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Probabilistic Analysis of Mine Slope Stability

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Abstract. In current geotechnical practice, slope stability calculations are usually carried out only based on deterministic methods, by obtaining a value of Factor of Safety (FS). These analyses can be seen as simplified as FS values do not consider the natural variability of soils and rocks properties. Disregarding the uncertainties inherent to geotechnical parameters may lead to unreliable results of slope safety. Probabilistic analyses, based on statistical concepts, have become recurrent in geotechnical practice, as they allow including the materials intrinsic variability. These analyses allow treating FS as a function and studying its properties. The probabilistic methods indicate the reliability index (β) and the probability of failure (PF) of the geotechnical structure to be analyzed. This work presents deterministic and probabilistic studies of a 200m high slope at the Cauê Mine, located in the Iron Region, in Itabira, Minas Gerais, Brazil, with basis on three standard deterministic methods (Bishop, Spencer and Morgenstern-Price) and three usual probabilistic procedures (FOSM, Point Estimates and Monte Carlo). The results indicate that fixing the critical deterministic failure surface usually gives values of β and PF similar to those achieved when the surface is free to vary [1]. It is also concluded that, with a fixed critical surface, β and PF results may change significantly, when different Limit Equilibrium methods are adopted. It is recommended to use probabilistic FOSM procedure with Morgenstern & Price technique in stability analyses similar to the one presented herein.

Keywords. FOSM, Point Estimate, Monte Carlo, Reliability Index, Probability of Failure, Cauê Mine.

1. Introduction

Due to the natural heterogeneity of geotechnical materials, the designation of soils and rocks properties is an arduous work. There are no analyses that accurately reproduce soil's *in situ* performance. Slope stability studies are usually developed by considering the average values of geotechnical properties for obtaining a Factor of Safety (FS), defined as the ratio between strength (R) and solicitation (S). Applying the average values of each variable's property when computing FS is defined as a deterministic technique. Since geotechnical properties are obtained by testing in distinct points of the soil mass, uncertainties about computed FS values are inherent to determinist analyses. This leads geotechnical experts to use their previous experience and judgement, when interpreting FS results.

In the 1980's, the advent of probabilistic concepts helped geotechnical specialists to achieve more confident practical decisions in their safety studies. Within probabilistic

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techniques, soil properties are treated as random variables. Examples of probabilistic approaches in geotechnical problems have been presented in [2], [3] and [4]).

The most common probabilistic methods in geotechnical engineering are FOSM, Point Estimate, and Monte Carlo. All three methods have been used in this work aiming at evaluating the effect of fixing a slope failure surface when applying a probabilistic method in a geotechnical problem. With this purpose, stability studies were carried out combining usual limit equilibrium methods (Bishop, Spencer and Morgenstern-Price) to each probabilistic procedure.

The geotechnical example considered in this study is an open pit mining slope in Southeast Brazil. In mine slope design, the risk conditions are variable, depending on its basic features. A high probability of failure (PF) is accepted when stabilization of the pit slope is more expensive than cleaning up or mining to flatter angles [5].

1.1. Reliability index (β) and probability of failure (PF)

The main objective of geotechnical probabilistic analyses is to obtain the Probability of Failure (PF) of a given problem. PF can be understood as the probability of FS to be lower than 1.0, or P(FS<1). Probabilistic methods offer PF as indirect result, after the Reliability Index (β) is computed from Eq. (1), considering that the random variables have a normal (Gaussian) distribution, as explained in [6] and [2]:

$$\beta = \frac{E[FS]-1}{\sigma[FS]} \quad (1)$$

where E[FS] denotes the expected value (or best estimate) of the Factor of Safety, and σ[FS] is the standard deviation of FS. As explained by Christian *et al.* [6], the reliability index represents the number of standard deviations separating the expected value of FS from its failure value of 1.0.

After computing the index β, the probability of failure (PF) may be easily estimated with Figure 1, by assuming the FS probabilistic distribution curve is Gaussian. Christian *et al.* [6] and Sayao *et al.* [2] show the relation between β and PF depends on FS distribution function for $\beta > 1.0$.

