From Research to Applied Geotechnics
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Bright Spark Lecture

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Marlísio was born and raised in São Paulo city, Brazil.

After graduating as a Civil Engineer from the Federal University of Santa Catarina - UFSC, he was granted his Master of Sciences degree on Geotechnical Engineering from the Polytechnic School of University of São Paulo - USP.

Since then, he worked for Figueiredo Ferraz, Bureau de Projetos and Tüv Süd, completing ten years of experience on design and consultancy of tunnels, retaining structures, slopes and dams. Currently, he is a PhD candidate at the University of Wollongong, Australia.

He published dozens of technical articles internationally. His awards include the best oral presentation at the 3rd Brazilian Tunnelling Congress, best technical article on the XV Panamerican Conference on Soil Mechanics and Geotechnical Engineering and finalist for the category “Young Tunneller of the Year” on the ITA-Awards.

Marlísio was the first president for the Young Members Group of the Brazilian Tunneling Committee. He engaged in the organization and scientific committees of international conferences. As a representative member of ISSMGE’s TC 204, he helped bringing their international symposium for the first time to the Americas, as the symposium vice-president.

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Personal Account on Reliability Analyses, from a Young Geotechnical Engineer

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Abstract. The use of deterministic stability analyses for geotechnical works is still common practice. Such analyses consider a single set of input parameters and therefore a single result is taken as definitive and compared to limits established by codes. However, a deterministic stability assessment taken as satisfactory may be associated with a probability of failure considered as high. This is why one approach should not suppress the other, they should rather be complementary. A similar comment can be made for the probability of failure, while a small value may not be accepted within a densely occupied urban scenario and a high value may be considered satisfactory within an uninhabited area. This is why the risk should also be evaluated rather than solely the probability of failure. Discussions concerning the risk of failure instead of mere deterministic approaches have significant importance, bearing in mind either insurance needs or the development of projects that are both more reliable and cost-effective.

Keywords. Tunnels, Dams, Uncertainty, Probability, Risk, Bayes theorem.

1. Introduction

The present paper is a brief account on the early stages of the author's professional career and was solicited after the Pan-American Bright Spark Lecture award had been granted by the Young Member Presidential Group - YMPG from ISSMGE. This award, on its debut, is intended to acknowledge the mature research and/or practice of young geotechnical engineers.

The establishment of groups devoted to the new generation of engineers is of paramount importance. They should have full support from our societies and associations, being comprised of recently graduated professionals and undergrad students, or even teenagers willing to start a graduation course yet not utterly decided about choosing Geotechnical Engineering as a future career.

Such groups are responsible for bridging the gap between different generations and empowering the youngers with voice and space within our geotechnical society.

One should not expect academia to transmit all knowledge to their students, leaving to senior professionals the responsibility of gathering protégés and enriching their minds with past experiences, expertise, and most importantly with guidance. Conversely, senior professionals must always bear an open-mindedness to innovation, usually conveyed by the youngers. This mutual relationship is beneficial for both ends and must always be encouraged.

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Fortunately, the author has been given the satisfaction of receiving a remarkable and excellence tutoring, in many diverse situations and during different extents of time, so that it would not be possible to reproduce a thorough citing. Nonetheless, bearing in mind this paper theme on Reliability Analyses, there are six names that must be praised in recognition to their contributions during the author's first twelve years of experience as a young geotechnical engineer. These outstanding Brazilian friends, inspiring mentors, are presented in Figure 1.

Arsenio Negro is a geotechnical designer and consultant who employed Marlísio with the main purpose of implementing in his company Bureau de Projetos a working group focused on non-deterministic studies. He has been successful at this hard and challenging task of gradually and continuously changing the deterministic culture a long time set in the geotechnical working practice.

Paulo Ivo B. Queiroz, kindly known as P.I. (π), is a Professor at the Aeronautical Technology Institute - ITA, expert on statistical treatment of data who has developed diverse probabilistic studies as a consultant for Bureau de Projetos, including health risk assessments of contaminated grounds. Together with Arsenio and Marlísio, he assessed probabilities of failure for several underground soil excavations.

Nelson Aoki is a Professor at the University of São Paulo - USP, specialized on foundations. He was elected to deliver the Milton Vargas Lecture 2011 titled "Probability of ruin and factor of safety for foundations" [1], as well as the Pacheco Silva Lecture 2016 titled "The factor of safety paradigm" [2],both cross-country lectures promoted by the Brazilian Association on Soil Mechanics and Geotechnical Engineering - ABMS.

Waldemar C. Hachich is a Professor at the University of São Paulo - USP, responsible for elective classes on geotechnical reliability. He is a member of ISSMGE's Technical Committee on Engineering Practice of Risk Assessment and Management (TC-304) and was elected to deliver the Milton Vargas Lecture 2018 titled "Safety, reliability and risks in geotechnical works" [3].

Tarcísio B. Celestino is a Professor at the University of São Paulo - USP, as well as the engineering manager at the company Themag. He permeates brilliantly between academia and practice, having supervised many post-graduate researches on reliability studies for the stability of underground rock excavations.

André P. Assis is a Professor at the University of Brasília - UNB, has supervised post-graduate researches and has given consultancy on reliability studies for assorted types of geotechnical works. He was elected to deliver the Pacheco Silva Lecture 2018 titled "Risk management in geotechnical works: consolidating theory into practice" [4]. Some of his works include probabilistic spatial characterization of rock masses discontinuities and risk assessments of iron ore tailing dams.

Figure 1. Inspiring mentors. From the left: Arsenio Negro, Paulo Ivo B. Queiroz, Nelson Aoki, Waldemar C. Hachich, Tarcísio B. Celestino, André P. Assis.

Evidently, many other world-wide experts could be mentioned for contributing to this paper theme. Some were pioneers on recognizing and stressing the importance of going non-deterministic, for instance, the internationally renowned late Professor Karl Terzaghi and late Brazilian Professors Milton Vargas and Victor de Mello.

Lastly, the following content shall not be taken as intended for teaching reliability methods, stating best procedures or defining state-of-art approaches. Instead, it merely stands as a humble collection of studies on reliability analyses, developed by a young engineer with the aid of brilliant senior tutors.

2. Why Going Non-deterministic?

A deterministic analysis considers a single set of input parameters and therefore yields a single result, which is taken as definitive and compared to limits established by codes or to best practice values. Normally, such parameters are mean or representative values obtained from tests, or even from previous experiences and literature.

Going non-deterministic simply means recognizing that the problem data is actually variable, thus considering at least one parameter as non-constant and analyzing all possible outcomes.

In that sense, a sensitivity analysis is often performed for its simplicity, evaluating how the change in one parameter affects the results. This is normally referred to as analyses with worst/best case scenarios.

However, a more refined analysis may be performed, which considers parameters as random variables represented by a certain probability distribution and having a possible dependency (correlation) amongst each other, the so-called probabilistic analyses.

In the case where the consequences of a certain event are also assessed, risk analyses are enabled. Such consequences are normally monetary-measured, related to the cost of repair/rebuilt, environmental damages, life losses, among others. Herein, the concept of risk is understood as the product of the probability of failure times the cost of its associated damages.

