_m) given by Eq. (4).

$$d\varepsilon^e_{sf} = f_m \cdot d\varepsilon^e_{sp} \quad (3)$$

$$f_m = \frac{2}{\pi} tan^{-1} \left[\left(\frac{q}{p} \right)^2 \right] \quad (4)$$

A hyperbolic relationship (Eq. (5)) between the paste's void ratio (e_p) and the mean normal stress (p) was assumed for dealing with the high compressibility of the paste,

$$e_p = \frac{N}{p^{\lambda}} - 1 \quad (5)$$

where, N represents the value of the paste's specific volume for a unitary value of stress. The yield function for the paste (f_p) is given by Eq. (6),

$$f_p = q_p - M[p^n(p_o - p)]^{(1/(1+n))} n^{(1/(1+n))} = 0 \quad (6)$$

A non-associative flow rule for the plastic strain rate of the paste material was adopted as given by Eq. (7),

$$\frac{d\varepsilon^p_{vp}}{d\varepsilon^p_{sp}} = \left(\frac{1}{1+n} \right) M[p^n(p_o - p)]^{\left(\frac{1}{1+n} - 1\right)} [p^{n-1} n(p_o - p) - p^n] \times \left\{ 1 + sin \left[\frac{\pi}{2\psi^{\beta}} \left(\frac{p}{p_o} \right)^{\beta} \right] 2n \right\}^{-1} \quad (7)$$

where, $\psi = n/(1+n)$, and corresponds to the p/p_o ratio in a critical state condition and $\beta = \ln(0.5)/\ln(\psi)$ makes $\sin[(\pi/2\psi^{\beta}) \times (p/p_o)^{\beta}] = 0$ for $p = p_o$. Further extensions to the model were made to account for changes in the properties of fibers, mass loss due to degradation and undrained stress-strain behavior of MSW [5, 6].

2.3. McDougall (2007)

The mechanical model for MSW proposed by McDougall [7] accounted for the elasto-plastic strain, creep strain and biodegradation induced strain to predict the overall landfill settlement. The load induced elasto-plastic strain was determined using a total stress plane strain formulation of MCC model with an associative flow rule and the classical plastic volumetric strain hardening rule of the MCC model [7]. The creep behavior was incorporated based on the "equivalent time" model proposed by Yin and Graham [8] and this model allows for the creep strain rate of an overconsolidated material and its hardening to be related to the normal consolidation line. The creep induced hardening was given by Eq. (8),

$$\frac{dh^c}{dt} = \sigma_C\left(\frac{\chi}{\lambda-\kappa}\right)\left(\frac{t - t_C + t_{eq} + t_{ref}}{t_{ref}}\right)^{\left(\frac{\chi}{\lambda-\kappa}-1\right)} \tag{8}$$

where, dh^c is increment in yield surface tip stress corresponding to a given period of creep strain, t_{eq} is equivalent time, t_{ref} is the reference time (refer Yin and Graham, 1989). McDougall and Pyrah [9] proposed a constitutive relationship between decomposition of solid degradable fraction (i.e. a change in solid phase volume, V_S) and the induced change in the void volume (V_V) using a degradation induced void change parameter (Λ) as given by Eq. (9) and was used in McDougall [7].

$$dV_V = \Lambda dV_S \tag{9}$$

Assuming that the changes in void ratio affect the waste in the same way as the soils, a biodegradation-hardening was defined as given by Equation 10,

$$dh^d = \Omega(e - \Lambda)\frac{dV_S}{V_S} \tag{10}$$

where, dh^d is the increment in yield surface tip stress due to biodegradation, Ω (a constant, in kPa) is a decomposition hardening multiplier that relates the magnitude of tip stress increments to increments of strain.

2.4. Hubert, Liu & Collin (2016)

The mechanical model adopted by Hubert, Liu & Collin [10] is a simplified version of the chemo-hydro-mechanical (CHM) model presented by Liu et al. [11]. A concentration parameter (α), varying between 0 and 1 based on the organic matter content at a specific time, was used to simulate the effect of the biodegradation on the mechanical behavior of MSW. Based on the value of the organic content and subsequently the concentration parameter the mechanical model determines the softening/hardening and thereby the MSW settlement. The strain rate for the calculation of the load induced settlement was expressed as the sum of an elastic reversible part and a plastic irreversible part. The elastic part was further decomposed into mechanical and chemical components. The elastic strain rate was given by the Hooke's law, while the chemical elastic strain rate was given by Eq. (11),

$$\dot{\varepsilon}_{ij}^{e,c} = \frac{1}{3} F_0 \beta_0 e^{\beta_0 (1-\alpha+\ln\alpha)} \left(\frac{1}{\alpha} - 1\right) \tag{11}$$

where F_0 and β_0 are material constants dependent on the soil and the chemical. The plastic strain rate was defined using three yield mechanisms (criteria) namely, pore collapse, frictional-cohesive failure and tensile failure given by f_1, f_2, and f_3, in Eq. (12), (13) and (14), respectively.