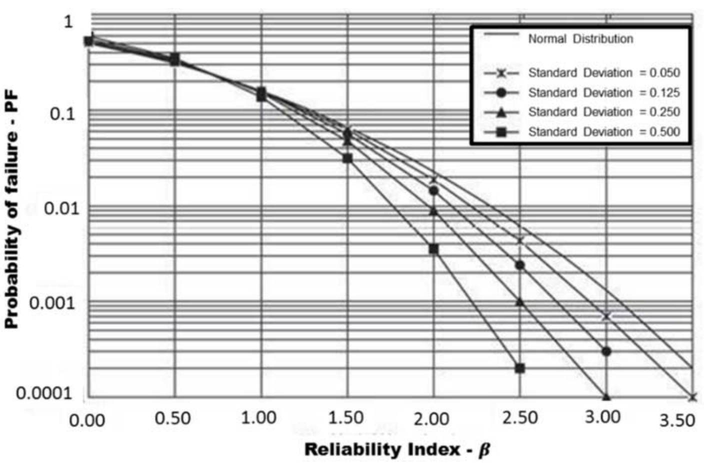

Figure 1. Probability of Failure as a function of the Reliability Index for different assumptions of FS distribution (adapted from [7]).

1.2. First Order Second Moment – FOSM

The initial method to be presented here is the First Order Second Moment (FOSM), an indirect probabilistic method for not requiring previous information on the probabilistic distribution curve of parameters considered as random variables. Estimates of mean and variance (V) of the dependent variable (FS) are made by applying Taylor Series around mean values of the independent variables [6]. Simplification of the Taylor Series to random variables that are not intrinsically correlated, leads to Eq. 2, which gives V[FS], as suggested by [8]:

$$V[FS] = \sum_{i=1}^{n} \left(\frac{\delta FS}{\delta x_i}\right)^2 . V[x_i] \tag{2}$$

In Eq. (2), V[FS] indicates variance of FS, and $V[x_i]$ is the variance of independent random variable (x_i). By definition, the variance of x_i is the square of its standard deviation ($\sigma[x_i]$).

Approximating the partial derivate member of Eq. (2) by the divided differences method (Eq. (3)) is the option suggested by Christian *et al.* [6] to solve the partial derivate components of that equation, without assumptions on the x_i functions.

$$\frac{\delta FS}{\delta x_i} = \frac{FS(x_i + \delta x_i) - FS(\overline{x_i})}{\delta x_i} \tag{3}$$

In Eq. (3), $\overline{x_1}$ = mean value of variable x_1 ; δx_1 = small variation of x_1 ; $FS(x_1 + \delta x_1)$ = Factor of Safety when only x_1 is varied; $FS(\overline{x_1})$ = Factor of Safety mean.

[3 and 2] reported to be satisfactory a positive (or negative) variation of 10% to each variable.

1.3. Point Estimates - PE

First described by [12], Point Estimates (PE) Method is also an indirect technique that enables approximating the mean and variance values of a random variable. It consists on a simple procedure for estimating the first and second probabilistic moments of a function F[X] without the need of knowing F[X] explicitly. That approach is possible out of a procedure that involves 2^n stability calculations, where n is the number of random variables within the analysis. The 2^n analyses are proceeded by incrementing or decrementing the independent random variable mean value by its standard deviation. The obtained results are combined in all possible ways to originate mean and variance values for the F[X] function. Supposing that FS is function of only two random variables, its first probabilistic moment (mean) can be disclosed by Eq. (4):

$$E[FS] = \frac{FS_{++} + FS_{+-} + FS_{-+} + FS_{--}}{4} \tag{4}$$

where notation FS_{++} refers to the value of FS when both random variables are simultaneously increased by their own standard variation values.