A survey carried out by ISSMGE's Technical Committee on Underground Construction in Soft Soil (TC-204) involved sending a questionnaire to practitioners dealing with the design and construction of urban tunnels in soil [5]. Based on the responses, they found that uncertainty and parameters variability are considered in tunnel projects by probabilistic analyses in only 6% of cases. Deterministic analyses with pessimistic soil parameters and adequate safety factors account for 40% of cases and deterministic analyses with averaged soil parameters and adequate safety factors predominate with 54% of cases.

It is important to point out that such responses are related exclusively to soil tunnels. For that reason, a new survey on risk assessment has been elaborated for a more comprehensive list of geotechnical works. It was created by the ABMS's Brazilian Technical Committee on Risk, a local chapter of the ISSMGE's TC-304. Although this study is unfortunately not published yet, preliminary outcomes indicate the same trend, in which practitioners still favors deterministic analysis.

Reasons for this were addressed by Ralph Peck in 1995, quoted by Whitman [6]: "Practitioners have not readily adopted reliability theory, largely because the traditional methods have been generally successful, and engineers are comfortable with them. In contrast, practitioners in environmental geotechnics require newer, more stringent assessments of reliability that call for a different approach".

Compared with other areas of knowledge, such as structural engineering, mechanics and economics, it is realized that geotechnical engineering lags behind in the use of reliability theories. It should be appraised that, apart from considering geotechnical parameters as random variables, the unpredictability of geological features and the ground spatial variability and heterogeneity should also be considered.

Then... after all, why going non-deterministic?

According to Harr [7], there is an increasing awareness that the inherent properties of geotechnical materials exhibit significant variability and that these uncertainties are not considered when value judgments concerning most likely scenarios are made.

Moreover, it is basically a matter of safety (either for ultimate limit state - ULS or serviceability limit state - SLS). A deterministic stability assessment taken as satisfactory may be associated with a probability of failure considered as high. This is the reason why one approach should not suppress the other, they should rather be complementary.

In order to ease the understanding of such a statement, different design approaches used for assessing safety should be explained:

- The Working Stress Design (WSD) approach, which is based on an overall factor of safety and has been consecrated since the beginning of geotechnical sciences.
- The Load and Resistance Factor Design (LFRD) approach, adopted mainly in North America, or the characteristic values and partial safety factor approach, widely used in Europe, which are the basis for modern design codes.
- The Reliability-based Design (RBD) approach, which considers as target a probability of failure or a reliability index.

With that in mind, Figure 2 illustrates how the use of non-deterministic analyses relates to a more proper assessment of safety. As an example, while a deterministic global factor of safety calculated by the WSD approach as FS = 1.50 is considered safe by different design codes, a probability of failure determined as $P_f = 10^{-2}$ may be considered inadmissible. Conversely, a deterministic FS = 1.25 normally not accepted by design codes may be related to a $P_f = 10^{-5}$, which might be considered as acceptable.

Therefore, it is **wrong** to make any statement towards the safety of geotechnical structures solely based on deterministic analyses (more commonly through the Global Factor of Safety). Due to the existence of uncertainties, it is not possible to assure absolute zero probability of failure, in practical and economic terms. Hence the need for evaluating safety by means of non-deterministic analyses.

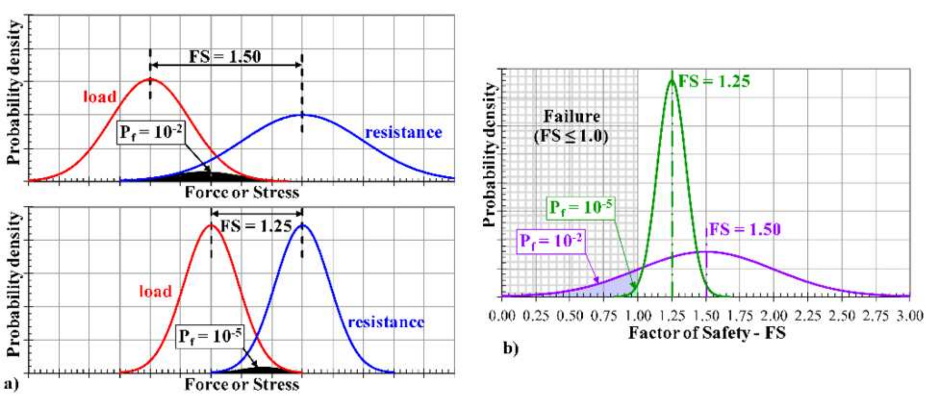

Figure 2. Non-deterministic assessment of safety, using a) LFRD approach and b) RBD approach.

Furthermore, a similar comment on *"relativeness"* can be made for the probability of failure. While a small value may not be accepted within a densely occupied urban scenario, a high value may be considered satisfactory within an uninhabited area. Therefore, the risk should also be evaluated, adding to the picture the failure consequences rather than solely the probability of failure. This may be better explained through Figure 3a, which presents a reduction of the risk by reducing the chances of failure (A), by minimizing the consequences (B), or by ideally both (C).

The monetary values in Figure 3a were deliberately not presented since the criteria for classifying the risk level is not yet common sense worldwide. Moreover, imposing a value to human life remains a very controversial topic [8].

Discussions concerning the risk of failure instead of mere deterministic approaches have significant importance, bearing in mind either insurance needs or the development of projects that are both more reliable and cost-effective.

The ideal risk-based project (design, construction, operation, etc.) should not try to eliminate risks since there is no such thing as an absolute zero probability of failure. Instead, the project should aim to minimize the expected cost (E), which is the mathematical expectation: the non-failure probability times the initial costs, plus the failure probability times the initial and the damages costs.

$$E = (1 - P_f).C_{ini} + P_f.(C_{ini} + C_{dam}) = C_{ini} + P_f.C_{dam} = C_{ini} + \text{Risk} \qquad (1)$$

By minimizing the expected cost, an optimum condition should be found in which the project presents adequate safety without spending unnecessary costs. As an example, a soil tunnel excavated without any ground conditioning presents a lower initial cost, yet the associated risk may be high thus increasing the expected cost; whereas the excessive execution of ground conditioning reduces the risk, however, increases excessively the initial cost and thus the expected cost may remain high.

As depicted by the hypothetical example from Figure 3b, an increase in safety may reduce the project risk, however, followed by an increase in the initial costs (quantity and redundancy of equipment, change in the construction methodology, preventive measures towards consequences, among others). Such relation normally presents a minimum expected cost, which is the objective for a project optimization. Nevertheless, the respective risk level for this optimum condition still must be evaluated, in order to decide whether it is acceptable or a further increase in safety is needed.

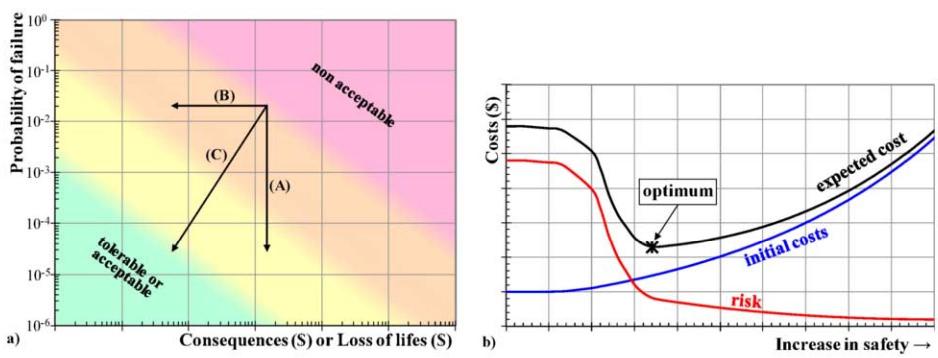

Figure 3. Illustrations for the concepts presented: a) risk as the product of the probability of failure times its consequences, and b) optimum expected cost for a risk-based project.

