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Fragility Curves of Flood Protection Levees Subject to Overtopping

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Abstract. In this paper, we represent uncertainty in the estimation of the parameters of an erosion law through different probability models including non-Gaussian models, and develop fragility curves for the surge-only overtopping erosion failure of an earthen structure with typical erosion parameters of a levee made of compacted fine-grained soil. Contrary to common practice in fragility analysis that assumes a prior probability model for the fragility curve, we found that the type of fragility curve depends largely on the type of probability models adopted for the design variables.

Keywords. Surface erosion model, surge-only overtopping, non-Gaussian probability models.

1. Introduction

Overtopping represents one of the most common cause of failure of flood protection levees worldwide [1]. This failure occurs when the shear stress at the levee's surface resulting from the free surface flow imposed by a storm, becomes higher than the critical shear strength of the levee material. The most severe damage occurs where higher shear stresses are concentrated. Often, this region is found somewhere along the downstream slope of the earthen structure.

Formally, the erosion problem of a levee can be addressed as a stress-strength problem of reliability theory. In reliability theory [2-3], a failure state occurs when the strength is exceeded by the stress and fragility is defined as the probability of finding the structure response on a failure state given that a certain value of the intensity of the stress has occurred. The functional relationship between fragility and intensity is called fragility curve [4].

Fragility curves are key elements in quantitative risk assessments of flood protection levees [5-7]. One of the most simple stress-strength models for the surge-only overtopping erosion failure of a levee can be defined in terms of five basic design variables: slope angle, Manning's coefficient, critical shear stress, erodibility coefficient, and intensity of the stress. The last variable represents the height that water reaches above the levee crest during surge-only overtopping.

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2. Fragility curve definition

Let $\mathbf{x}^T = [X_1, X_2,\ldots,X_N]$ be a state vector that contains random basic design variables X_1, $X_2,\ldots,X_N$ with N -variate joint probability density, represented by $f_{\mathbf{X}}(\mathbf{x})$. For simplicity, the safety and failure events against a specific mechanism are described in terms of a state function $g(\mathbf{x})$, written such that: $\{g(\mathbf{x}) \leq \xi\}$ represents the safety event, i.e., $\{\mathbf{x} \in S\}$, and $\{g(\mathbf{x}) > \xi\}$ represents the failure event, i.e., $\{\mathbf{x} \in \Im\}$, where ξ is a threshold that delimits the safety and failure regions.

The failure probability given a specific intensity value h is the fragility, and it can be expressed as [2]:

$$F_H(h) = P[\mathbf{x} \in \Im | H = h] = \int_{\Im|h} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{1}$$

The curve given by $F_H(h)$ is the fragility curve, which expresses the probability of the capacity being lower than that required given a stress intensity h. The failure domain depends on the intensity; as the intensity increases, the failure probability approaches unity.

3. Fragility analysis

Analyzed in the following illustrative example is the fragility of a flood protection levee made of compacted fine-grained soil with 5 m in height and 4 m wide, subjected to surge-only overtopping in the interval: $0 \leq h \leq 2$ m.

3.1. Limit state function

The limit state function for the surge-only overtopping erosion failure of a protection levee made of compacted fine-grained soil can be written as follows:

$$g(\mathbf{x}; h) = \xi - k_d(\tau_0 - \tau_c) \cdot t \tag{2}$$

where ξ: erosion [mm], t: time [h], k_d: erodibility coefficient [mm/h/Pa], (τ_c: critical shear stress [Pa], and τ_0: shear stress at the soil-water interface [Pa] [8]:

$$\tau_0 = \rho g h_0 \sin\theta \tag{3}$$

where ρ: water density [kg/m^3], g: acceleration of gravity [m/s^2], θ: landward-side levee slope angle [°] and h_0: water height perpendicular to the slope [m]:

$$h_0 = \left[(2/3)^{3/2}\sqrt{g}h^{3/2}\right] / \left[\sqrt{\sin\theta}/n\right]^{3/5} \tag{4}$$

where h: overtopping height [m], and n: Manning coefficient [s/m$^{1/3}$].

