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Influence of the Variability of Geotechnical Parameters in Cantilever Retaining Walls Design

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Abstract. Geotechnical design of earth retaining structures, such as retaining walls, traditionally has been based on a deterministic methodology using fixed Factors of Safety (FS), which does not take into account the intrinsic variability of the soil's behavior. In order to estimate the uncertainty in the soil's response, an analysis of the effect of the variability of the geotechnical parameters in the design of retaining walls has been developed, finding that, due the high variability of the geotechnical parameters of the retained and foundation soils, a deterministic analysis may underestimate or overestimate the loads and resistances considered in the process of the retaining wall design. Based on the above consideration, a simplified alternative design approach, based on failure probabilities (instead of a fixed FS), is presented. This approach can lead to better retaining wall designs, which could be less expensive while maintaining the required level of safety and reliability.

Keywords. Retaining wall, probabilistic design.

1. Introduction

The geotechnical design of retaining walls is generally based on the geotechnical parameters of the soils that interact with it (i.e. retained soil and foundation soil). The nature of the estimation processes of these geotechnical parameters does not allow obtaining exact values of these since there is variability, which according to Baecher [1], can be natural or epistemic.

According to the traditional approaches for the design of retaining walls, it is necessary to use deterministic values of the geotechnical parameters (usually "average" values, or "more unfavorable" values), which do not take into account the variability of the parameters, nor the influence that these could have on the performance of the structure. Due to the fact that it is increasingly necessary to design safer and less expensive structures, a probabilistic analysis may be an adequate alternative, in the sense that it allows to optimize the design according to the importance of the structure, consequences of failure, construction costs, etc.

In that sense, this paper presents a brief introduction to the simplified probabilistic design of retaining walls, including equations that allow the development of such probabilistic analysis based on a deterministic analysis. Due to the great variety of soil

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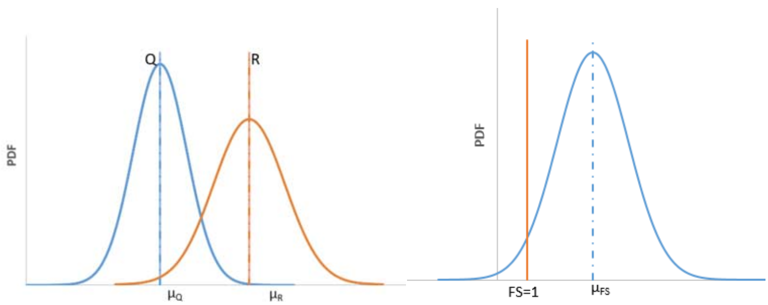
types and existing walls, only a particular case was analyzed. Cantilever retaining walls, founded over a cohesive over-consolidated soil, and retaining a granular fill, have been chosen for this study.

2. Basic concepts for probabilistic design

2.1. Reliability and factor of safety

In the deterministic design approach, the parameters that express the risk condition of a structure are usually the Factors of Safety (FS), which are defined as the quotient between the resisting forces of a structure, and an estimated value of acting forces or effects, for certain stability conditions. In the case of retaining walls, the usual stability conditions analyzed are sliding, overturning and bearing capacity failure [2]. A limitation of the FS lies in that it does not take into account the variability of the input parameters, nor the importance of the structure.

On the other hand, if we take into account that the values of the acting forces and resisting forces in a structure are not deterministic, we can affirm that they may be described by means of probability density functions, as shown in Figure 1.



(a) (b)

Figure 1. (a) Probability density function corresponding to Loads (Q) and Resistances (R). (b) Probability density function corresponding to the FS.

As shown in Figure 1 (a), the fact that the average values of resistance are greater than the average values of imposed loads, does not imply that there is no probability of failure, which in this case is defined as the probability of Q being greater than R. Also, Figure 1 (b) shows the probability density distribution of the FS; in this case the probability of failure may be defined as the probability that the FS is less than one.

In that sense, if we assume that the probability distributions of the loads, resistances, or FS, can be described by means of a Normal Distribution, the probability of failure could be estimated by means of the quantification of the mean and its corresponding standard deviation.