3. The Influence of Uncertainty

According to [9], there are basically two types of uncertainties: the intrinsic ones (natural variability) and the epistemic ones (lack of data, phenomenon comprehension or modeling ability). While the latter can be reduced by either gathering or improving information, the former must be dealt with and ideally accounted for.

It is highly intuitive the understanding that the less you know and comprehend a certain geotechnical work, the more unreliable the outcomes would be. This could also be translated as the more variable/uncertain the input parameters, or the less accurate the problem depiction, the wider the range of possible results from the analyses; therefore, the higher the number of possible undesired results and hence the higher the probability of failure.

In order to illustrate that, the author and others analyzed how the variability of geotechnical parameters affects the probability of failure of underground excavations [10], and results are following discussed. This study has been awarded the best oral presentation in the 3rd Brazilian Congress on Tunnels and Underground Structures [11].

Two soil tunnels were selected as study cases, both presented in the book "Tunnelling in Brazil" published by the Brazilian Tunneling Committee [12]: the *Alto da Boa Vista Tunnel*, a water tunnel built experimentally in 1978 in Sao Paulo city to link two water treatment plants [13]; and the *Paraíso Tunnel* from Line 2-Green of Sao Paulo metro, built in 1989 near Paulista Avenue to accommodate an additional track for maneuvering trains [14]. For both cases, the geological conditions were comprised by soils of the Neogene/Paleogene period, with groundwater level below the tunnel floor. No instability was noted during construction of both tunnels.

Two different analytical solutions were utilized to assess the underground excavations stability:

- Anagnostou and Kovári, Figure 4a.
  A solution by [15] based on the limit equilibrium method, which mechanism represents a global excavation failure comprised of a wedge and a prism. Similar solutions have been presented by [16], [17], [18] and [19]. The wedge is located at the tunnel face and is limited by a plane inclined at ω degrees to the vertical and approximates the tunnel cross-section by a rectangle or a square. The prism height is equal to the tunnel cover, its width is equal to that of the tunnel and its thickness depends on the angle ω. The soil shear resistance acting on the failure surfaces are calculated considering the Mohr-Coulomb failure criteria. The Simplex optimization algorithm was used to determine the wedge angle ω, in order to minimize the Factor of Safety - FS by maximizing the acting forces and minimizing the resistant forces.
- Modified Mühlhaus, Figure 4b.
  A solution by [20] based on the lower bound theorem of plasticity, modified by [21] and [22] to include gravitational forces. Its mechanism approximates the ground by a thick-walled sphere in a plastic state representing global failure, with an outer surface with radius R_e tangent to the ground surface and an inner surface with radius R_i corresponding to the unsupported excavation length. The excavation stability FS is defined from the internal limiting pressure P_{lim} that would cause collapse, the internal acting pressure P_{int}, the surface load σ_s and the octahedral stress at the tunnel crown σ_{oct}. Recently, the solution was reviewed by [23] enabling the representation of a multi-layered soil profile and accounting for seepage forces due to the groundwater flow.

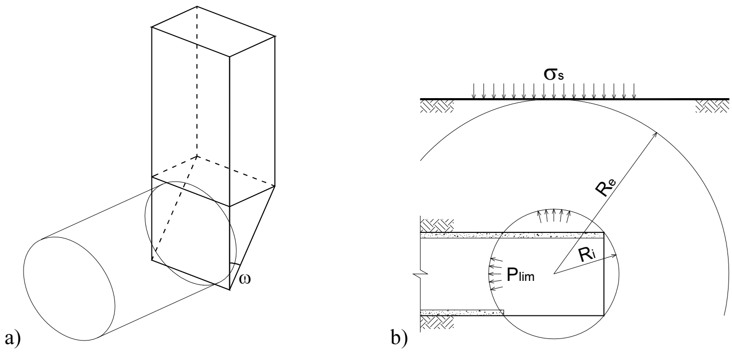

Figure 4. Failure models considered for each analytical solution: a) Anagnostou and Kovári b) Mühlhaus.

The probabilistic analyses performed for the study utilized both first-order approximations and Monte Carlo simulations, following discussed. The analyses applied the analytical solutions presented and changed the standard deviation (variability) of the geotechnical parameters.

- First-order approximation
  Given a function which represents the safety (for instance the analytical solutions presented), a Taylor series expansion may be applied to the function around its mean value and approximated at its first-order term, as discussed by [24] and [25]. Thus, the mean value of the function's results is equal to the result calculated with the mean values of each random variable (i.e. the mean FS is equal to the deterministic FS); whereas the function's standard deviation is calculated from the standard deviations of each random variable. Then, the probability of failure is calculated from these two statistical moments (mean value μ, and standard deviation σ), assuming a normal probability distribution for the results.
- Monte Carlo simulations
  For this approach, the function is repeatedly calculated by varying its random variables and analyzing the overall result. The technique allows any kind of problem to be approached with no restrictions on the number of variables or on the complexity of the function. The success in using it is related to the ability to perform a large number of simulations, which depends on the available computational capacity. Understandably, the simulation is known as a "brute force method", requiring a very large number of results, such that the probability to be assessed approaches the "exact value". Small probabilities of failure require a higher number of calculations. Typically, the simulation is taken as an "exact method" because, in theory, the result tends to exactness when the number of simulations tends to infinity.

After all analyses, it was observed that when considering lower standard deviations for the geotechnical parameters (less variability), the calculated results of FS - Factor of Safety presented a lower dispersion, yielding a lower probability of failure (higher reliability). Conversely, higher standard deviations for the parameters lead to higher probabilities of failure (up to $P_f = 13\%$). This is intuitively expected, as explained previously in Figure 2. However, more important and unsettling is the founding that these high probabilities of failure were associated with deterministic factors of safety (calculated with mean values parameters) that are usually considered satisfactory.

Among several results obtained, Figure 5 presents one typical example of how safety is affected by the variability of the geotechnical parameters.

After *Kolmogorov-Smirnov* adherence tests, the normal distribution of probabilities was chosen as the best fitted. It was noted that the analytical solution by Mühlhaus provided closer adherence to the normal distribution then the Anagnostou and Kovári solution. This can be attributed to the minimization algorithm used by the latter for obtaining FS, yielding a singularity that could be better represented by a bi-modal distribution instead (FS ≈ 1.4 in Figure 5).

Moreover, the study also indicates that the deterministic result not necessarily is equal to the mean value of all results (in Figure 5, $FS_{determ} = 2.07$ and $\mu_{FS} = 1.66$). In such case, the use of first-order approximations may lead to probabilities of failure far from the "exact" value (in Figure 5, $P_f = 0.1\%$ for first-order and $P_f = 4.7\%$ for Monte Carlo).