After knowing the mean, the variance of FS may be computed by Eq. (5).

$$V[FS] = [FS^2] - (E[FS]^2) \quad (5)$$

Knowing the first and second probabilistic moments of FS (mean and variance), β and PF may be obtained as described in item 1.1.

1.4. Monte Carlo Simulation - MC

First described by John Von Neumann and Stanislav Ulam [9], the Monte Carlo Simulation (MC) probabilistic method is usually applied on multiple fields of Science. It is a statistical sampling technique that works by the generation of as many random values as necessary to describe F[X] attempting to approximate the function description to reality. The results are more precise as the number of generated values approximate to infinite. Facing the impossibility of performing infinite simulations, it was suggested, by [8], a minimum number (N) of simulations to be accomplished in order to have satisfactory results. N is, commonly, very large and therefore demands uncountable procedure repetitions (iterations). Hence, it didn't use to be simple to apply the Monte Carlo Simulation method by the times when there was no practical software to support the procedure. Thus, the nature of Monte Carlo simulation method justifies why it needs to be carried out with computational support.

In Geotechnical Engineering, this method yields a satisfactory replication of a dependent variable's behavior (i.e. FS) and also allows the obtainment of PF for a geotechnical structure as long as its geometry, mean, standard deviation, and the characteristic curve of each random variable (e.g. Φ', ɣ, c', NA) are known. Among the 3 probabilistic methods presented herein, the MC method demands having the knowledge of the probability density function of each random variable involved in the situation.

When applying the MC simulation in their geotechnical researches, [10], [11], [9], [12] and [13] have positioned themselves for the FOSM and/or PE when treating about probabilistic methods.

2. Study Case

Probabilistic analyses were herein carried out for computing the probability of failure for an iron open pit mine slope located in the Iron Region, a major iron, gold and gemstone producer of Brazil. It accounts for about 40% of the country's total gold production in the last five centuries, and it used to comprise the two iron tailings dams which were object of two of the largest world mining industry tragedies (Fundão and B1, respectively in Mariana and Brumadinho).

The mine studied herein is known as Cauê Mine, and it is part of the Itabira Complex, in central Minas Gerais State, in Brazil. It is located in the NE region of the Iron Region (Figure 2). The Cauê Mine (Figure 3) was one of the biggest iron mines in the world in the 1970's.

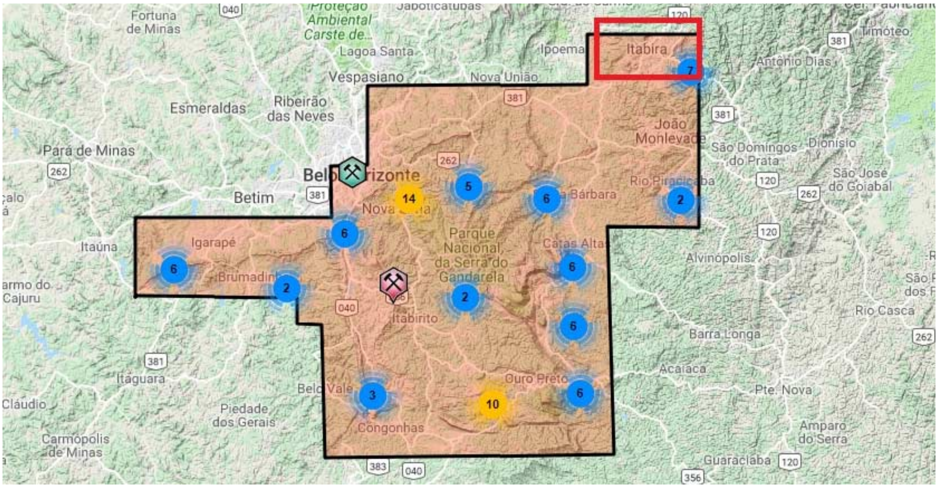

Figure 2. Itabira’s location in the Iron Region, emphasized in a red rectangle. The numbers indicate how many mines are included on the pinned regions (source: Google Maps [14]).

Figure 3. Open pit Cauê Mine [15].