2.2. Monte Carlo Method

The concept of this method starts from the basic concept of probability, because it calculates the probability of occurrence of an event performing the "experiment" a sufficient number of times, and determining the distribution of the dependent random variable as a density function of the results obtained in the "experiments" carried out [3]. In other words, if the probability distribution of the random input variables (X1, X2, ..., Xn) is known, a random sampling of those variables is performed, and the dependent variable "F" is evaluated deterministically; then, the previous process is repeated a sufficiently large number of times to achieve the convergence of the probability distribution of the dependent random variable.

3. Deterministic and probabilistic design of retaining walls

3.1. Characteristics of the retaining walls considered in this study

Due to the wide variety of soil types and existing retaining walls, cantilevered retaining walls have been chosen for this study, with the following characteristics (see Figure2).

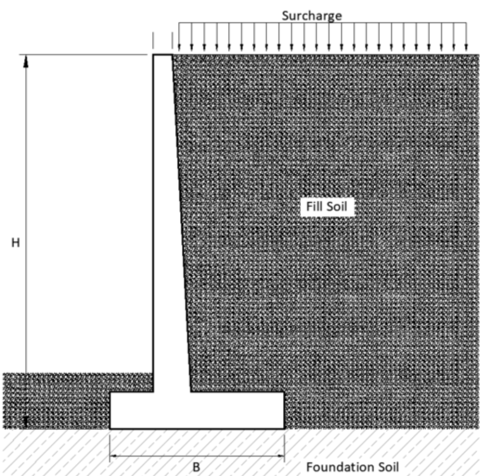


Figure 2. Characteristics of the structure under study.

- Cohesive foundation soil.
- Granular retained soil.
- Heights (H) of 4, 5, 6, and 7 m.
- Values of the undrained shear strength correspondent to the foundation soil (Su), between 98 and 294 kN/m^2.
- Stiff and over-consolidated foundation soil (settlement is not analyzed).
- Surcharge value of 2.5 kN/m^2.
- Specific weight of the retained soil (γ) equal to 18.6 kN/m^3.
- Deep water table.

3.2. Deterministic design

The deterministic design of the retaining walls has been developed in order to determine the average value of the width “B” of the wall foundation, which satisfies the Factors of Safety for the analyzed stability conditions. It is important to clarify that, for the present analysis, gravity forces, earth pressure (using the Rankine theory for active and passive earth pressure), and the effects of surcharge have been considered.

As indicated by Bowles [4] [5], the stability of the walls has been verified considering the following criteria:

- Overturning Stability. The stability of the wall has been verified against the forces that can cause it to rotate. The considered criterion of deterministic stability is $FS_v \geq 1.5$. (FS_v = Factor of Safety against failure by overturning).
- Sliding stability. The stability of the wall has been verified against the forces that can cause it to slide. The considered criterion of deterministic stability is $FS_d \geq 1.5$. (FS_d = Factor of Safety against sliding failure).
- Verification of bearing capacity. The stability of the wall has been verified to ensure that the foundation conditions are met, that is, that the stresses transmitted to the ground are lower than its allowable stress (defined by its shear strength). It is important to note that in this section, the bearing capacity has been estimated using the Meyerhof equations, considering the corrections for load inclination and eccentricity as non-superimposed effects. That is, the effects of the horizontal load and the eccentricity are taken into account separately; therefore, the deterministic stability criteria can be written as: $FS_{bc-e} \geq 3$, and $FS_{bc-i} \geq 3$. ($FS_{bc-e / i}$ = Factor of Safety against bearing capacity failure, eccentricity and inclination).
- It is important to mention that as part of the present work, there have not been verifications by settlements or global stability failures, which are outside the scope of the present investigation.

Taking into account the stability criteria mentioned lines above, 100 deterministic designs were developed, considering:

- Heights (H) of 4, 5, 6, and 7m.
- Values of retained soil friction angle of 30°, 32°, 34°, 36° and 38°.
- Values of the undrained shear strength (correspondent to the foundation soil) of 98, 147, 196, 245, and 294 kN/ m^2.

