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Analytical Solutions for Tunnel Excavation Stability Considering Linear Variation of Undrained Shear Strength with Depth

Luis Eduardo SOZIO[a,1]
[a]*Civil Engineer*

Abstract. Tunnel collapses are often reported to occur during excavations, or shortly thereafter. Yet it is felt that excavation stability analyses are rarely performed in tunnel design. This paper presents analytical solutions for stability analyses applicable to conventional and mechanized tunnel excavation methods, focusing on a linearly increasing undrained shear strength with depth. These models are derived from equilibrium equations, in cylindrical or spherical coordinates (for 2D or 3D analyses), and Mohr Coulomb strength criterion, being applicable to both collapse and blow out types of failure. The main limitations of the analytical models are outlined. Comparisons with published and numerical method results are included, and an example application is given comparing the analytical solution to Finite Element analyses. The analytical model is straightforward and adequate to be used at preliminary stages of a design. Due to its limitations the analytical model is recommended to be complemented by more accurate numerical methods as design progresses into detailed stages.

Keywords. Tunnel, excavation, stability, undrained shear strength

1. Introduction

Tunnel excavation accidents often result in serious consequences, including loss of life and major disruptions during construction. A brief investigation into available tunnel stability methods reveals lack of agreement on a standard method. The absence of an established methodology could explain the fact that stability analyses are seldom included as a tunnel design criteria. However, a general simple analytical model initially developed by Caquot [1] has shown to hold results that are in reasonable agreement with those obtained from numerical methods, such as the Finite Element Method, and approximately concurring with laboratory physical model tests. Several Authors have followed and further developed the original Caquot model, as in Jaeger & Cook [2], Atkinson & Potts [3] Davis et al. [4], Mühlhaus [5], Sozio [6, 7], and Carranza [8].

A specific terminology for this model is usually employed, namely Thick Wall Cylinder (TWC) or Thick Wall Sphere (TWS), for 2D or 3D arrangements respectively.

The objective of this paper is to present a particular form of the analytical model, which is applicable to a linearly increasing undrained shear strength with depth, for collapse (cave in) and blow out (heave) types of failure, in both two-dimensional (2D) and three-dimensional (3D) geometrical configurations.

[1] Corresponding Author, Av. Washington Luis 1576, BL. E Ap.111, 04662-002 Sao Paulo Brazil.

2. Development of the analytical model

Caquot's [1] original model is applicable to a general cohesive frictional material $(c, \emptyset)$ for a two-dimensional configuration. Further developments of this method (by authors quoted at the introduction) included a three-dimensional emulation of a tunnel heading, for both drained $(c', \emptyset')$ and undrained (Su) behavior. Radial fluid flow was later incorporated into the model to allow for an effective stress analysis including groundwater seepage gradients (of interest in $c', \emptyset'$ analyses). Sozio [7] showed that different material layers could be included into the analytical model, and solved through a simple spreadsheet. Thus, such model provides stability estimates for collapse or blow out types of failure, in 2D and 3D arrangements, drained and undrained layers, including groundwater flow effects (for drained layers). A specific development for linearly increasing undrained shear strength is the objective of this paper. A geometrical 3D arrangement is shown at Figure 1, and it should be noted that for 2D analyses, $R_i = 0.5D$.

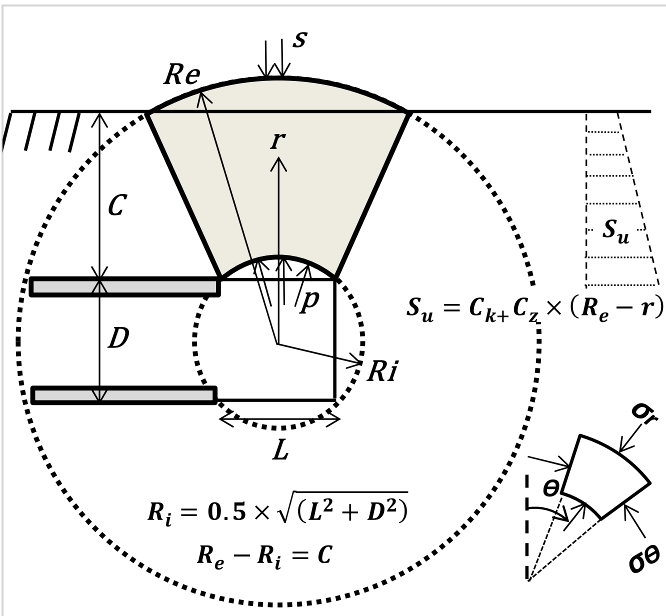

Figure 1. Geometry and symbols, analytical model, 3D arrangement.

