The paper was published in the proceedings of XVI Pan-American Conference on Soil Mechanics and Geotechnical Engineering (XVI PCSMGE) and was edited by Dr. Norma Patricia López Acosta, Eduardo Martínez Hernández and Alejandra L. Espinosa Santiago. The conference was held in Cancun, Mexico, on November 17-20, 2019.

N.P. López-Acosta et al. (Eds.)

doi:10.3233/STAL190106

Effect of Unsaturated Soil Zone on Slope Stability

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Abstract. An unsaturated zone or a negative pore-water pressure state in the soil is present in slopes exposed to the environment. The objective of this study is to demonstrate the changes in the computed factor of safety of a slope when the unsaturated soil zone is considered or ignored. The proposed analysis is conducted using the general limit equilibrium method. The analysis considered a two-dimensional slope of a typical embankment section assuming a simplification of pore-water pressure as hydrostatic distribution, defined through a conventional seepage analysis. The shear strength is evaluated using a non-linear equation for unsaturated soils and the Mohr-Coulomb criterion for saturated soils. The results show that the factor of safety is underestimated when the negative pore-water pressure of unsaturated soil is neglected. In contrast, if the unsaturated zone is involved, the factor of safety is higher because soil suction increases the shear strength of the soil, improving the slope stability.

Keywords. Factor of safety, slope stability, soil suction, limit equilibrium method, unsaturated soil.

1. Introduction

Evaluating the stability of a slope depends on the shear strength of the soil. When the soil is saturated, the effective shear strength parameters (cohesion and internal friction angle) are used to describe its behavior. However, when the soil is unsaturated, the effective shear strength parameters, as well as the constitutive laws for saturated soils, are insufficient. A particular case is a steep slope with a deep phreatic surface that, when having low shear strength parameters, remain stable.

Several studies note that in unsaturated soils, the water forms menisci between the contacts of the solid particles of the soil. These menisci induce additional contact forces that increase the shear strength of the soil [1]. To date, some works have demonstrated the application of unsaturated soil mechanics in geotechnical engineering [2] and the effect of negative pore-water pressure on earth structures [3]. However, in engineering practice, neglecting the effects of negative pore-water pressure is common, where slope stability analyses are not the exception. Fredlund et al. (2012) [4] provide some explanations for why engineers may not feel comfortable considering negative pore-water pressure when performing slope stability analyses. Consequently, negative pore-water pressure may be omitted because: i) shear strength theory for unsaturated soils is

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not well known, ii) commercial program codes do not usually include the behavior of unsaturated soils, and iii) a perception exists that negative pore-water pressure cannot be maintained in the long term.

Based on these explanations, this study evaluates the effect of negative pore-water pressure on slope stability analyses through discretization of the soil shear strength components. The negative pore-water pressure distribution is simplified by assuming a hydrostatic distribution. In addition, performed analyses are entirely deterministic. The results highlight the differences in the factors of safety when the unsaturated soil zone is either considered or neglected by applying unsaturated or saturated shear strength criteria, respectively.

2. Theoretical concepts

2.1. Shear strength of soils

The shear strength of saturated soils can be described by the Mohr-Coulomb criterion as given by the following equation [5]:

$$\tau_f = c' + (\sigma - u_w)_f \tan\varphi' \tag{1}$$

where τ_f is the shear strength, c' is the effective cohesion, $(\sigma\text{-}u_w)_f$ is the normal effective stress, σ is the normal stress, u_w is the pore-water pressure, and φ' is the effective angle of internal friction.

In unsaturated soils, the shear strength can be described using one of two approaches: a) effective stresses [6-8] or b) independent stress state variables (i.e., net stress and matric suction stress) [9, 10, 11, 12].

A general criterion describing the shear strength of an unsaturated soil from the Soil-Water Characteristic Curve (SWCC) is given by the following equation [10].

$$\tau_f = c' + (\sigma - u_a)_f \tan\varphi' + (u_a - u_w)_f \Theta_{n_V} \tan\varphi' \tag{2}$$

where $(u_a\text{-}u_w)$ is the matric suction, u_a is the pore-air pressure and Θ_{nV} is the normalized volumetric water content.