$$f_1 = q^2 + M^2(p + p_s)(p - p_0) = 0 \quad ; \quad p \geq (p_0 - p_s)/2 \tag{12}$$

$$f_2 = q - M(p - p_s) = 0 \quad ; \quad \sigma_t < p \leq (p_0 - p_s)/2 \tag{13}$$

$$f_3 = p + \sigma_t = 0 \tag{14}$$

where, p_s is a parameter related to the cohesion and σ_t is the limit tensile strength. The long-term large deformation due to biodegradation was considered by introducing chemical softening controlled by α. As the organic content decreases the α value increases to a value closer to 1 which leads to an increase in the chemical softening induced. The parameter α is linked to both the preconsolidation pressure (given by Eq. (15)) and the cohesion parameter (given by Eq. (16)) in order to describe the biodegradation effect on f_1 and f_2 mechanisms, respectively.

$$p_0(\alpha) = p_0^* S(\alpha) \tag{15}$$

$$p_s = p_s^* + k_\alpha \alpha \tag{16}$$

where p_0^* and p_s^* is value of the p_0 and p_s for initial organic content ($\alpha = 0$), $S(c) = \exp(-a\alpha)$ is the chemical softening function, a is a constant governing the decrease of p_0 with the increase in α, k_α is a model constant.

2.5. Feng, Cao, Bai & Yin (2016)

The constitutive model proposed by Feng, Cao, Bai & Yin [12] incorporates the effect of biodegradation on the volumetric strain of MSW by incorporating the biodegradation induced void change parameter (Eq. (9)) first proposed by McDougall and Pyrah [8]. The extent of degradation in MSW (D_d) was calculated as the difference between the initial relative cellulosic content (defined as the ratio of the total mass of cellulose and hemicellulose to the mass of lignin) denoted by C_i and the C at any time t to the C_i. Based on the constitutive relationship given in Eq. (9) and D_d, the biodegradation induced volumetric strain was determined. The elastic and plastic strain induced by load from the overburden waste was based on the MCC model with the stress dilatancy effect. Thus, the expression for the total volumetric strain including the elastic, plastic and biodegradation components is expressed by Eq. (17),

$$d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p + d\varepsilon_v^b = \frac{\kappa}{1+e_0}\frac{dp'}{p'} + \frac{\lambda-\kappa}{1+e_0}\left(\frac{dp'}{p'} + \frac{2\eta d\eta}{M^2+\eta^2}\right) - \frac{(1+\Lambda)m_i}{V_i \rho C_i} dC \tag{17}$$

The total volumetric strain was further expressed in terms of its volumetric and deviatoric components along with the strain components corresponding to the biodegradation effects to express the total strain in a matrix form that can be readily implemented in a numerical method solver [12]. It should be noted that the authors used a first order decay type function to depict the variation of the relative cellulosic content (*C*) with time as the MSW biodegrades.

2.6. Lu, Xue, Huang & Lim (2018)

The constitutive model proposed by Lu, Xue, Huang & Lim [13] for elasto-plastic deformation incorporated a nonlinear yield function with a shear hardening flow rule to accurately simulate the stress-dilatancy behavior typically exhibited by MSW under applied stresses. The yield function was given by a nonlinear function (Eq. (18)),

$$f = q + Mp^{\xi} = 0 \tag{18}$$

where, M is the hardening function, ξ is the strength parameter that defines the nonlinearity of the yield function. The shear hardening rule for the evolution of the state parameters under the plastic loading was given by Eq. (19),

$$\varepsilon_s^p = \frac{p}{h_s G}\left[M + M_f \ln\left(1 - \frac{M}{M_f}\right)\right] \tag{19}$$

where, G is the elastic shear modulus, M_f defines the condition at critical state, ε_s^p is plastic shear strain, h_s is a fitting parameter. The plastic potential that defines the flow rule is given by Eq. (20),

$$Q = q - \frac{AM_c p}{1-A}\left[1 - \left(\frac{p}{p_0}\right)^{-(1-A)}\right] = 0 \tag{20}$$

where, A is a material constant obtained by fitting the experimental curve of volumetric strain and axial strain, M_c is a stress-dilatancy parameter that corresponds to the transition of volumetric behavior from compressive to dilative.