Three important facts have emerged from the study [10] and [11]:

i) The geotechnical parameters variability directly affects the probability of failure. Consequently, ground investigations should not assess solely mean values of properties but also their dispersions and coefficients of correlation. The lower the parameters' standard deviations (the lower the uncertainty), the greater the project's reliability;

ii) Adequate global factors of safety might be associated with a non-negligible probability of failure. It is believed that, for future geotechnical projects, factors of safety recommended by standards and accepted in practice should be associated with acceptable levels of probability of failure, with targets also recommended by design codes;

iii) The use of first-order approximations is attractive for its simplicity; however, it should be used with caution and preference should be given to more rigorous, robust reliability-based methods.

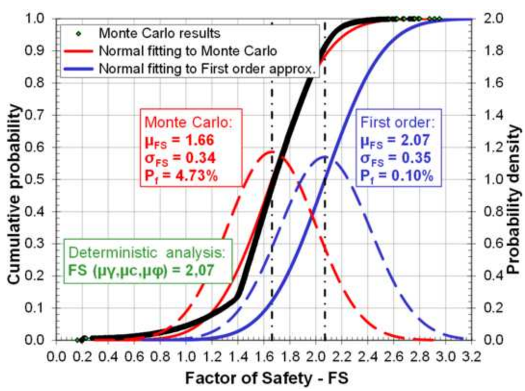

Figure 5. One of several results from the study of [10] and [11]: Alto da Boa Vista tunnel, solution by Anagnostou and Kovári, high values of standard deviation for the geotechnical parameters.

4. The Assessment of Variability

Geotechnical parameters are often characterized by mean values (μ), with no mention of their standard deviation (σ). In order to estimate their dispersion (variability) whenever data is insufficient, published values can be useful, conveniently expressed in terms of the coefficient of variation (V), which is defined as $V = \sigma / \mu$.

Values of coefficients of variation for some geotechnical parameters were compiled by [7], [9], [24], and [26], and are reproduced in Table 1 in terms of maximum and minimum bounds. The values shown cover a wide range, providing just a crude reference with which to estimate the standard deviation.

Although this stands as a possible alternative, one must bear in mind the variability effects, as previously discussed. Hence, the importance of its correct characterization or its reduction based on observations (Bayesian updating), both cases following presented.

Table 1. Values of coefficient of variation (V) for some geotechnical parameters.

γ specific weight (kN/m³)	**c' eff. cohesion (kPa)**	**φ' eff. friction angle (°)**	**Cc compression index**	**k coeff. conductivity (m/s)**
3% to 7%	13% to 100%	7% to 12%	10% to 37%	80% to 240%

4.1. The use of available data

For cases when sufficient data is made available, the natural variability of certain property may be assessed by means of classical statistical analyses, whether or not considering its spatial and/or time dependency.

The author has performed such analysis unfortunately only during rare occasions since the availability of abundant data is not usual. For instance, the reliability assessment of an iron ore tailing dam was carried out based on the statistical analysis of some laboratory tests results [27], as presented in Figure 6.

Four different geotechnical materials were assessed: first and second stages earthfills (embankment), and residual and saprolitic soils (foundation).

The normal distribution of probabilities was chosen to represent the occurrence of the parameters specific weight (γ) and effective friction angle (φ'), while the log-normal distribution was chosen for the effective cohesion (c').

A slightly negative correlation was observed between the cohesion and the friction angle (coefficient of correlation from ρ = -0,134 to ρ = -0,245). Conservatively, both parameters were considered as independent of one another (ρ = 0) for the four materials.

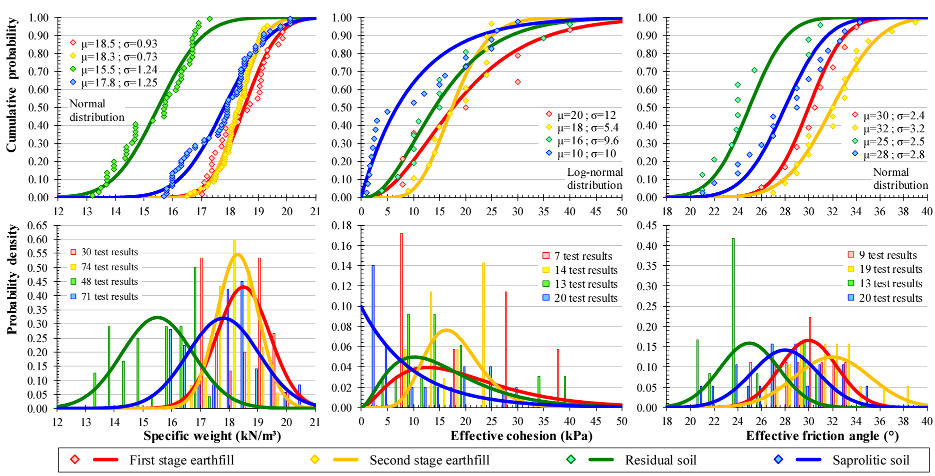

Figure 6. Statistical analysis of laboratory tests results for the assessment of parameters variability [27].

4.2. Reduction of uncertainty through observations

For those cases when properties have to be estimated due to the lack of available data, the high variability (uncertainty) associated with such properties may be reduced through the observation of some particular behavior.

Observational methods are usual for geotechnical engineers, in order to confirm or change idealized models and hypothesis made, as well as re-evaluate the parameters adopted, among other reasons, during the project execution or after its completion.

Back-analyses are the most common approach to evaluate parameters based on some performance, requiring one observation (for instance FS = 1.0) for each parameter to be back-analyzed. These deterministic back-analyses, however, are not capable of providing information concerning the parameters variability.

On the other hand, Bayesian probabilistic back-analyses provide the probability of occurrence of each back-analyzed parameter. Moreover, the number of performance observations does not need to be equal to the number of back-analyzed parameters, as imposed by the deterministic approach. For these reasons, the Bayesian updating poses as a powerful tool, which should be used more often.

It consists of applying the Bayes theorem of conditional probabilities, in which the likelihood of a prior event is updated given that a later event has occurred.

For instance, the probability distribution (type and moments) of geotechnical parameters may be updated conditioned to the realization of some observation, such as a collapse, a change on the expected behavior or any other type of performance measurement. Since more information is added, the updated probability relates to a more specific scenario with higher certainty associated with its occurrence and, therefore, it presents less variability than the previous one (smaller standard deviation).

The Bayesian probabilistic back-analysis methodology presented herein is based on [28] and [29] and considers a normal probability distribution for both the variables to be updated and the performance observations.

The back-analysis starts by acknowledging the parameters to be updated (initial state variables), with a vector of mean values {s'} and a matrix of covariance Cov[s']. Following, the performance observations are also represented by a vector of mean values {P} and a matrix of covariance Cov[P].

The updating is based on the comparison of the performance observations and the predicted values of performance, i.e. {P} – {p}, where the latter is calculated using any recognized method (analytical solutions, numerical simulations, etc.).

The methodology considers a linear relationship between the initial state variables {s'} and the predicted values of the performance {p}:

$$\{p\} = [A].\{s'\} + \{B\} + \{v\} \tag{2}$$

where [A] is the linear coefficients matrix and {B} is the independent terms vector for the hyperplane adjusted to the predicted values of performance (for instance, using least squares method). A vector of errors {v} may also be included, related to a possible error trend from the prediction method or the observation method (systematic errors).