The source of the shear strength is the attractive forces that act among the surface of the soil particles. An understanding of the soil shear strength involves knowing the factors that influence the interaction between two surfaces at their points of contact. When the interaction effect is proportional to the normal force that is pushing two soil particles together, it is named frictional strength (second term of the equations 1 and 2). When shear resistance between particles is independent of the normal force, it is said there is a cohesion strength between particles (first term of the equations 1 and 2). This cohesion can develop between soil particles that have remained in stationary contact over a long period. In some cases, this cohesion strength can be very important, as when cementation turns sand into sandstones. However, the magnitude of cohesion strength between particles usually is very small, and its contribution to the total shear strength is small. Finally, there is an apparent cohesion resulting from the water tension between the particles that pulls the particles together when the soil tends to dry. This apparent cohesion is in fact a part of the friction resistance, as the effective stress is enhanced by the tension in the water (third term of the equation 2) [13, 14].

2.2. General limit equilibrium method

The general limit equilibrium method (GLE) is a general theory used to analyze slope stability. This method modifies the shear force equation to consider the matric suction of the unsaturated soil zone. Therefore, the factor of safety with respect to moment and force equilibriums can be defined by the following two equations, respectively:

$$FS_m = \frac{\sum(c\beta + N\tan\varphi' - u_a\beta\tan\varphi')R}{A_L a_L + \sum Wx - \sum Nf} \tag{3}$$

$$FS_f = \frac{\sum(c\beta + N\tan\varphi' - u_a\beta\tan\varphi')\cos\alpha}{A_L + \sum N \operatorname{sen}\alpha} \tag{4}$$

where c is the total cohesion, β is the sloping distance across the base of a slice, N is the total normal force mobilized on the base of each slice, R is the radius for a circular slip surface, $A_{(L,R)}$ is the resultant external water forces; $a_{(L,R)}$ is the perpendicular distance from the resultant external water force to the center of rotation, W is the total weight of a slice, x is the horizontal distance from the centerline of each slice to the center of rotation, f is the perpendicular offset of the normal force from the center of rotation and α is the angle between the tangent to the center of the base of each slice and the horizontal.

3. Application

The assessment of the factor of safety considered a deterministic pseudo-coupled seepage-slope stability analysis. The SVFlux and SVSlope [15, 16] softwares were used in this study to analyze seepage and slope stability, respectively. Seepage analysis was employed to define the water table and establish a hydrostatic distribution of pore-water pressure in the slope. The proposed analysis considered a two-dimensional slope of a typical embankment section with a height of 20 m and an angle inclination of 30° (Figure 1). Hydraulic heads of 25 m and 15 m were assigned upstream and downstream of the embankment geometry, respectively, to define the water table position. A zero flux at the base and a potential seepage face onto the slope surface were applied.

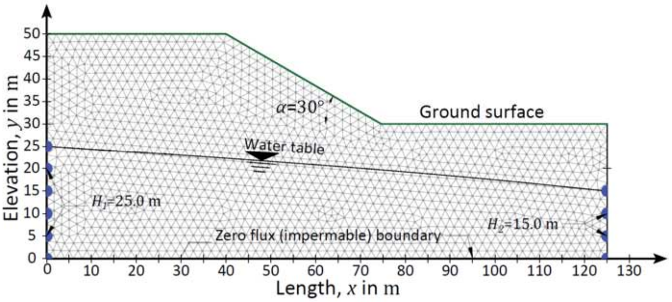


Figure 1. Geometry and boundary conditions of a typical embankment.