2.1. Equilibrium equation and strength criterion

- Equilibrium equation in radial direction, for cylindrical (2D) and spherical (3D) arrangements, refer to Poulos & Davis [9] Chapter 1 – Fundamentals Definitions and Relationships, including a radial body force emulating gravity:

$$\frac{d\sigma_r}{dr} + (n-1)\frac{(\sigma_r - \sigma_\theta)}{r} = -\gamma \tag{1}$$

where σ_r=radial stress, σ_θ=tangential stress, γ=radial body force, emulating soil unit weight; and n=2 or 3 for cylindrical (2D) or spherical (3D) arrangements respectively.

- Undrained shear strength linearly increasing with depth:

$$S_u = C_k + C_z \times (R_e - r) \tag{2}$$

where C_k=initial shear strength value, applicable at surface level (distant R_e from tunnel center); C_z=rate of variation of shear strength with depth (where depth is given by (R_e-r).

- Mohr-Coulomb strength criterion in terms of principal stresses:

$$\sigma_\theta = \sigma_r \pm 2\big(C_k + C_z \times (R_e - r)\big) \tag{3}$$

where ± applicable to collapse or blow out failures, respectively.

- Combination of Eqs. (1) and (3), and rearrangement:

$$d\sigma_r + (n-1)(dr/r)\big(\mp 2(C_k + C_z \times R_e)\big) + (\pm 2C_z \times n + \gamma)dr = 0 \tag{4}$$

where ± applicable to collapse or blow out failures, respectively.

- Integration of Eq. (4):

$$|\sigma_r|_{R_i}^{R_e} + (n-1)\big(\mp 2(C_k + C_z \times R_e)\big)|ln|_{R_i}^{R_e} + (\pm 2C_z \times n + \gamma)|r|_{R_i}^{R_e} = 0 \tag{5}$$

where σ_r=s is the surface stress at r=R_e, σ_r=p is the minimum stress (collapse), or maximum stress (blow out) at r=R_i and p is the minimum support pressure or maximum blow out pressure in the tunnel.

2.2. Limiting pressure equation

Eq. (5) is rewritten to include the established boundary conditions:

$$p = s + (\pm 2C_z \times (n-1) + \gamma) \times (R_e - R_i) \mp \\ \mp (n-1)\big(2(C_k + C_z \times R_e)\big)ln\left(\frac{R_e}{R_i}\right) \tag{6}$$

where ± (before $2C_z$) is applicable to collapse or blow out failures, respectively; and ∓ (before n-1) is applicable to collapse or blow out failures, respectively.

3. Analytical model limitations

3.1. Geometrical and gravitational deviations

Eq. (6) is the result of a simplified model where body forces emulating gravity are radial, as opposed to true vertical gravity. Furthermore, it can be seen from Figure 1 that equilibrium is only applicable to a sector of cylinder or sphere, but not to the whole soil mass bounded by horizontal surface. Therefore, it cannot be stated that this analytical model is a true Lower Bound Solution, and although its results are generally conservative, there is no guarantee that its limiting pressures are always on the safe side.

3.2. Sidewall and Bottom Failure - Collapse

It can be depicted from Figure 1 that the limiting pressure p is balanced at tunnel crown ($p = p_{crown}$) and such arrangement is considered to be appropriate for blow out mechanisms. However, collapse mechanisms involving not only tunnel crown but also its sidewalls and bottom may not be adequately represented, particularly for a very low and uniform shear strength profile with depth, and a constant tunnel support pressure. Davis et al. [4] considered an average pressure balanced at tunnel axis, which is implicit from their definition of Stability Number N, and equivalent to adding $(\gamma \times D/2)$ to $p = p_{crown}$, i.e. $p_{axis} = p_{crown} + (\gamma \times D/2)$. This hypothesis was also considered by many of the Authors quoted in the introductory chapter, and is maintained herein whenever a uniform shear strength profile and a constant support pressure is considered.