The Cauê Mine slope (Figure 4) which is reported in this study is 200 m high, has an average inclination of 34°, and it is basically a saprolitic material from weathered iron quartzite. The phreatic line is shown as a tilted line going from the slope’s base up to a point about 80 m below the crest. This same slope was originally reported by [2], and its geotechnical and statistical characteristics were obtained from 50 direct shear test results. The soil properties are presented in Table 1.

Using these results, the slope safety was studied by the probabilistic methods FOSM, PE and MC, combined with the deterministic methods Bishop, Spencer e Morgenstern-Price, with the *Slide 6.0 software* from *RocScience*.

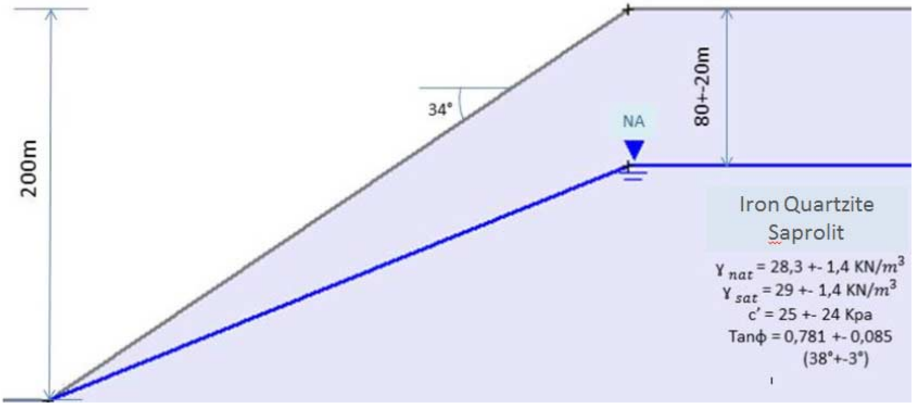

Figure 4. Typical cross section of a slope from the Cauê Mine.

Table 1. Geotechnical and statistical slope properties.

Property	Mean ($\bar{x_i}$)	Variance ($V[x_i]$)	Standard Deviation ($\sigma[x_i]$)
Natural unit weight - γ_{nat} (KN/m^3)	28.3	1.96	1.4
Saturated unit weight - γ_{sat} (KN/m^3)	29.0	1.96	1.4
Cohesion - c' (KPa)	25.0	590.0	24.0
Tangent of Friction Angle– tg (Φ')	0.781	0.0072	0.09
Pore-pressure – u	120.0	400.0	20.0

3. Results

FOSM procedure [16] was applied considering the five properties listed in Table 1 as random variables. As the FOSM method gives the relative influence of each property in the PF result, it may be perceived (Figure 5) that, for this specific case, natural and saturated unit weights (γ_{nat} and γ_{sat}) are not relevant to the slope safety, and may be considered as constant values (non variable). After this conclusion, the PE and MC methods were carried out with only three random variables (properties Φ', c' and u) from Table 1. Alonso reported in [17] that the uncertainties in pore-pressure and strength parameters are very relevant in risk analyses.