3.1. Characterization of the material

A homogenous and isotropic soil with a unit weight of 20 kN/m^3 was assumed. The unsaturated zone in the slope was described by an SWCC curve associated with a fine material (Figure 2). The following equation of Fredlund and Xing (1994) [17] was used to model the SWCC curve:

$$\theta_w(\psi) = \theta_{sat}\left[1 - \frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{\psi}{h_r}\right)}\right]\left\{\frac{1}{\ln\left[e+\left(\frac{\psi}{a_f}\right)^{n_f}\right]^{m_f}}\right\} \tag{5}$$

where $\theta_w(\psi)$ is the Soil-Water Characteristic Curve, θ_w is the volumetric water content, and ψ is the soil suction. The θ_{sat}, which is the saturated volumetric water content, was set to 0.41 m^3/m^3, and a_f, which is a fitting parameter closely related to the air-entry value (ψ_{aev}), was set to 100 kPa. Finally, n_f and m_f are fitting parameters with a 1.0 value, and h_r is another fitting parameter related to residual soil suction (ψ_{res}) and was set to 1000 kPa.

The shear strength of the soil was established using the non-linear model of Vanapalli et al. (1996) for unsaturated soils (see equation 2) [10] and the classical Mohr-Coulomb criterion for saturated soils as given by equation 1. The effective shear strength parameters were assumed to be 10 kPa and 34° for the effective cohesion and effective angle of internal friction, respectively. Figure 2 shows the relationship between the SWCC curve and suction strength using the third term in equation 2. Because of the effects of negative pore-water pressure, the shear strength estimated by equation 2 increased gradually until the residual suction (ψ_{res}) was reached. This increase represents the apparent cohesion resulting from water tension between the soil particles that pulls the particles together. When the value of the residual suction was exceeded, the suction strength of the soil decreased until the Mohr-Coulomb envelope was obtained. In this case, because the Mohr-Coulomb criterion does not consider negative pore-water pressure, the shear strength remained constant for all suction values.

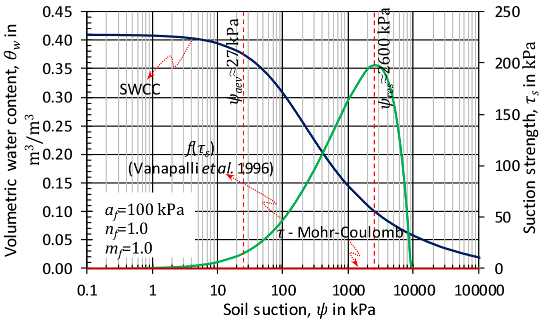
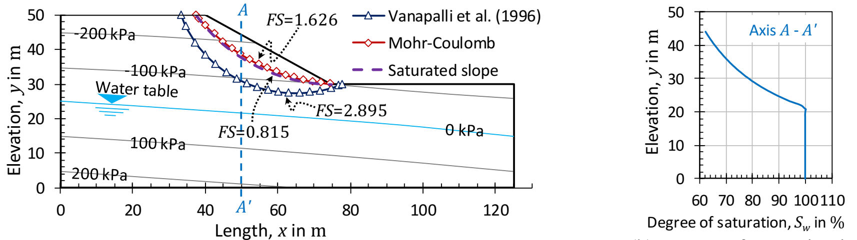


Figure 2. Relationship between soil-water characteristic curve and suction strength.

3.2. Critical slip surface

Figure 3 illustrates the minimum factors of safety and critical slip surfaces evaluated using the equations given by Vanapalli et al. (1996), the Mohr-Coulomb criterion and assuming a fully saturated slope. In this study, the minimum factor of safety estimated by Vanapalli et al. (1996) was 2.895 and 1.626 when using the Mohr-Coulomb criterion. The minimum factor of safety given by Vanapalli et al. (1996) increased almost twice as much as the minimum factor of safety estimated by the classical Mohr-Coulomb criterion. On the other hand, when the slope is assumed to be fully saturated (i.e. the water table is equal to natural ground surface), the factor of safety was reduced to 0.815.