3.3. Probabilistic design

According to the aforementioned, the Monte Carlo Method uses as input data the probability distributions of the input values involved in the design. Then, using random sampling, it simulates the behavior of the system by generating a large number of values; that is, many deterministic designs are generated using values of input probability distributions. Then the data resulting from the multiple deterministic designs is adjusted to a probability curve. With this probability curve, the standard deviation of the FS can be estimated, which, by means of Eq. (1), allows us to estimate the probabilities of failure associated with each stability criterion.

Table 1. Ranges of coefficient of variation values for the considered soil parameters.

Parameter	Coefficient of variation	Source
Specific Weight (γ)	4% - 8%	[6]
Undrained Shear Strength (Su)	11% - 45%	[7]
Friction Angle (ϕ)	3.7% - 9.3%	[8]

Due to this paper intending to analyze the influence of the variability of geotechnical parameters on the variability of design FS, the probability curves of the following input parameters have been considered in the probabilistic model: the specific weight of the retained soil (γ), the value of the retained soil friction angle (ϕ), and the value of the undrained shear strength for the foundation soil (Su). The ranges of variability of these parameters are shown in Table 1.

Probabilistic models have been developed based on the deterministic designs described in the previous section, with the purpose of obtaining expressions (through regressions of the obtained data) that would allow to directly estimate the coefficients of variation (i.e. the values of the standard deviations normalized by the mean values) of the FS. It is important to indicate that these expressions can be used with two main objectives:

- In the design of retaining walls: if the coefficients of variation of the soil parameters are estimated, and "acceptable" failure probabilities are defined, the average values of the FS can be estimated, which can be used in the calculation of a deterministic design. In other words, a probabilistic model could be developed based on a deterministic one.
- In the verification of the stability of existing retaining walls: the probability of failure can be estimated by estimating the coefficients of variation, and the average values of the FS.

With the purpose of reducing variables and obtaining good approximations in the previously mentioned expressions, probabilistic simulations were carried out, introducing the probabilistic distribution of only one variable (γ, Su or ϕ), while the two other variables remained constant. These simulations were called "Individual Simulations". Individual simulations were carried over the 100 deterministic designs, considering the cases shown in Table 1 and resulting in a total of 1500 individual probabilistic simulations for each type of FS (overturning, sliding, bearing capacity-eccentricity and bearing capacity-inclination).

On the other hand, to estimate the variability of the FS, simulations were carried out in which the probabilistic distributions of the three variables were taken into account. These simulations were called "Group Simulations", and as in the case of "Individual Simulations", 1500 probabilistic group simulations were developed for each type of FS evaluated.

4. Expressions for the simplified probabilistic design of retaining walls

In order to obtain the expressions that would allow the estimation of the coefficient of variation of the FS (based on the variability of geotechnical parameters and the characteristics of the retaining wall), numerical regressions were used, taking as premise to obtain simple expressions and to reduce the number of variables, without this implying a significant decrease in the correlation coefficients.

4.1. Nomenclature used in the results of individual and group simulations

Due to the fact that obtained expressions (shown in section 4.2) include the results of two different types of simulations (individual and group), and to avoid confusion, the following nomenclature is used:

- cv_{γ} : Coefficient of variation of specific weight.
- cv_{Φ} : Coefficient of variation of the retained soil friction angle.
- cv_{su} : Coefficient of variation of the undrained shear strength of the foundation soil.
- $cv_{v\gamma/v\Phi}$: Coefficient of variation of the FS_v, obtained in the individual simulation, considering only the variation of the specific weight, or the variation of the friction angle of the retained soil.
- cv_v : Coefficient of variation of the FS_v, obtained in the group simulation.
- $cv_{d\gamma/d\Phi/dsu}$: Coefficient of variation of the FS_d, obtained in the individual simulation, considering only the variation of the specific weight, the variation of the friction angle of the retained soil, or the variation of the undrained shear strenght of the foundation soil.
- cv_d : Coefficient of variation of the FS_d, obtained in the group simulation.
- $cv_{be\gamma/be\Phi/besu}$: Coefficient of variation of the FS_{bc-e}, obtained in the individual simulation, considering only the variation of the specific weight, the variation of the friction angle of the retained soil, or the variation of the undrained shear strength of the foundation soil.
- cv_{be} : Coefficient of variation of the FS_{bc-e}, obtained in the group simulation.
- $cv_{bi\gamma/bi\Phi/bisu}$: Coefficient of variation of the FS_{bc-i}, obtained in the individual simulation, considering only the variation of the specific weight, the variation of the friction angle of the retained soil, or the variation of the undrained shear strength of the foundation soil.
- cv_{bi} : Coefficient of variation of the FS_{bc-i}, obtained in the group simulation.

4.2. Results and regressions of individual and group simulations

As indicated in the previous sections, for each type of simulation (i.e. individual and group simulations), 1500 probabilistic simulations were obtained for each type of FS, totalizing 6,000 probabilistic simulations (for the 4 FS types analyzed); likewise, each probabilistic simulation is made up of 5,000 deterministic iterations. Therefore, the simulations developed to obtain the expressions that allow estimating the variability of the FS are based on 60,000,000 deterministic estimates.

Expressions obtained through regressions of the individual simulations results, and which allow the estimation of the values of the "partial" coefficients of variation of the design FS, are shown below:

$$cv_{v\gamma} = 1.24 \cdot 10^{-4} \cdot \varphi - 7.96 \cdot 10^{-4} \cdot H + 0.194 \cdot cv_{\gamma} \quad (1)$$

$$cv_{v\emptyset} = 10^{-2} * (-0.22 * H^2 + 3.18 * H + 21.59) * e^{(0.03*H^2 - 0.44*H + 5.91)*10^{-2}*\emptyset} * cv_{\emptyset} \quad (2)$$

$$cv_{d\gamma} = 7.56 \cdot 10^{-2} - 6.29 \cdot 10^{-4} \cdot \varphi + 4.77 \cdot 10^{-5} \cdot Su + 2.10 \cdot 10^{-3} \cdot H + 0.45 \cdot cv_{\gamma} \quad (3)$$

$$cv_{d\emptyset} = 0.1 * e^{0.073*\emptyset} * cv_{v\emptyset} + (2.354 * H - 2.222 * \emptyset + 121.33) * 10^{-3} \quad (4)$$

$$cv_{dsu} = 4.98 \cdot 10^{-2} - 1.63 \cdot 10^{-3} \cdot \varphi + 1.21 \cdot 10^{-4} \cdot Su + 4.81 \cdot 10^{-3} \cdot H + 0.786 \cdot cv_{su} \quad (5)$$

$$cv_{be\gamma} = -4.23 \cdot 10^{-3} - 1.26 \cdot 10^{-4} \cdot \varphi + 4.70 \cdot 10^{-5} \cdot Su - 2.11 \cdot 10^{-4} \cdot H + 0.812 \cdot cv_{\gamma} \quad (6)$$

$$cv_{be\varphi} = -2.73 \cdot 10^{-1} + 8.50 \cdot 10^{-3} \cdot \varphi + 1.05 \cdot 10^{-4} \cdot Su - 7.62 \cdot 10^{-3} \cdot H + 2.97 \cdot cv_{\varphi} \quad (7)$$

$$cv_{besu} = 4.03 \cdot 10^{-3} + 2.81 \cdot 10^{-5} \cdot Su + 0.918 \cdot cv_{su} \quad (8)$$

$$cv_{bi\gamma} = 8.46 \cdot 10^{-3} - 6.84 \cdot 10^{-4} \cdot \varphi + 7.92 \cdot 10^{-6} \cdot Su + 2.32 \cdot 10^{-3} \cdot H + 0.75 \cdot cv_{\gamma} \quad (9)$$

$$cv_{bi\varphi} = -6.54 \cdot 10^{-2} + 2.59 \cdot 10^{-3} \cdot \varphi + 4.30 \cdot 10^{-5} \cdot Su - 5.67 \cdot 10^{-3} \cdot H + 1.12 \cdot cv_{\varphi} \quad (10)$$

$$cv_{bisu} = 2.48 \cdot 10^{-3} - 1.05 \cdot 10^{-3} \cdot H + 0.954 \cdot cv_{su} \quad (11)$$