3.3. No information on displacements up to failure

This analytical model is based on Equilibrium equation in cylindrical or spherical coordinates and Mohr Coulomb strength criterion, only. It does not include stress strain and strain displacement (compatibility) relationships, which, together with equilibrium and strength criterion would characterize a complete solution, as in Finite Element methods. Therefore, the analytical model does not provide information on induced displacements in soil mass. This limitation is important to be considered when tunneling may influence settlement sensitive buildings and structures.

3.4. Spherical or cylindrical geometry emulation

Tunnel excavation geometries do not match cylindrical or spherical arrangements, with the possible exception of a plane strain (2D) circular tunnel. In general, the geometry considered in the analytical model should envelope the actual tunnel excavation geometry, as illustrated at Figure 1, where a theoretical sphere envelopes an unsupported length of a tunnel. L is the unsupported length, and D is the diameter that envelopes a non-circular tunnel cross section (it matches the tunnel diameter in case of a circular section). Such provision is likely to be a source of conservativeness in the analytical model, particularly for 3D arrangements.

3.5. Local collapse

Local collapse mechanisms involving tunnel heading or their side lengths are not accounted for in the analytical model, and this can be construed as an additional consequence of limitations (*3.1*) and (*3.2*). Reference can be made to Davis et al. [4] local collapse mechanisms as function of the non-dimensional ratio $\gamma D/S_u$. Solutions for the critical height of a vertical cut may be alternatively used for preliminary estimates, e.g. $\gamma D/S_u \leq 3.8$.

3.6. Continuum media

These models are applicable to continuum media. Discontinuum mechanisms such as falling blocks and sliding wedges should be treated in accordance with limiting equilibrium statics established in the field of rock mechanics, as discussed by Sozio [10].

4. Support pressure comparisons between different methods

Wilson et al. [11] published a comprehensive investigation on the undrained stability of a circular tunnel (2D) where shear strength increases linearly with depth. The authors used Finite Element limit analysis to establish lower and upper bound estimates, and succeeded to bracket the true collapse load to within 5% for a series of soil cover to diameter ratios and various shear strength vs. depth relationships, including constant shear strength with depth (C_z=0), which is a particular case of Eq. (2).

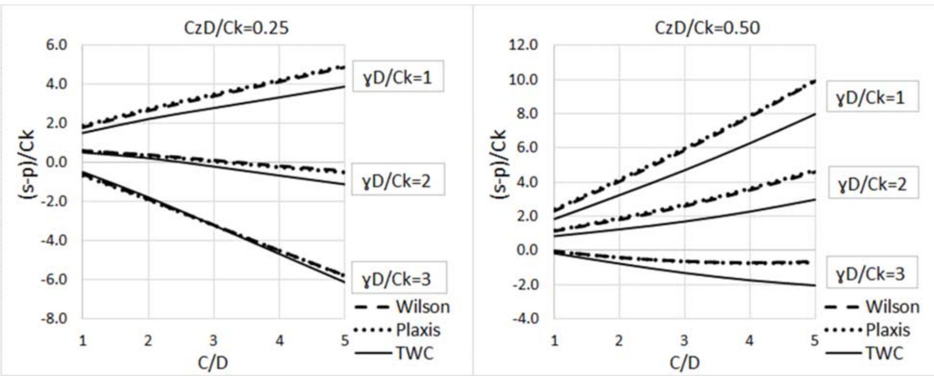

Figure 2. Comparative evaluations, Wilson et al., Plaxis, and analytical (TWC) models. For symbols, refer to Figure 1 and Eq. (6).