In Eq. (2), k_d and τ_c are properties that depend on the levee material and are determined in field and laboratory erosion tests. k_d determines the rate with which the levee erosion takes place, and τ_c establishes the threshold from which the levee erosion begins. As illustrated in Figure 1, the erosion begins when $\tau_0 > \tau_c$, and it will occur at a rate given by k_d.

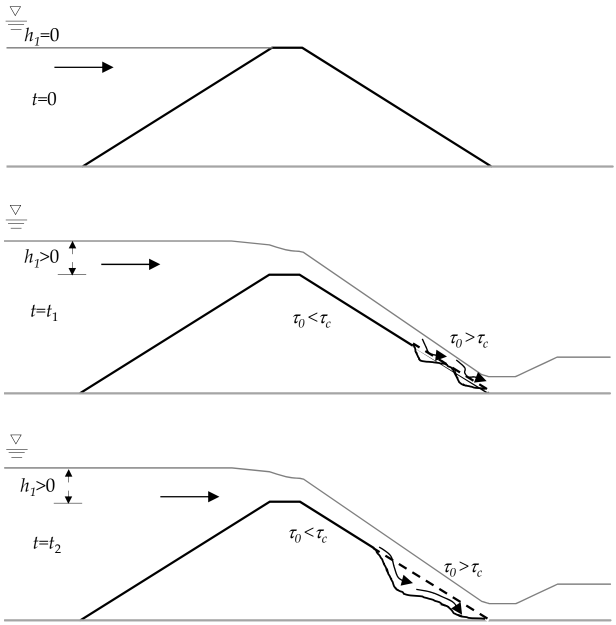

Figure 1. Illustration of the levee erosion process.

3.2. Uncertainty modeling

A practical situation is considered in which the designer faces uncertainty in the estimation of four basic design variables: θ, n, τ_c and k_d. Then, the fragilities are determined from different samples of the random variable H reflecting the water level fluctuations in the reservoir.

The results of the laboratory tests on samples of compacted fine materials reported by [9-10], are considered in the estimation of the expected values and dispersions of τ_c (critical shear stress) and k_d (erodibility coefficient). When constructing the histogram of the values reported for τ_c, it was found that the lognormal density satisfactorily describes the test results. It was also observed that the parameter k_d fluctuates over several orders of magnitude and that its minimum value is always greater than zero. Therefore, the Weibull density was adopted for representing the uncertainty in this parameter. In practice, an expected value of the Manning coefficient n is selected based on the type of levee material, and it is accepted that it can vary within a known interval [8]. In this case, the uncertainty was modeled through the Beta density described in [11]. For simplicity, the possible variations of the landward-side levee slope angle θ were represented through a Gaussian random variable. The adopted parameters of the set of random basic design variables are indicated in Table 1.

Table 1. Probabilistic parameters of the basic design variables.

Parameter	Unit	$E\{X\}$	CV	Min.	Max.	Type
θ	rad	0.197396	0.1	----	----	Normal
n	s/m$^{1/3}$	0.0158	0.1	0.01	0.035	Beta
τ_c	Pa	16	6	0.0	----	Lognormal
k_d	mm/h/Pa	2.5	6	0.1	----	Weibull

Here, it has been considered that there is a correlation between the three parameters that depend on the type of the levee material (n, τ_c and k_d). This observation is not subjective; it is based on the results of the laboratory erosion tests reported by [9-10]. A negative correlation (-0.7) was considered between τ_c and k_d to represent that when a variable tends to increase, the other variable tends to decrease, as observed in the laboratory tests reported by [9]. A negative correlation (-0.8) was also considered between τ_c and n to represent that as the Manning coefficient n increases, the critical shear stress τ_c decreases. This tendency is based on the fact that the compacted soils with higher sand content often exhibit higher Manning coefficients n, but are less resistant to erosion. The numerical values assigned to the correlation coefficients mentioned above (-0.7 and -0.8), reflect the degree of dependence partially subjectively assigned by the designer to reflect such tendencies. As mentioned above, this decision is based on the observed behavior of these parameters in laboratory tests reported in the literature. To be consistent with our reasoning, there must be a correlation between n and k_d, but opposite in sign (0.8).