Likewise, expressions obtained by means of regressions of the group simulations results (also based on the "partial" coefficients of variation), which allow the estimation of the "total" coefficients of variation of the design FS, are shown below.

$$cv_{v} = 0.123 \cdot cv_{v\gamma} + 0.992 \cdot cv_{v\varphi} \quad (12)$$

$$cv_{d} = -4.11 \cdot 10^{-2} - 5.57 \cdot 10^{-4} \cdot H + 0.152 \cdot cv_{d\gamma} + 0.608 \cdot cv_{d\varphi} + 0.902 \cdot cv_{dsu} \quad (13)$$

$$cv_{be} = 0.265 \cdot cv_{be\gamma} + 0.596 \cdot cv_{be\varphi} + 0.818 \cdot cv_{besu} \quad (14)$$

$$cv_{bi} = 0.206 \cdot cv_{bi\gamma} + 0.333 \cdot cv_{bi\varphi} + 0.937 \cdot cv_{bisu} \quad (15)$$

As an example, developed by using the above expressions, Figure 3 shows the estimated Factors of Safety for a certain cantilever retaining wall ($H = 5\,m, \mu_{\varphi} = 32°, \mu_{Su} = 196\,kN/m^2$) under two variability scenarios: Scenario 1 - $cv_{\gamma} = 4.0\%$, $cv_{\varphi} = 3.7\%$, and $cv_{Su} = 11.0\%$. Scenario 2 - $cv_{\gamma} = 6.0\%$, $cv_{\varphi} = 6.5\%$, and $cv_{Su} = 28.0\%$. It can be seen that, for the same structure and the same probability of failure, the FS can be reduced if the variability of the parameters is reduced, leading to an optimization in the design and for instance, lower construction costs.

It also worth mentioning that, as seen in Figure 3, when comparing FS values for different stability conditions, a higher FS value does not necessarily mean lower probability of failure.

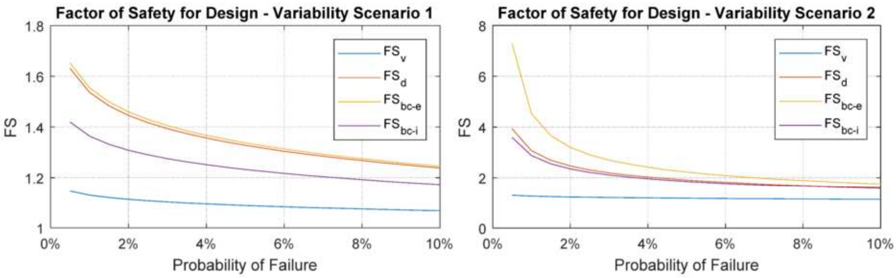


Figure 3. Estimated FS values for a certain cantilever retaining wall, under two variability scenarios.

5. Conclusions

- This paper introduces a simplified probabilistic approach for the design of cantilever retaining walls, including equations that allow the estimation of the coefficient of variation of the Factors of Safety for three different stability conditions. This procedure may allow the development of probabilistic designs based on deterministic designs, and also the estimation of the probability of failure of existing walls.
- By quantifying the variability of the FS, and defining an "acceptable failure probability", it is possible to estimate "adequate" FS values, which allows an optimization in the design. This optimization could be developed based on the importance of the structure, and the variability involved in the design.
- Deterministic analyzes may not be adequate to optimize the design of a retaining structure, since they do not quantify the variability of the design parameters.
- A higher FS for a certain stability condition does not guarantee that its probability of failure would be lower compared to another stability condition with a lower FS.

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