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A new end-bearing capacity equation of piles in crushable soils

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ABSTRACT

The significant impact of grain size distribution on soil mechanics behaviour has been realized long ago; however, only recently, has the newly developed breakage mechanics theory and derived breakage constitutive models incorporated such an obvious essential factor in a novel mathematical way, whilst obeying the thermodynamic framework. Applying a simple breakage model in the context of a piled foundation leads to a new end bearing capacity equation of piles in crushable soils, being simple and also efficient. This novel equation not only highlights the important role of initial grain size distribution in altering the end-bearing capacity, but also includes the impact of friction angle, elastic modulus and Poisson's ratio. All factors are included in a simple mathematical expression that only possesses three parameters, i.e., internal friction angle, Poisson's ratio and a critical comminution (isotropic compression) pressure. This makes the application of the new equation into engineering practice easy and straightforward. As an illustration, an example is given to show a quick and accurate assessment of pile tip resistance using the new equation compared to using other existing counterparts.

Keywords: granular materials, GSD, breakage mechanics, end bearing capacity, piles

1 INTRODUCTION

End-bearing capacities of piles in weak (or crushable) soils do not generally obey the laws of widely-used conventional bearing capacity equations, such as Terzaghi (1943) and Meyerhofl (1976) where the calculated pile tip resistance is seen to increase with increasing friction angles of soils. In most cases, although possessing relatively high friction angles, crushable soils (e.g., calcareous and carbonate sands) have shown much lower tip resistance in engineering practice than those calculated from the conventional methods (Poulos and Chua 1985; Golightly and Hyde 1988; Yamamoto et al. 2009). The reason has been generally acknowledged to be due to soil crushability which governs the mechanical response of crushable soils (Datta et al 1980; Poulos and Chua 1985; Alba and Audibert 1999; Lade et al.1996; McDowell and Bolton 2000).

When crushable soils are subject to various loadings (compression, shearing or a combination of both), the constituent granular particles tend to be fractured, resulting in significant change/evolution of the grain size distribution (GSD) – particularly for those sediments containing large amounts of shell fragments and with high carbonate contents. The GSD's have proven very essential, from both experimental and theoretical standpoints, to assist with understanding of the mechanical behaviour of crushable soils (Datta et al.1979; Einav 2007a). For example, GSD's (along with other elastic soil properties) have been shown to have significant impact on the end-bearing capacities of piles and stress distributions in calcareous and carbonate soils and silica sand in recent investigations (Zhang et al. 2013; Zhang et al. 2014).

The investigations of Zhang et al. (2013) further established a new end-bearing capacity equation for piles penetrating into crushable soils, which results in better predictions when compared to a number of other approaches.

The purpose of this paper is to provide a comparison of results from application of the proposed new equation with results from existing more commonly employed approaches to predicting pile tip resistance.

By revisiting the equation originally proposed, this paper also provides a simpler and more practical solution to accurately estimate the tip resistance, which may be found beneficial for engineering practice.

2 EXISTING END-BEARING CAPACITY EQUATIONS

The end-bearing capacity of piles, q_p is usually expressed as a function of a dimensionless bearing capacity factor (N_q or N^*_q) and initial effective vertical stress, σ_{v0} or alternatively initial effective mean stress, p_0 , as shown in the following:

$$q_p = N_q \sigma_{v0} \tag{1}$$

$$q_p = N_q^* p_0 \tag{2}$$

where N_q or N^*_q may be associated with each other through initial horizontal stress coefficient K_0 :

$$N_q^* = \frac{3}{1+2K_0} N_q \tag{3}$$

Therefore, when investigation of q_p is reported, this is usually in the form of the end-bearing capacity factor, i.e., N_q or N^*_q. The following discussion herein focuses on the equations used to determine such a dimensionless factor.

2.1 Classic/traditional equations

One of classic equations to assess pile tip resistance factors was proposed by Prandtl (1921), in which an end-bearing capacity factor was derived for flat strip surfaces punching through idealised weightless incompressible media by assuming a general shear failure type. His equation is expressed as:

$$N_q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) e^{\pi \tan\phi} \tag{4}$$

where Φ is the internal frictional angle of soil. The assumption of a general shear failure, however, does not consider local shear failure which is commonly observed for a pile driving into compressible soils. In this regard, Terzaghi (1943) suggested a modified equation by adopting a reduced friction angle $\phi_r = \tan^{-1}\left(\frac{2}{3}\tan\phi\right)$ to account for the local shear failure pattern for a pile tip foundation in compressible soils:

$$N_q = \tan^2\left(\frac{\pi}{4} + \frac{\phi_r}{2}\right) e^{\pi \tan\phi_r} \tag{5}$$

However, equation (5) is known to give conservative answers for piles (Vesic 1973), along with another fact that it does not reflect the pressure dependance of the bearing capacity factor with increasing pile penetration depth.