The updated mean values vector of the state variables {s"} is calculated by:

$$\{s''\} = \{s'\} + Cov[s'].[A]^T.([A].Cov[s'].[A]^T + Cov[P])^{-1}.(\{P\} - \{p\}) \tag{3}$$

The updated covariance matrix of the state variables Cov[s"] is attained by:

$$\mathrm{Cov[s'']} = \mathrm{Cov[s']} - \mathrm{Cov[s'].[A]^T.([A].Cov[s'].[A]^T + Cov[P])^{-1}.[A].Cov[s']} \quad (4)$$

A practical example on the use of this methodology was presented by the author and others [30], in which values of earth coefficient at rest (K_0) and of deformability modulus at 50% of failure (E_{50}) were back-analyzed for fine soils found along the tunnels from Line 3 of Santiago Metro, Chile.

The mean values and variances of the initial state variables were defined as: i) for K_0, the lower and upper limits of 0.40 and 1.60 were considered according to previous experience, then confidence levels of 5% and of 95% were respectively adopted; ii) for E_{50} (MPa), laboratory and *in-situ* tests results were used. The initial state variables were considered independent from one another (zero covariance), resulting in:

$$\{s'\} = \begin{Bmatrix} K_0 \\ E_{50} \end{Bmatrix} = \begin{Bmatrix} 1.00 \\ 37.64 \end{Bmatrix} \quad \text{and} \quad \mathrm{Cov[s']} = \begin{bmatrix} 0.365 & 0 \\ 0 & 18.49 \end{bmatrix}$$

The performance observations were based on the field monitoring installed during the construction of Line 2 of Santiago Metro, whose tunnels were excavated through the same type of fine soils under study. A total of 38 monitoring sections were evaluated, using the following observations: logarithm of the surface settlement at the tunnel symmetry axis (Ln ρ_{sup}); logarithm of the calculated maximum transversal angular distortion at surface (Ln β); lining displacements in the vertical direction at the tunnel crown (δ_v) and in the horizontal direction at the tunnel springline (δ_h); and the radial ground stresses acting onto the tunnel lining at the crown (σ_v) and at the springline (σ_h). The statistical analysis of these observations yielded the following mean values and covariances (displacements in mm and stresses in MPa):

$$\{P\} = \begin{Bmatrix} \mathrm{Ln}\ (\rho_{sup}) \\ \mathrm{Ln}\ (\beta) \\ \delta_{h\ left} \\ \delta_v \\ \delta_{h\ right} \\ \sigma_v \\ \sigma_h \end{Bmatrix} = \begin{Bmatrix} 1.62 \\ 7.76 \\ 0.38 \\ 2.50 \\ 0.43 \\ 0.04 \\ 0.06 \end{Bmatrix} \quad \text{and} \quad \mathrm{Cov[P]} = \begin{bmatrix} 0.25 & -0.20 & -0.07 & 0.08 & 0.36 & 0 & 0 \\ -0.20 & 0.29 & 0.50 & 0.81 & -0.63 & 0 & 0 \\ -0.07 & 0.50 & 4.44 & 0.17 & -0.38 & -0.07 & 0.04 \\ 0.08 & 0.81 & 0.17 & 6.91 & -2.02 & -0.03 & 0.01 \\ 0.36 & -0.63 & -0.38 & -2.02 & 2.81 & -0.04 & 0.02 \\ 0 & 0 & -0.07 & -0.03 & -0.04 & 0 & 0 \\ 0 & 0 & 0.04 & 0.01 & 0.02 & 0 & 0 \end{bmatrix}$$

The predicted values of the tunnel performance were obtained using a numerically derived model based on 2D and 3D finite-elements analysis [31]. A systematic error of -0.046 MPa was accounted for the pressure cells because they tend to underestimate ground stresses acting on tunnel linings. The linear relationship between the initial state variables and the predicted values of performance was found to be represented by the following linear coefficients matrix and independent terms vector:

$$[A] = \begin{bmatrix} -4.00 \times 10^{0} & -2.86 \times 10^{-2} \\ 5.15 \times 10^{0} & 2.86 \times 10^{-2} \\ 7.72 \times 10^{0} & 3.84 \times 10^{-3} \\ -5.76 \times 10^{0} & -8.79 \times 10^{-3} \\ 7.72 \times 10^{0} & 3.84 \times 10^{-3} \\ 1.41 \times 10^{-1} & 1.42 \times 10^{-3} \\ 7.03 \times 10^{-2} & -3.73 \times 10^{-4} \end{bmatrix} \quad \text{and} \quad [B] = \begin{Bmatrix} 6.59 \times 10^{0} \\ 2.06 \times 10^{0} \\ -7.26 \times 10^{0} \\ 7.14 \times 10^{0} \\ -7.26 \times 10^{0} \\ -8.51 \times 10^{-2} \\ 6.49 \times 10^{-2} \end{Bmatrix}$$

The back-analyzed parameters found for the Santiago fine soils are $K_0 = 0.84$ and $E_{50} = 40.1$ MPa, with standard deviations of 0.06 and 15.7 MPa, respectively. These values were incorporated into the design of Line 3 of Santiago Metro, despite both being higher than what was used previously for the Line 2 design. A higher K_0 enabled the tunnel lining to be optimized, whereas a higher stiffness was favorable regarding ground settlements, reducing potential damages to nearby structures.

5. Case Study on Reliability-based Design

A reliability analysis was performed by the author and others to assess the probability of failure for the main tunnel of Universidad de Chile Station, Line 3 of Santiago Metro [32].

The excavation was completed in 2016, crossing beneath an existing track tunnel from Line 1, built in the 1970s. The surface area is densely occupied, with historical and government buildings. Two heavy traffic roads are also on site, one on the ground level parallel to the track tunnel, while the other is an underground passage parallel to the station tunnel (see Figure 7a).

Due to such complexity, Santiago Metro required during the detailed design stage that the probability of failure for the Line 3 excavations was minimized.

5.1. Site description

The ground is a thick deposit of fluvial, well graded and dense gravel with a finer cohesive matrix. As presented in Figure 7a, the stratigraphy is comprised by topsoil (1.0 m thick earthfill), over the second deposition (3.0 m thick) and the first deposition of gravels from the Mapocho River.

The cross section for the station main tunnel is presented in Figure 7b. The central pillar was chosen for the stability analyses since its excavation area (70.48 m²) is larger than that of each side drifts.

To assess the excavation stability, the analytical solutions by Anagnostou and Kovári [15] and by Mühlhaus [20] modified by [21] and [22] were used. Both solutions were previously discussed in section 3 (see Figure 4). The tunnel cross-section presented in Figure 7b also indicates the projection of the inner surface of the thick-walled sphere for the modified Mühlhaus failure model (blue circle) and the excavation face for the Anagnostou and Kovári failure model (red rectangle).