Based on the experiences reported by [12], an arbitrary permissible erosion scale is adopted in the present work as follows:

- Minor damage: if $\xi > 0.3$ m.
- Intermediate damage: if $\xi > 0.6$ m.
- Severe damage: if $\xi > 0.9$ m.

3.3. Fragility curve development

The levee fragility against overtopping is analyzed for each level of damage specified above and for two overtopping residence times: 4 h and 20 h. The first period represents the typical time of analysis, and the second period represents an extreme condition. Each point of the empirical fragility curve was determined based on 500,000 simulations using the Monte Carlo method [13]. In this context, an empirical fragility curve (point by point) is defined for different values of h and the theoretical version of this curve is obtained through a fitting procedure.

3.4. Results and discussion

Figures 2(a)-2(c) show with symbols the empirical fragility curves corresponding to the fragility analysis. Note that as the overtopping duration increases, the failure probability increases. In addition, as the failure indicator ξ increases, the failure probability decreases because the occurrence frequency of a more severe failure is lower if the analysis times are kept constant. For an overtopping close to 2 m in height that lasts 20 h, the levee reaches failure with a probability very close to one. In accordance with [14], the fragility curves in Figures 2(a)-2(c) can be interpreted as the probability that the levee will at a minimum suffer the damage specified when subjected to an overtopping with intensity h. The failure probabilities, even for the typical period of analysis of 4 h, are, in fact, high. Considering that representative values of the erosion properties of compacted soils have been used in the analyses, these probabilities are then also representative of a levee subjected to overtopping and can explain the observations published by some authors.

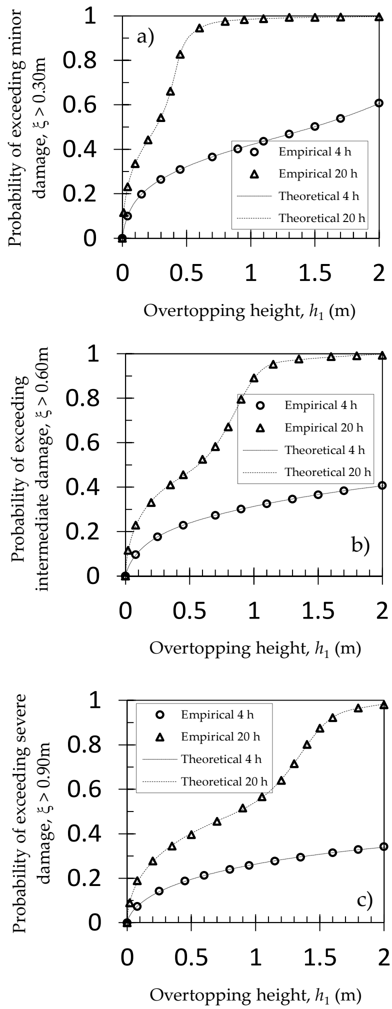

Figure 2. Fragility curves for different failure conditions: a) Fragility curve for a minor damage level, b) Fragility curve for an intermediate damage level, c) Fragility curve for a severe damage level.

For example, [15] have reported erosion time initiation. They found that erosion of compacted clayey levees usually begins between 10 h and 20 h after its rupture. The results of the present fragility analysis can also help to explain the occurrence frequency of failures in dams due to overtopping (48.4%) and piping (46.1%) [1]. Recall the erosion of the landward-side slope favors the piping of the compacted fine embankment materials subjected to seepage. To increase levees' reliability against overtopping, some authors show the advantages of planting grass over the levee [16] or, even better, increasing the erosion resistance of the planted grass using a geogrid [10].