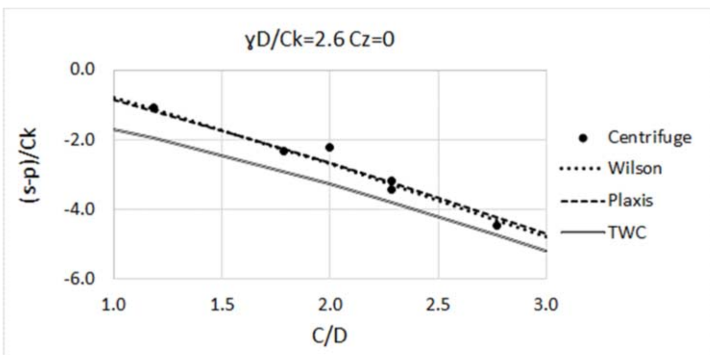

Figure 3. Wilson et al., Plaxis, and analytical (TWC) compared to centrifuge data from Mair [13], for uniform shear strength profile and constant support pressure.

Wilson et al. [11] average upper and lower bound uniform pressure values were compared to $p = p_{crown}$ from Eq. (6), (TWC, n=2, collapse) and to Finite Element code Plaxis 3D [12], which was run in plane strain mode to simulate 2D conditions. Curves representing the above three methods are shown at Figure 2, for cover to diameter ratios ranging from C/D=1 to 5, $\gamma D/C_k$=1, 2 and 3, $C_z D/C_k$=0.25 and 0.50 (values larger than 0.50 appear to be unlikely for normally or slightly over consolidated clays). A uniform tunnel pressure p was adopted in Plaxis [12], as considered in Wilson et al. [11] models. The graphics ordinate and coordinate axes were chosen as adopted in Wilson et al. [11] paper. Lower ordinate values are on the safe side, as these represent either greater supporting pressures or smaller surface loads.

The main points noted from these analyses are:

- Wilson et al. [11] and Plaxis [12] results are practically coincident for all cases.
- The analytical model TWC tends to be conservative as the rate of variation of shear strength with depth ($C_z D / C_k$) is increased.
- This conservativeness tends to reduce and may even reverse slightly to the unsafe side as the rate of variation of shear strength with depth is reduced, (i.e. *Cz* tends to zero) approaching a uniform shear strength profile. Such a condition may require balancing supporting stresses at axis level, as discussed at limitation *3.2*.

5. Comparisons between different methods and centrifuge tests by Mair [13]

As pointed out by Wilson et al. [11] the centrifuge data of Mair [13] are one of the most comprehensive sets of experimental results available. Those tests were performed in a soil with a constant undrained shear strength, $C_z = 0$ and $\gamma D / C_k = 2.6$.

Centrifuge uniform support pressure test results are plotted at Figure 3, and compared to Wilson et al. [11] average upper and lower bound values, to Plaxis [12] plane strain (2D) outputs, and to the analytical model. Considering the uniform shear strength profile of this case and constant tunnel support pressure, TWC results from Eq. (6) are balanced at tunnel axis, $p_{axis} = p_{crown} + (\gamma \times D/2)$. As before, Wilson et al. [11] and Plaxis [12] values are practically coincident, and TWC results are on the safe side.

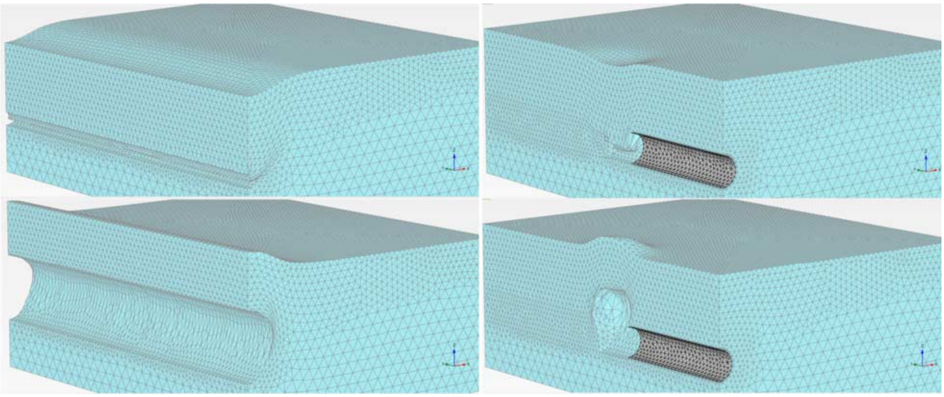

Figure 4. Plaxis exaggerated deformed meshes, 2D at left, 3D at right, collapse at top, blow out at bottom. Rigid lining shown as dark outline, right side.