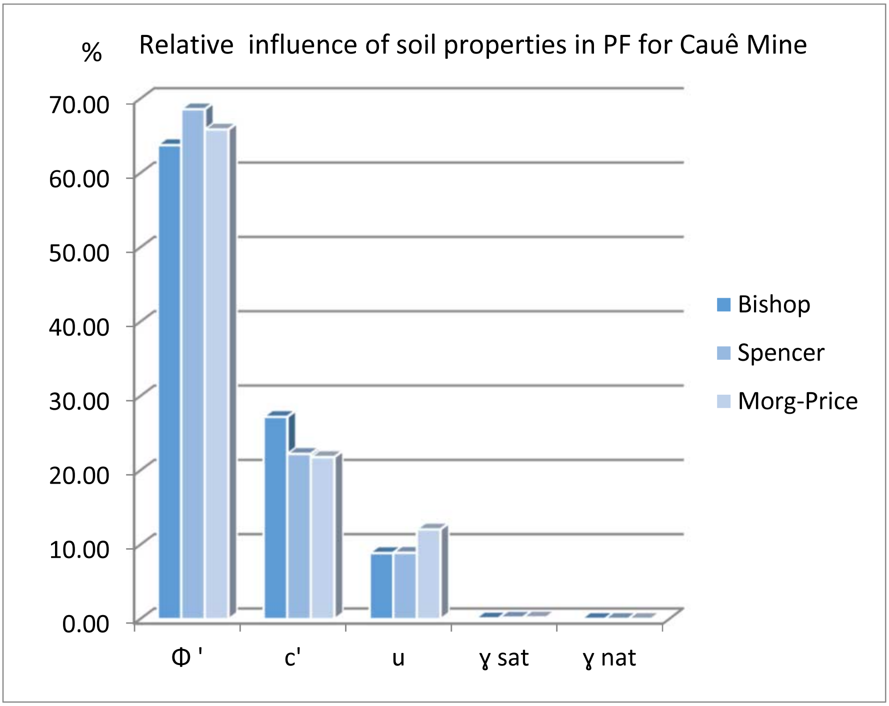

Figure 5. Relative influence of each variable in the Safety of the Cauê Mine slope.

Each probabilistic procedure (FOSM, PE and MC) was considered combined with three Limit Equilibrium methods (Bishop, Spencer and Morgenstern-Price) for two scenarios: fixed and unfixed potential failure surface. These combinations led to a total of 6 analyses for each probabilistic procedure. The results are summarized in Table 2.

The random variables from Table 1 were assumed to have normal distribution functions and to be independent of each other, hence β was computed from Eq. 1, and PF was estimated from the correlation in Figure 1, considering FS to be normally distributed.

Table 2. Deterministic and probabilistic geotechnical analyses results.

		Probabilistic Method											
		FOSM		PE		MC		FOSM		PE		MC	
Limit Equilibrium Method	FS	β						PF					
		Fix surf	Free surf	Fix surf	Free surf	Fix Surf	Free surf	Fix surf	Free surf	Fix surf	Free surf	Fix surf	Free surf
Bishop	1.26	1.62	1.62	1.61	1.44	1.87	1.87	1:20	1:20	1:20	1:14	1:43	1:43
Spencer	1.26	1.59	1.61	1.48	1.21	1.88	1.85	1:17	1:20	1:14	1:9	1:44	1:40
Morg-Price	1.26	1.54	1.56	1.61	1.43	1.87	1.84	1:16	1:17	1:19	1:13	1:44	1:40

4. Final Considerations

Defining the degree of FS reliability is an essential step for ensuring the safety of slopes and providing engineers a chance to take better decisions, for minimizing the possibility of disasters such as the recent ones that have affected the mining industry in Mariana, Brumadinho and Mount Polley. Consideration of probabilistic procedures as a guide on studies of slope mine stability is well evidenced.

The main distinctions among FOSM, PE and MC methods are the number of analyses involved in each of them, and the need of previous knowledge of the probability density function of each random variable involved in the problem.

Deeply understanding each step involved in the probabilistic procedure is necessary to guarantee the methods efficiency. This work presents the study of how fixing or not fixing slope failure surface affects the magnitude of β and PF results.

Analyzing the results from this study (Table 2), it may be noted that fixing the critical failure surface simplifies the procedure and gives β and PF results similar to those reached when the surface is free to vary. It was also noted that fixing the critical surface, β and FS results change significantly, depending on the adopted Limit Equilibrium method.

In conclusion, it is suggested to use probabilistic FOSM analysis with fixed potential failure surface, with a rigorous stability method, such as Spencer or Morgenstern & Price, in a procedure similar to the one presented herein. In cases of homogeneous materials, the critical surface tends to be circular, indicating, also, the adequacy of using the Bishop method, which is commonly used in limit equilibrium safety analysis.

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