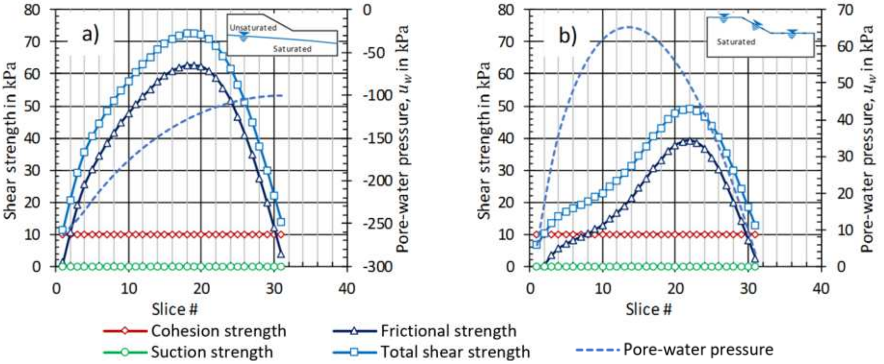


(a) Critical slip surfaces and hydrostatic pore-water distribution

(b) Degree of saturation in axis A-A'

Figure 3. Analyzed embankment section.

Discretization of the shear strength along the slip surfaces allowed to analyze the effect of the unsaturated soil zone on slope stability. Thus, Figure 4a shows that the total shear strength estimated using the Mohr-Coulomb criterion mainly depends on the frictional strength. Therefore, the critical slip surface tends to be superficial. Because the cohesive strength was low, it did not contribute considerably to slope stability. Consequently, the factor of safety was underestimated because of the classical Mohr-Coulomb criterion does not consider the negative pore-water pressure distribution of the unsaturated zone. When the slope is assumed to be saturated, the critical slip surface is approximately the same as that estimated by the Mohr-Coulomb criterion; however, due to the positive pore-water pressures that reduce the effective stress of frictional strength component, the total shear strength and the factor of safety are considerably reduced (Figure 4b).



(a) Saturated-unsaturated slope

(b) Saturated slope

Figure 4. Shear strength estimated by Mohr-Coulomb equation at the base of the slices of the critical slip surface.

Figure 5 shows the shear strength acting along the critical slip surface as estimated by the model of Vanapalli et al. (1996). According to this model, the slip surface is deeper as a result of the additional cohesive forces that occur in the unsaturated soil zone in the slope. The results obtained using the equation of Vanapalli et al. (1996) revealed that the slip surface is deeper than that derived from the Mohr-Coulomb criterion because of the additional cohesive forces that develop in the unsaturated soil zone in the slope. Thus, the effect of the unsaturated zone (i.e. the suction strength) on slope stability is evident in terms of the factor of safety value, which better represents the actual conditions of the slope.

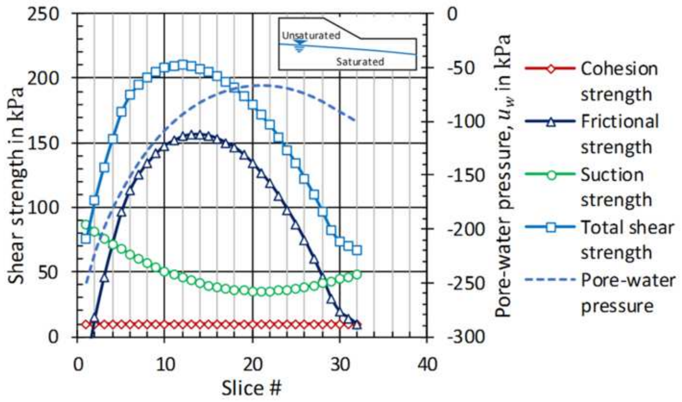


Figure 5. Shear strength estimated by the equation of Vanapalli et al. (1996) at the base of the slices of the critical slip surface.

3.3. Fixed slip surface

Figure 6 shows a fixed slip surface analyzed using the criteria of Mohr-Coulomb and Vanapalli et al. (1996), where the factors of safety are 2.269 and 3.045, respectively.

Figure 7 shows the discretization of the shear strength of the soil acting along a fixed slip surface. As in the previous case of a critical slip surface, the negative pore-water pressure may or may not be included depending on the considered equations. Thus, because the Mohr-Coulomb criterion does not consider the effects of the unsaturated soil zone, the shear strength is underestimated, and the factor of safety tends to be lower.

