The paper was published in the proceedings of XVI Pan-American Conference on Soil Mechanics and Geotechnical Engineering (XVI PCSMGE) and was edited by Dr. Norma Patricia López Acosta, Eduardo Martínez Hernández and Alejandra L. Espinosa Santiago. The conference was held in Cancun, Mexico, on November 17-20, 2019.

From Research to Applied Geotechnics
N.P. López-Acosta et al. (Eds.)

doi:10.3233/ASMGE190029

Technical Session #17
"Rock Mechanics"

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From Research to Applied Geotechnics
N.P. López-Acosta et al. (Eds.)

doi:10.3233/ASMGE190030

Analytical and Numerical Study of the Stability of Shallow Circular Cavities in Weak Rocks

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Abstract. This keynote lecture addresses the problem of determining the mechanical stability of shallow circular cavities in cohesive-frictional ground (weak rocks and soils) assumed to obey a Mohr-Coulomb shear failure criterion. Methods traditionally used to analyze stability of shallow excavations are reviewed first. A method based on the application of limit analysis and shear strength reduction technique in full numerical analysis is chosen for evaluating stability conditions of shallow cavities. Application of a scalar factor of safety for shallow tunnels is introduced and dimensionless groups of variables controlling the stability of the openings are identified. The stability of shallow circular cavities in purely cohesive ground and in cohesive-frictional ground are discussed and observations of practical interest are highlighted. Comparison of stability results obtained with the proposed analytical equations, with full numerical analyses, and with approaches used by other authors in the published literature are discussed. The effect of water in the ground and inside the excavation on the obtained factor of safety is reviewed. Similarities of controlling groups of variables for the cases of shallow tunnels and slopes are highlighted.

Keywords. Stability of shallow circular cavities. Factor of Safety. Limit Analysis. Limit Equilibrium. Finite Difference Method. Shear Strength Reduction Technique. CAES system.

Extended Abstract

This keynote lecture presents a study of mechanical stability of shallow circular cavities carried out as part of a multidisciplinary project that looked into the feasibility of using abandoned underground openings (drifts and shafts) from iron ore mining in northern Minnesota (USA), for Compressed Air Energy Storage (CAES) applications (Fosnacht et al. [1]; Carranza-Torres et al. [2]).

The lecture addresses the problem of establishing the stability conditions of shallow cylindrical or spherical openings excavated in cohesive-frictional ground, and subject to either decreasing or increasing internal pressure, associated with the process of 'contraction' or 'expansion' of the cavities during operation of a CAES system (Succar and Williams [3]). It is worth noting that the problem of *contracting* cavities not only has applications in the design of CAES systems, but has broad applications in civil

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engineering, when assessing support requirements for shallow tunnels in soils for subway/metro projects (Carranza-Torres et al. [4]).

There exists several methods to assess the stability conditions of shallow cavities in geotechnical engineering. Potts and Zdravkovic [5] provide a general classification of available methods as follows: *i)* closed-form; *ii)* limit equilibrium; *iii)* stress field; *iv)* lower bound (or statically admissible); *v)* upper bound (or kinematically admissible); *vi)* beam-spring; and *vii)* full numerical analysis methods.

Contracting and expanding cavities in CAES systems have been traditionally analyzed using Terzaghi's type of limit equilibrium methods (Terzaghi [6]; Sofregaz U.S. Inc. [7]).

An alternative approach to assess stability, which is the one discussed in this lecture, is to use a combination of statically admissible solutions derived from the lower bound theorem of plasticity, and full numerical analysis solutions.

The statically admissible solution presented in this lecture is based on a classical model presented by Caquot [8], that allows to compute a conservative estimate of the internal support pressure for a section of shallow cylindrical tunnel, or spherical cavity, when the ground is assumed to obey a Mohr-Coulomb shear failure criterion. The full numerical analysis solution considered in this lecture is based on the application of the shear *Strength Reduction Technique* implemented in the finite difference code FLAC (Itasca, Inc. [9]).

In this study, the model by Caquot is adapted to express the stability of the cavity in terms of a single scalar value, i.e., a factor of safety, as traditionally done to assess stability of slopes in geotechnical engineering. The model is also adapted to consider both, internal pressure values below the stress existing prior to excavation (the *in situ* stress at the crown of the cavities), to represent the case of *contracting* cavities; and internal pressure values above the in situ stress, to represent the case of *expanding* cavities (Carranza-Torres et al. [2, 10]). In the extended model, all input variables representing geometry (e.g., radius and depth of the cavity), loading conditions (e.g., internal pressure, surcharge at the ground surface) and material properties (unit weight, cohesive-frictional properties of the Mohr-Coulomb ground) are scaled and expressed in dimensionless form, with the objective of obtaining equations to relate the conservative estimate of the resulting factor of safety with the scaled input variables. For example, the scaled depth of the cavity is defined as the depth of the cavity axis divided by the cavity radius. Introducing a 'converted' unit weight, defined as the unit weight of the ground multiplied by the cavity radius, the scaled internal pressure is defined as the internal pressure divided by the *converted* unit weight; the scaled ground surcharge is defined as the surcharge acting on the ground surface divided by the *converted* unit weight; the scaled cohesion is defined as the cohesion of the ground divided by the *converted* material unit weight, etc.