The originally proposed soil conditioning, presented in Figure 8a, consists of forepoling in the tunnel roof (injected self-drilling bolts, Ø40/16mm, Ø70mm boring, 6m long with 3m overlapping) and soil nailing in the tunnel face (glass fiber bars injected with resin, Ø22mm, Ø123mm boring, 8m long with 4m overlapping). The reliability study led to a change in the soil conditioning design, as presented in Figure 8b: the forepoling was extended from 6 to 12m long with 8m overlapping, and the three upper layers of the frontal soil nailing were extended from 8 to 10m long with 6m overlapping.

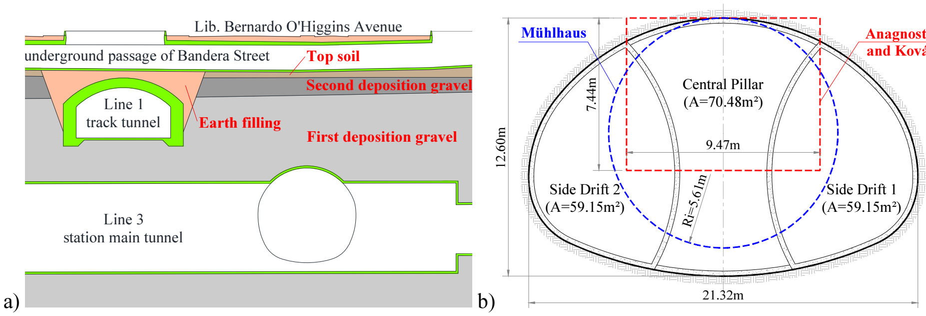

Figure 7. The main tunnel for the Universidad de Chile Station: a) longitudinal section, with soil stratigraphy and surrounding structures, b) cross-section geometry [32].

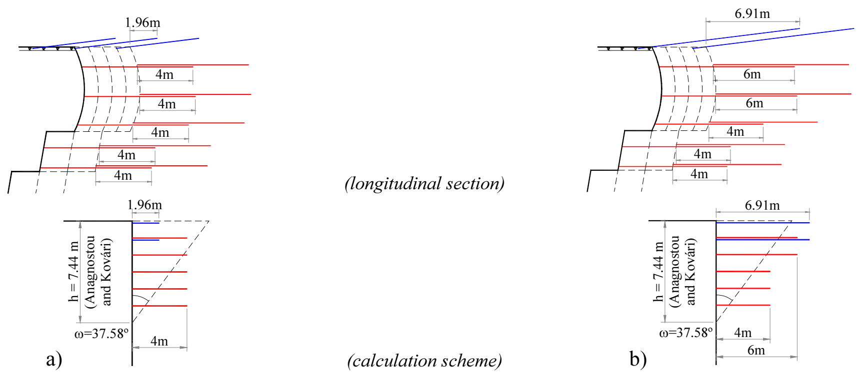

Figure 8. Soil conditioning for the station main tunnel: a) original design and b) design change [32].

5.2. Reliability analyses

The reliability analyses were carried out with Monte Carlo simulations, considering only the variability of the geotechnical parameters.

A set of different analyses was required due to the variations on the tunnel cover (15.6 m below the underground passage and 5.0 m below the existing Line 1 track tunnel), for the two analytical solutions and to represent a possible encounter of two excavation headings. This resulted in six analyses, as illustrated in Figure 9. The representation of the two headings encounter is only possible using the failure model of Anagnostou and Kovári, for the Mühlhaus failure model imposes a spherical symmetry.

The internal pressure P_{int} was determined by the mobilization of the soil conditioning (only the tension force in the bolts was considered and the shear and bending strengths were neglected), by the reaction of the open shell lining foundation (calculated according to [33]) and by the stabilizing horizontal pressure provided by the frontal core (berm).

The parameters used for the deterministic tunnel design were taken as the mean values for the probabilistic analyses. The standard deviations for γ and φ' were determined after coefficients of variation from literature; for c' laboratory test results were used; for K_0 values from a probabilistic back-analysis presented by [34] were adopted.

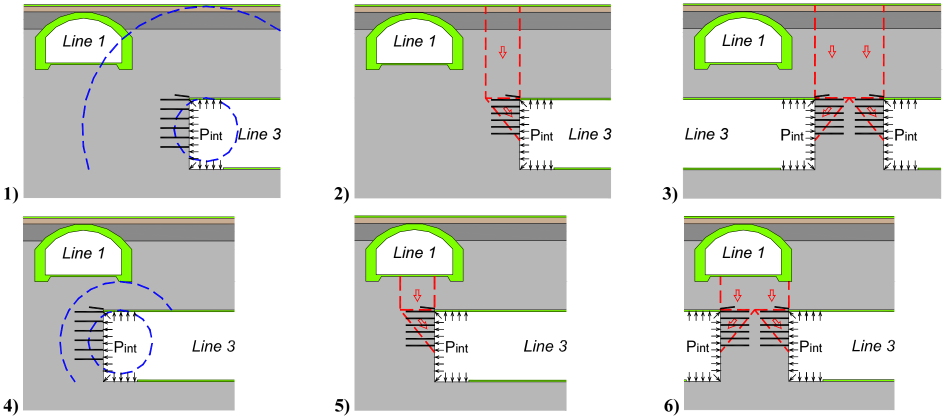

Figure 9. An illustrative summary of the analyses performed [32].

Table 2 presents the probabilities of failure calculated for each one of the reliability analyses. The results from the initial analyses, considering the originally designed soil conditioning, indicate that the excavation of the Central Pillar of the University of Chile Station main tunnel presented a probability of failure up to 6.44%. It was also noticed that the encounter of two tunnel headings increases the excavation probability of failure.

An important observation is that the modified Mühlhaus analytical solution yields higher probabilities of failure than the Anagnostou and Kovári solution, as expected. One should keep in mind that these solutions do not provide an exact value for safety. The lower bound theorem of the Mühlhaus yields safer values, whereas the Anagnostou and Kovári limit equilibrium method can roughly act as an upper bound approximation (the system energy balance and a viable kinematic motion are not ensured), what can be unsafe.

Other analyses were following performed, this time changing the soil conditioning as depicted by Figure 8, whose results are also presented in Table 2. The encounter of the two excavation headings below the existing Line 1 track tunnel presented a 0.23% probability of failure, which led to the design specification that such an encounter should be executed away from this region. For all remaining analyses, the probability of failure was considered negligible (< 0.00). This is because there was not even one case of failure observed throughout the 30,000 simulations carried out for each analysis ($P_f < 3.3_x10^{-5}$).

Table 2. Probabilities of failure (%) calculated for the different analyses [32].

Soil conditioning	1	2	3	4	5	6
original design	4.74	< 0.00	0.03	6.44	0.09	1.37
changed design	< 0.00	< 0.00	< 0.00	< 0.00	< 0.00	0.23

6. Case Study on Risk Assessment

A risk assessment was performed by the author and others for an iron ore tailing dam [27]. Due to restraints towards confidentiality, the object of this study shall be herein referred to as Dam X. This risk assessment has been granted the *José Machado Award* for the best Brazilian geotechnical project of the biennium 2017-2018, by ABMS.

Tailing dams are complex structures in which the material accumulated in the reservoir presents no trivial geotechnical behavior. This is an important aspect especially when dam embankments are raised by the upstream or centerline methods, for the tailings that once acted just as a load start to act also as the embankment foundation.