Contrary to common practice in fragility analysis that assumes a lognormal probability model for the fragility curve, we found that the theoretical fragility curves for the problem stated through Eqs. (2)-(4) are given by a mixture Weibull distribution [18] as follows:

$$G_X(x) = \sum_{j=1}^{3} w_j F_X(x) \tag{5}$$

where $G(x)$: mixture distribution function, w_j: weight associated with the j-th Weibull distribution, and $F_X(x)$: Weibull distribution function.

The theoretical fragility curves associated with the empirical curves were determined using the least squares technique [17]. In the work by [13], the scale and shape parameters of that curves were reported. Here, the theoretical fragility curves are shown with dashed lines in Figures 2(a)-2(c). Note that the theoretical curves exactly describe the empirical curves. Given the shape of the theoretical curve, it can be concluded that the probability density of the parameter k_d (erodibility coefficient) has the greatest influence on the shape of the fragility curve. Furthermore, the influence of n (Manning coefficient) and τ_c (critical shear stress) is lower than that of k_d. The shape of the theoretical curve given by Eq. (5) can be explained by the non-linearity of the erosion joint model given by Eqs. (2)-(4). The calculations performed during the present work showed that the correlation between the design variables modifies the parameters of the theoretical curve but has no influence on the density type. Some authors have suggested the use of exponential densities for the erosion parameters of soils [19]. Another possibility is to consider lognormal probability densities to represent the uncertainty in the estimation of these parameters. However, in the case of the erodibility coefficient, a lognormal density seems to be unrepresentative of the physical behavior of such a parameter. Indeed, the erodibility of the compacted fine materials cannot be zero; it starts from a minimum value greater than zero and moves away from zero as the uncertainties in the quality of the borrow bank materials and the placement procedures of the materials are higher. Given the effect of the k_d density type on the shape of the fragility curve, it is recommended that the consequences be assessed in a probabilistic risk analysis in each particular case.

3.5. Further comments

In a comprehensive analysis performed by [20], the sensibility of the erosion failure to all variables involved was investigated. They found that surface erosion of the levee is significantly more sensible to uncertainty in Manning's coefficient than to any other parameter and that uncertainties in the critical shear stress and landward-side levee slope angle on such failure are negligible. [20] stressed that the influence of the erodibility coefficient of the levee material over the erosion failure is considerably weaker than that of the Manning coefficient, yet still influences the type of fragility curve. Thus, they conclude that the probability density function of the variable with the strongest impact on the erosion failure of the levee does not determines the type of fragility curve necessarily. Therefore, within a more realistic context of non-Gaussian uncertainties, it is not correct to assume a prior theoretical model for the fragility curve ignoring the distributions of the basic variables because it is not possible to anticipate its type based only on the distribution of the random variable with the strongest incidence on the component's failure.

4. Conclusions

In dealing with predictions of the potential damage of earthen structures subject to overtopping, geotechnical engineers often postulate an erosion model and estimate its parameters within a context of pronounced uncertainty. Owing to this uncertainty, predictions made with erosion models are also uncertain. Therefore, the use of a probabilistic approach is necessary to add a higher degree of realism to such predictions. Within a probabilistic framework, a failure state occurs when the strength is exceeded by the stress and fragility is then defined as the probability of finding the structure response on a failure state given that a certain value of the intensity of the stress has occurred. In this context, the concept of fragility curve emerges as the functional relationship between fragility and intensity. The present article has shown that the fragility curves for the erosion failure of a protection levee subjected to surge-only overtopping differ significantly from the lognormal distribution when a non-Gaussian probabilistic model is used for the erodibility coefficient of the levee material. This will result in important practical implications when the fragility curve is combined with the curve associated with the specific threat and consequences for quantifying the risk. Therefore, it is recommended that the effects of the non-Gaussian uncertainties on the risk be assessed in all cases.

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