In contrast, when the equation of Vanapalli et al. (1996) is applied, the unsaturated zone is involved. Therefore, the shear strength and factor of safety are higher than those estimated when using saturated shear strength models.

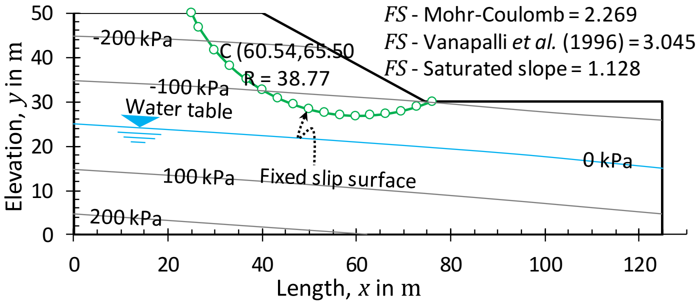


Figure 6. Fixed slip surface and hydrostatic pore-water distribution into the analyzed slope.

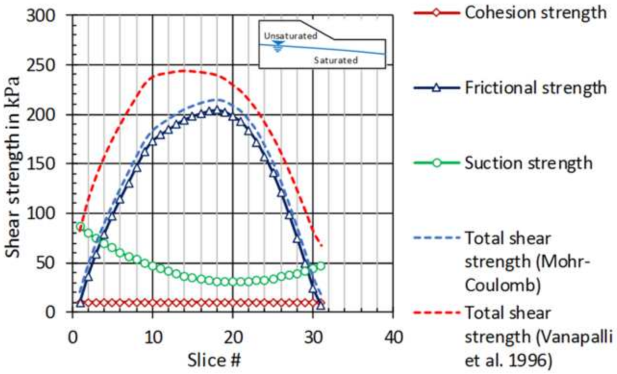


Figure 7. Shear strength estimated using the equations of Mohr-Coulomb and Vanapalli et al. (1996) for a fixed slip surface.

4. Conclusions

In this study, slope stability analyses were performed considering the negative pore-water pressure of the unsaturated zone of the soil. In these analyses, the pore-water pressure distribution was simplified by assuming a hydrostatic distribution. A typical embankment section of a two-dimensional slope with a hydrostatic pore-water pressure distribution defined through a conventional seepage analysis was evaluated. Based on the results obtained, the following conclusions stand out:

- The shear strength of saturated soil has two properties: i) frictional strength, and ii) cohesion strength. In contrast, an unsaturated soil, in addition to having the two properties mentioned above, has the property of suction strength, which depends on the negative pore-water pressure that develops in the soil.
- Neglecting the effect of negative pore-water pressure on slope stability analysis with a deep phreatic surface underestimates the factor of safety, which is unrepresentative of the real conditions developed in the soil structure. In this case, the unsaturated soil theory can provide a more realistic factor of safety.
- Soil suction or negative pore-water pressure directly affects the shear strength of unsaturated soils, which in turn affects the factor of safety in slope stability analyses.
- Suction strength is generally not included in common engineering design, but it can be essential to analyze and understand some geotechnical problems that cannot be described by classical theory of saturated soils.
- Soil suction is present in earth structures. Therefore, its inclusion in a study of any geotechnical structure related to an unsaturated zone of the soil provides results that are more accurate and realistic.
- In slope stability analyses of steep slopes and deep water table levels, the shear strength may be influenced by the suction strength. Therefore, conducting these analyses based on unsaturated soil mechanics concepts is desirable. Analyses using classical shear strength criteria can provide results that do not fully represent these concepts.
- This work highlighted the effect of soil suction on slope stability as a product of the distribution of hydrostatic pressures. However, soil suction may be derived

from different environmental conditions such as rainfall, evaporation, vegetation cover, evapotranspiration, and solar radiation. This issue should be investigated in future studies. In addition, the effect of changes in infiltration due to rainfall on factor of safety must be further studied. These changes could be considered in the analyses by means of stochastic techniques.

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