Another particularity of tailing dams concerns the variation of their conditions throughout their lifespan. Typically, the dam is not built to its final height, being raised as the reservoir volume is depleted. This is especially due to fluctuations in the ore market value, which is responsible for the mine operation rhythm and for the consequent tailing disposal plan. Consequently, a single dam can be built at different times by different constructors, based on designs also elaborated by different companies.

Such complexity, translated in terms of material properties and of construction and operation histories, requires a geotechnical risk management elaborated specifically for this type of structure.

The mining company owning Dam X had the initiative of elaborating the so-called Geotechnical Risk Management project, whose purpose is to assess the safety condition of their geotechnical structures. It allows the latter to be managed, having a monetized risk as a guide for preventive or improvement actions. The methodology elaborated for this project, despite being based on consecrated theories, has pioneer application in the world, developed with the collaboration of an international consultants panel.

The Dam X was selected as a case study in order to present this risk assessment methodology. It is a compacted earthfill dam, has 71 m of height, 810 m of crest length and 130 million m³ of reservoir volume. Its design was developed to be constructed in three stages, using the downstream raising method. Currently, only the first and second stages were constructed, finished by 1981 and 2016 respectively. The tailings are disposed of in a single point at the reservoir rear end, by gravity.

Figure 10 presents the cross-section selected for analysis, which refers to the dam's maximum height. The field monitoring counts with 19 piezometers (PZs), 8 water level indicators (INAs), 6 reservoir level rulers, 2 flow meters and 21 topographical landmarks. The piezometric level acting on the cross-section was interpreted after the evaluation of the instruments installed. Distinct piezometric levels were observed for the foundation (red line) and for the embankment (blue line), justified by the different hydraulic conductivities.

The embankment (both stages) was built with a clayey earthfill extracted from the abutments, which was originally comprised of residual soils from gneiss and micaxist. The earthfill presented fines content > 50% and plasticity index > 30%.

The foundation (at the cross-section) is comprised of residual and saprolitic soils, weathered from shales. The residual soil was characterized as sandy-silt, with fines content from 30% up to 64% and plasticity index from 11% up to 37%.

The Geotechnical Risk Management methodology has five steps for each structure: i) data acquisition and consolidation; ii) probability of failure calculation; iii) hypothetical dam-break study; iv) assessment of consequences; and v) risk calculation.

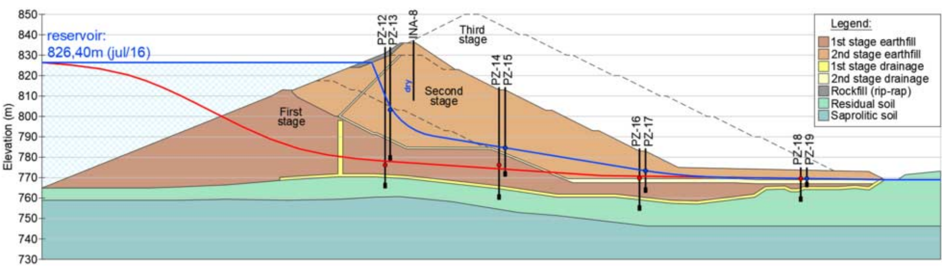

Figure 10. Cross-section with piezometric levels interpreted for the **foundation** and the **embankment** [27].

6.1. Probabilities of failure

Recent studies by [35], based on compilations from *ICOLD*, *UNEP* and *US Department of Interior*, validate the definition of four main failure modes for tailing dams: overtopping, internal erosion, embankment instability, and liquefaction.

The probability for each failure mode is calculated using quantitative approaches, which reduces the results subjectivism by minimizing the qualitative aspects along the process.

Overtopping

The analysis begins with the flood routing simulation for all existing upstream water bodies, resulting in the affluent and effluents hydrographs of the dam's reservoir. Moreover, several rainfall events are analyzed, each one associated with a certain duration and Return Period (RP). The probability of failure is then calculated as the inverse of the return period (1/RP) for the event that causes the overtopping.

For the Dam X, the overtopping was not verified even for the maximum flood caused by the Probable Maximum Precipitation (PMP), remaining a freeboard of 1.90 m. Therefore, the probability of overtopping was neglected.

Internal erosion

The methodology for evaluating the internal erosion potential combines Event Trees with Fault Trees and has been used since the late '90s by USACE[36], USBR [37], some Australian organizations, among others, still under development in the world. It is based mostly on [38], [39] and [40], bearing the appropriate adaptations and specificities for tailing dams. Its use has the advantage of an in-depth reflection on the failure mode progression and the factors that influence each of the phenomenon stages.

There are basically four main internal erosion mechanisms: regressive erosion with a piping formation, suffusion (internal instability), erosion due to concentrated flow and erosion on the contact between different materials. All possible initiating events likely to occur must be evaluated (piping through the dam or its foundation, erosion on the spillway concrete/soil contact, etc.).

An **Event Tree** is elaborated for each initiating event, with the nodes: (a) initiation; (b) continuation; (c) pipe formation; (d) pipe progression; (e) process detection and intervention; and (f) failure mechanism formation. The probability of occurrence of each node is determined after specific **Fault Trees**, which depend on the type of mechanism and the initiating event. The Fault Trees combine evaluations on soil properties, hydraulic gradients, the presence of filters, the ability to detect and intervene in the process, etc.

Figure 11 presents the Event Tree for the worst scenario evaluated for Dam X, piping through the left abutment, with $\mathbf{P_f = 1_x10^{-5}}$. All Fault Trees were evaluated for every single node of all the analyzed Event Trees, however, they are not herein presented.

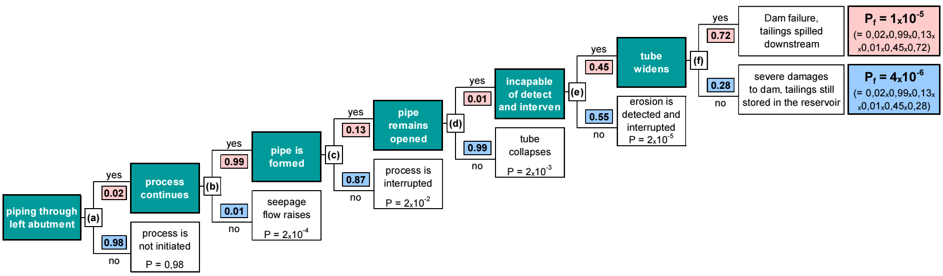

Figure 11. Event Tree for piping through the left abutment, with the nodes' probability of occurrence [27].

Embankment instability

The stability analyses were performed using the limit equilibrium method. The geotechnical parameters were considered as random variables, with probability distributions determined after statistical analyses of laboratory tests results, previously presented in Figure 6. The piezometric levels are presented in Figure 10.

The probability of failure for the embankment instability was assessed through Monte Carlo simulations. A total of 500,000 analyses were performed with non-fixed failure surfaces, i.e. a new critical surface was determined for every single analysis.

The total number of simulations was considered adequate since the mean value and the standard deviation of the Factor of Safety had converged to steady values (μ_{FS}=1.55 and σ_{FS}=0.11 respectively). However, the frequentist probability of failure (number of FS<1.0 observations divided by the number of simulations) did not converged, what would require a greater number of simulations, but not viable in practice.