The stability of shallow cavities for the case of purely cohesive ground is discussed first. A closed form solution to compute factors of safety for both *contracting* and *expanding* cavities is presented. With all other scaled input variables being the same, it is shown that contracting and expanding *spherical* cavities do have always a value of factor of safety that is twice the value of the factor of safety for the corresponding *cylindrical* section of tunnel. For both, contracting and expanding cavities, the factor of safety is shown to increase with increase of the scaled cohesion of the ground, and to decrease with increase of (scaled) depth of the cavity, when the internal pressure is null or the internal pressure is a fixed ratio of the *in situ* stress, respectively (Carranza-Torres et al. [10]). For the case of contracting cavity, it is shown that the factor of safety becomes a minimum when there is no internal pressure, and becomes infinite when the internal pressure is equal to the in situ stress prior to excavation. A similar but opposite behavior is shown to occur for the case of expanding cavities, with the factor of safety being infinite for internal pressure equal to the in situ stress, and the factor of safety decreasing when the internal pressure increases. In particular, it is found that for a scaled internal pressure equal to two times the scaled in situ stress prior to excavation, the factor of safety for the expanding cavity is the same as for the contracting cavity with null internal pressure. This implies that the range of factors of safety for cavities in a CAES system can be bounded to a minimum prescribed design value, provided the expanding cavity does not have a scaled internal pressure that is larger than two times the scaled value of the in situ stress prior to excavation.

Results obtained with the extended Caquot's model, for both *contracting* and *expending* cavities, are compared with results obtained with the finite-difference code FLAC (Itasca, Inc. [9]). Numerical (FLAC) models confirm the behavior of the factor of safety with the scaled input variables observed earlier on. Comparison of both methods show that the values of factors of safety obtained with the lower bound solution are within 10% (on the conservative side) of the values of factors of safety obtained with the numerical models

With the extended Caquot's model and numerical (FLAC) models providing equivalent results of factor of safety, a comparison of results obtained with the extended Caquot's solution and with other approaches published in the literature is presented.

The Terzaghi's type of equilibrium models (Terzaghi [6]; Sofregaz U.S. Inc. [7]) are evaluated first. The comparison shows that for purely cohesive ground, limit equilibrium models can lead to both over conservative (i.e., too safe) and nonconservative (i.e., unsafe) factor of safety values, for both, *contracting* and *expanding* cavities, depending on the scaled depth of the cavity (the ratio of the depth to the axis of the cavity and the cavity radius). In general, the factor of safety is highly conservative for very shallow cavities, and this degree of conservatism decreases with the scaled depth of the cavity. With all scaled input variables being the same, at the scaled depth (axis depth vs cavity radius) equal to ~18:5 for *contracting* cavities, and equal to ~3:5 for *expanding* cavities, both extended Caquot's solution and limit equilibrium solution yield the same factor of safety. For cavities with scaled depths larger than the mentioned values, the factor of safety values obtained with limit equilibrium methods become now nonconservative (i.e., unsafe).

Results obtained with the proposed limit analysis solution are also compared with the results obtained with a semi-analytical stress-field solution presented by Davis et al. [11]. The solution by these authors allows prediction of the required support pressure for a circular cylindrical tunnel in purely cohesive ground. Obtaining a prediction of the required support pressure using the extended Caquot's solution introduced in this lecture, requires considering a factor of safety equal to one, and solving for the internal pressure at the critical state of equilibrium. Comparison of the results reported in Davis et al. [11] and those obtained with the proposed limit analysis solution are shown to be in good agreement —Carranza-Torres and Reich [12].

An extension of the proposed limit analysis model to account for water in the ground is presented next. Cases of cylindrical sections of tunnel in purely cohesive ground with a water surface at or above the ground surface, and limiting conditions of flooded and dry openings are considered. The analytical model shows that for the case of flooded cavity, the factors of safety are typically two times higher than for the same cavity in dry ground, provided the cohesion values of the saturated and dry ground are similar. For the limiting case of dry cavity and hydrostatic water pressure in the ground surrounding the cavity, the factors of safety are typically 80% to 90% lower than for the same cavity in dry conditions, again for cohesion values of saturated and dry ground being similar, and provided that no tensile failure occurs (i.e., when a mechanical support pressure that is at least equal to the hydrostatic pressure in the ground on the periphery of the cavity is considered).

Finally, the lecture presents the solution of factor of safety for *contracting* and *expanding* cavities in cohesive-frictional ground. The stability formulation for cohesive-frictional ground uses the same compact scaling law introduced by Hoek and Bray [13], when analyzing stability of slopes using limit equilibrium methods —see also, Carranza-Torres and Hormazabal [14]. It is shown that as for the cases of slopes considered by Hoek and Bray, the factor of safety divided by the tangent of the internal friction angle, when the 'converted' material unit weight introduced earlier on is used, depends on the scaled depth of the cavity, the scaled internal pressure, the scaled ground surcharge (with the scaling law as introduced earlier on), and the scaled material cohesion (i.e., the cohesion of the ground divided by the *converted* unit weight) divided by the tangent of the internal friction angle.

As for the case of cavities in purely cohesive ground discussed earlier on, several observations of theoretical and practical relevance on the dependence of the resulting factor of safety for *contracting* and *expanding* cavities, with the scaled input values introduced earlier are discussed. Comparison of results obtained with numerical finite difference (FLAC) models and with the proposed extension of Caquot's limit analysis solution for cohesive-frictional ground are presented. The comparison suggests that the analytical solution can provide a reasonable conservative measure of the stability conditions for the cavities.

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