6. Example application, 2D and 3D arrangements, Analytical and Plaxis models

Plaxis [12] models were employed in 2D and 3D arrangements, in order to allow comparison to analytical 2D (TWC) and 3D (TWS) models, Eq. (6). Tunnel diameter is D=4.5m, and an unsupported tunnel length L=6.0m was adopted for 3D analyses. Such geometry is intended to simulate a slurry shield excavation; for this case, the tunnel pressure was applied as a fluid pressure in Plaxis [12], with γ_{slurry} = 10kN/m³. The pressure value from Plaxis [12] is referenced to tunnel crown elevation.

Undrained shear strength is S_u= 8.0+1.0z, kPa, z being the depth in m below surface level, and soil unit weight is γ_{soil} = 16kN/m³. Illustrations of magnified (exaggerated) deformed Plaxis [12] meshes are given at Figure 4, for 2D and 3D arrangements, for a cover to diameter ratio *C*/*D*=2.

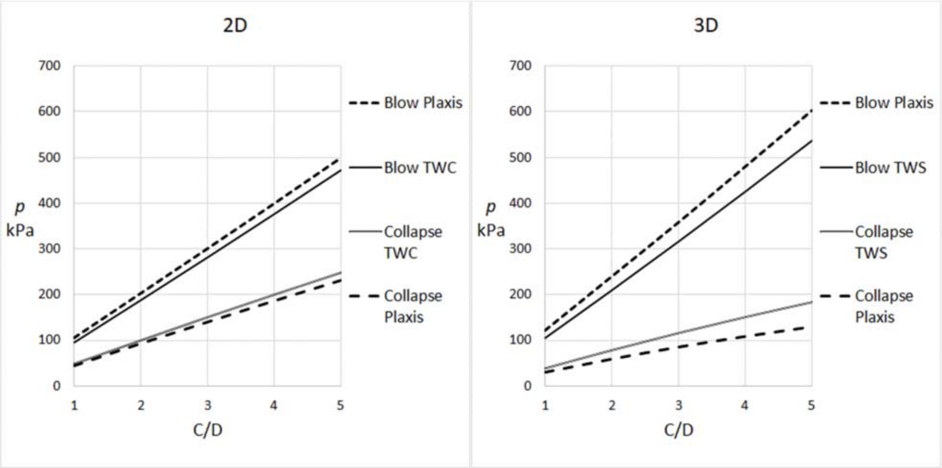

Figure 5. Collapse and Blow Out fluid pressures (*p*) referenced to tunnel crown, for D=4.5m, unsupported tunnel length L=6.0m (in the 3D analysis), C/D being the soil cover to tunnel diameter ratio.

Limiting fluid pressure curves for all analyses are shown at Figure 5, and the main aspects to be observed are:

- Analytical curves (TWC and TWS) are inside Plaxis [12] curves (larger margin for 3D analyses), which means that analytical model is conservative for the parameters of this example.
- The operational range between collapse and blow out limiting pressures is significantly reduced as tunnel becomes shallower, indicating an increasingly higher risk of both collapse and blow out for very shallow tunnels (*C*/*D*<1).

7. Conclusion

A simple analytical model was developed to enable quick evaluations of collapse and blow out limiting tunnel pressures, for 2D and 3D geometrical arrangements, in undrained soils allowing a linear increase of shear strength with depth. However, in view of the analytical model limitations as described in chapter 3, its use should be restricted to preliminary design assessments.

Two and Three Dimensional (2D and 3D) numerical methods (such as Finite Element) are complete solutions able to overcome analytical model limitations. Embedded software procedures such as arc length control enable a robust numerical approach to failure loads. Furthermore, a complete displacement field can be observed up to failure. Numerical methods are thus considered a more accurate solution than analytical models, being recommended to be deployed as design advances into detailed stages.

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