In order to avoid the influence of a limited number of analyses, the probability of failure was determined by adjusting a probability distribution to the results, according to Figure 12. After *Kolmogorov-Smirnov* adherence tests, the Beta distribution was selected as best fitted, yielding $\mathbf{P_f = 4_x10^{-6}}$.

For this assessment, the deterministic FS was found to be higher than μ_{FS}, what is the same trend discussed in section 3 with outcomes from [10] and [11].

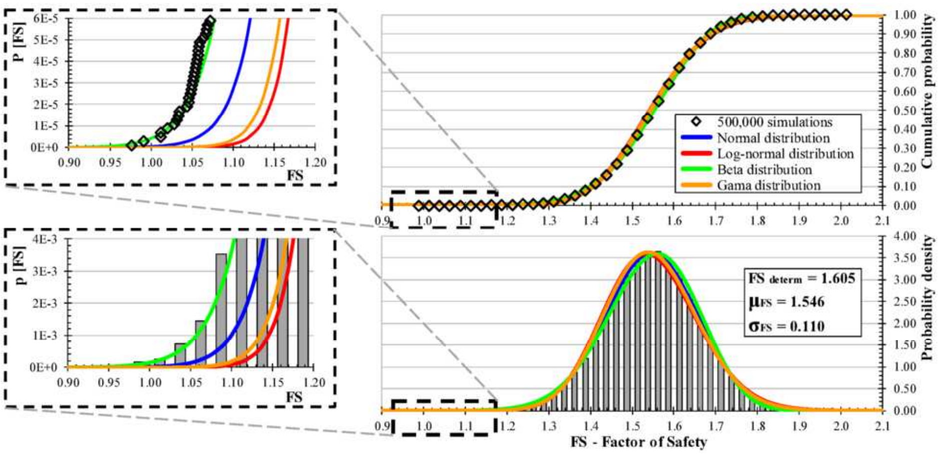

Figure 12. Results of Factor of Safety obtained from the 500,000 simulations [27].

Liquefaction

This phenomenon has been society's great concern. Its occurrence is subjected to materials being susceptible and to the deflagration of a trigger.

To be considered susceptible to liquefaction, the material must be non-cohesive, saturated, and present in-situ void ratio higher than the critical value (contractile behavior during an undrained failure).

The trigger can be associated with static or dynamic events, such as excess pore water pressure due to rapid loading (overloads, dam raising, reservoir level increasing), natural seismicity or induced vibrations (heavy equipment traffic, detonations, adjacent structures failures), as well as sudden increase of shear stresses (material removal from the dam toe, foundation differential movement), among others.

The present methodology assesses the liquefaction potential based on [41], [42] and [43], using results from CPTu and/or SPT tests. The stability is assessed using the limit equilibrium method, considering an undrained strength ratio (s_u/σ'_{v0}) for the susceptible materials and maintaining the assigned drained strength (c' and φ') for the remaining materials. The probability of failure is then assessed in the same way as to the embankment instability, using Monte Carlo simulations.

Liquefaction analysis is a topic that demands contributions worldwide since the geotechnical practice still relies on the limit equilibrium method. Advancements are needed towards the determination of the undrained strength (strongly dependent on the test type and on the soil in-situ state, which is difficult to characterize), as well as the approach used for the stability assessment (the analyses must be effective stresses oriented, considering the soil stress-strain behavior with post-peak softening coupled to a consolidation theory able to predict the dissipation of the excess pore water pressures).

The probability of failure due to liquefaction was neglected to the Dam X because the tailings do not act as the foundation (downstream raising) and the clayey soil used for the embankment is not susceptible.

6.2. Hypothetical Dam-break study

The dam-break study is elaborated simulating different scenarios, which include rainy or rainless day, with or without the dam failure. Its main objective is to estimate the influence of the failure, delimiting the flooded area.

The study is developed following the stages: definition of the failure hydrograph, depending on the failure mode; elaboration of the valley's geomorphological model; hydrological characterization of watercourses; flood wave propagation; flood mapping.

The dam-break study for Dam X yielded a flood damage potential up to 155 km downstream of the dam. More expressive inundations were identified along the first 50 km, striking two municipalities with houses and urban infrastructure close to the watercourse.

6.3. Assessment of consequences

The monetized values for the consequences caused by an eventual dam failure are assessed within the flood area, partitioned into six categories: i) economic; ii) health and safety; iii) social; iv) environment; v) regulatory agencies; and vi) company image.

Eight scenarios are evaluated, considering a nocturnal or diurnal failure, during a rainless or a rainy day, with an alert issued by sirens at the failure or 4 hours earlier.

The methodology details the assessment of costs for each category and scenario, with thorough procedures not presented herein. The population at risk was differentiated as diurnal or nocturnal according to the activities developed in the inventory area.

The costs of the consequences caused by the Dam X failure are summarized in Table 3, concerning the scenarios of nocturnal failure. The four remaining scenarios of diurnal failure are not presented, for their outcomes were less critical.

Table 3. Cost of consequences (BR$) for different scenarios of nocturnal failure [27].

rainless day, no alert	rainless day, with alert	rainy day, no alert	rainy day, with alert
40,464,626,894	13,516,793,074	44,357,513,702	15,651,634,166

6.4. Risk calculation

The monetized risk for Dam X is presented in Figure 13, the so-called risk panel. Only the rainless day scenarios are presented since the failure probabilities for the rainy-day scenarios must be multiplied by the probability of a decamillennial rainfall (10^{-4}).

The risk panel allows clear visualization of the risks and facilitates decision-making actions towards their mitigation, by reducing either the probability failure or its consequences.

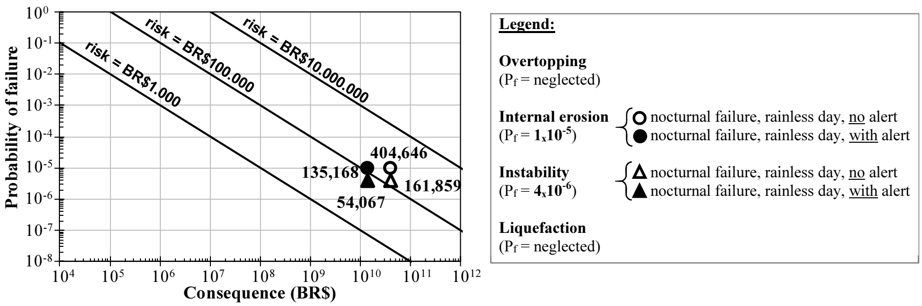

Figure 13. Risk panel for Dam X [27].

7. Final Remarks

Reliability analyses are more often recognizably needed, and it is imperative that its use becomes more common practice. Such an approach must not suppress the conventional, deterministic methods, instead, they should complement each other. As discussed, it is a matter of safety, for a deterministic outcome taken as adequate may correspond to a high probability of failure.

The quantification of the risk, besides minimizing the use of qualitative evaluations which are essentially subjective, represents a fundamental tool for the management of geotechnical structures, allowing the allocation of investments to be optimized.

Computational limitations are no longer a plausible excuse for avoiding going non-deterministic. In fact, brilliant young geotechnical engineers are out there waiting to push boundaries and introduce innovative practices, provided that they are assisted and encouraged by senior mentors.

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