3. Discussion on model limitations

The constitutive models discussed above have all proposed the mathematical interpretation of the mechanical behavior of the waste with varying complexity. However, these models do have some limitations based on the assumptions made in regard to the actual behavior of MSW. For example, Babu et al. [3] incorporated the creep and biodegradation effects into the settlement prediction. But, the fundamental formulation of those aspects itself was based on simplified empirical behavior (exponential functions) with the model parameters of no physical meaning. Thus, the general applicability of the model is questionable. The model proposed by Machado et al. [4], although being more comprehensive about the strength and elasto-plastic deformation, is quite parameter intensive which makes it impractical to determine their values and also complex for numerical implementation let alone the various assumptions made with regard to the fiber and paste components of the MSW. Moreover, the mass loss of organic solids and the corresponding relationships of the variation of the fiber characteristics with time were based on simplified assumption of the first order decay model. McDougall [7] assumes that MSW in landfills is mostly in the drained condition, which may not be true in cases that involve leachate injection. Although the landfill settlement is a large strain problem, but the model accounts for only small strains with a 1D settlement for creep and biodegradation. Further, the model doesn't exclusively address the effect of degradation on strength characteristics of the waste. Hubert et al. [10] used the CHM model that was proposed for soils with chemical constituents. However, the authors did not justify its applicability to predict the mechanical behavior of MSW by validating it with any experimental data. The model also doesn't account for creep induced strain in the settlement prediction. Further, the variation of model parameters with degradation were assumed to follow simple functions with no sound basis. Feng et al. [12] utilized the fundamental phase relationship developed by McDougall and Pyrah [8] to define the effect of biodegradation on the physical properties. But it doesn't really capture the typical characteristic response of the MSW (concave upwards curve at large strains) in a triaxial compression test. This may be due to inadequate formulation of the nonlinearity in the strain hardening behavior of MSW. Lu et al. [13] did incorporate a nonlinear shear hardening model but focused essentially on the elastoplastic settlement with no regard to the long-term compression due to creep and biodegradation of MSW.

4. Summary and key challenges

It is observed from the review, that a good model for MSW should adequately address the elasto-plastic behavior, the creep and the biodegradation of the MSW for accurate representation of the mechanical behavior of the MSW. It is also found important that the constitutive model incorporates the interrelated behavior of shear strength and deformation of MSW. In particular, the MCC model as observed from the review and from the validation of the model with the experimental data is found to be a reasonably good model as a first step to simulate the elasto-plastic behavior of MSW. However, a non-associative flow rule and a non-linear hardening rule like the one proposed by Machado et al. [4] and Lu et al. [13] is much needed to capture the characteristic stress-strain curve of MSW. The fundamental phase relationship based on the depletion of organic solids in waste with time which was developed by McDougall and Pyrah [9] appears to be a novel yet substantiated approach to address the effects of biodegradation on the long-term compression characteristics of waste pertaining to biodegradation. In this regard, the approach used by McDougall [7] of integrating the hardening effect induced by degradation into the volumetric strain is a more realistic approach. The creep behavior is an essential component of the mechanical model of MSW and there was only one model [7] that comprehensively defined and integrated the creep behavior into their mechanical model. The equivalent time model which was developed and used for clay soils is also found suitable for MSW and could readily be used to simulate the creep settlement.

From a practical standpoint, there are many challenges in order to develop and realize the right model for accurate simulation of the mechanical behavior of MSW. It is quite essential that the developed model be less parameter intense and be developed based on readily obtainable properties of MSW, while still be reasonably able to capture the stress-strain response of MSW. The model parameters that do not hold any specific physical meaning should have a database developed based on its application to different laboratory and field experimental data to ensure their practical validity. The model should be transparent and user friendly since it is finally anticipated to be used by landfill design professionals and practitioners. The numerical implementation of the model and the computational efforts (costs and time) are other broader challenges that must be addressed as the model is developed. Further to help develop the understanding of the modeling of the mechanical behavior of MSW, a modeling challenge similar to the one proposed by Beaven et al. [14] would trigger the importance of the modeling of this aspect, exchange and evaluate the modeling approaches from different research groups and identify areas that need further investigation.

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