Vesic (1973) attempted to modify Prandtl's equation (1), by considering the soil compressibility through the introduction of a reduction compressibility factor ξ_{qc} :

$$N_{qc} = N_q \xi_{qc} \tag{6}$$

$$\xi_{qc} = \exp\left[(3.07\sin\phi)\frac{\log(2I_r)}{1+\sin\phi} - 3.8\tan\phi\right] \tag{7}$$

where N_q is taken from equation (4), the sand rigidity index $I_r = G/(\sigma_{v0} \tan\phi)$ was introduced to account for the influence of both the initial effective vertical stress σ_{v0} and the shear modulus G .

Vesic (1975) proposed another equation, an alternative to his own equations (6) and (7), as shown:

$$N'_{qc} = N'_q \xi'_{qc} \tag{8}$$

Where

$$N'_q = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)e^{\left(\frac{\pi}{2}-\phi\right)\tan\phi} \tag{9}$$

$$\xi'_{qc} = \frac{3}{3-\sin\phi}(I_{rr})^{\frac{4\sin\phi}{3(1+\sin\phi)}} \tag{10}$$

$$I_{rr} = I_r / \left(1+\bar{\varepsilon}_v I_r\right) \tag{11}$$

where $\bar{\varepsilon}_v$ indicates an average volumetric strain in the plastic zone, which is difficult to obtain accurately and therefore it is recommended to take $\bar{\varepsilon}_v = 0$ for very dense soils to give an upper bound estimate of pile tip resistance.

Houlsby et al (1988) proposed an empirical formula to estimate end-bearing capacity of piles in uncemented calcareous sand as shown below:

$$q_p = 38 p_a \left(\frac{p_0}{P_a}\right)^{0.6} \tag{12}$$

where P_a is atmospheric pressure, typically being taken as 100 kPa; therefore the end-bearing capacity factor can be expressed as:

$$N_{qh} = \frac{38 p_a}{p_0^{\ 0.4} P_a^{\ 0.6}} \tag{13}$$

The above equations share some similarity, i.e., an end-bearing capacity factor is mainly (or exclusively) a function of the internal friction angle of a soil, and increases as the angle increases. However, this conclusion has proven not to apply to crushable soils, as previously observed in terms of experimental observations and many engineering projects, due to the lack of consideration of soil crushability. A new end-bearing capacity equation was therefore developed to account for soil crushablity *via* capturing the evolution of GSD’s, while totally obeying the framework of the rules of thermodynamics.

2.2 Proposed new equation

The new end-bearing capacity equation has been presented in Zhang et al (2013) which takes into account the impact of granular crushability along with other critical factors such as internal friction angle and elastic moduli (or alternatively Poisson ratio).
The proposition of the new equation originates from a micromechanics-based continuum theory of breakage mechanics (Einav 2007a; Einav 2007b; Einav 2007c; Einav 2007d) which links the evolving cumulative GSD due to grain crushing to mechanical behaviour of granular materials within the framework of thermodynamics.

A simple breakage model of breakage mechanics was implemented into finite element models of a pile penetrating into crushable soils, thus enabling a sensitivity analysis of pile tip resistance as a function of varying contributing factors including initial GSD, friction angle and elastic moduli (or Poisson ratio).

As a consequence of a series of finite element analysis, the new equation to predict the end-bearing capacity of piles has been derived as:

$$N_q^* = \alpha \left(\frac{p_c}{p_0} \right)^{2\beta} \tag{14}$$

where for uncemented calcareous and carbonate sands, it suggests:

$$\alpha = M^3 + 14\left(\frac{G}{K} \right)$$

$$\beta = 0.42$$

and

$$p_c = \sqrt{2KE_c / \vartheta}$$

The above model possesses five physically identifiable mechanical parameters:

1) elastic shear modulus *G*;
2) elastic bulk modulus *K*;
3) friction coefficient $M = q_f / p_f$, indicating the friction ratio at critical state between the ultimate mean and triaxial shear stresses, p_f and q_f respectively;
4) critical breakage energy (with the dimension of stress) E_c, indicating the yielding condition due to breakage dissipation;
5) ϑ is called the 'criticality proximity parameter', which measures how far the initial GSD is from the ultimate GSD (Einav 2007a) which conveniently can be assumed to be a fractal pattern. For the readers understanding, ϑ can be simply considered, although not absolutely strictly, as an indicator of crushability ϑ for a granular soil in a general sense. This is because the higher the value of ϑ , the more uniformly graded particles exist in a granular soil, and therefore the soil is easier to crush.

The parameters in 1) to 4) are typically measured and calibrated from a single standard isotropic compression test and a subsequent drained shear test. The parameter ϑ can be calculated using a statistics moment (Einav 2007a) for a known initial cumulative GSD and an estimated fractal ultimate cumulative GSD.

The aforementioned traditional end-bearing capacity factors have been compared with the proposed new equation based on results of a model pile test for a typical calcareous soil, Chiibishi Sand, from Kuwajima et al. (2009), as shown in Figure 1. It is shown that the proposed new equation gives the best estimate of the end-bearing capacity factor at a deep pile penetration (*S*) of three pile diameters (*D*) such that ultimate pile tip resistance might be closely approximated. The behaviour of pressure dependence of the bearing factors is also noted in, and reproduced by, the formulae of Vesic (1973; 1975) and Houlsby (1988), although they either overestimate or underestimate the experimental result.

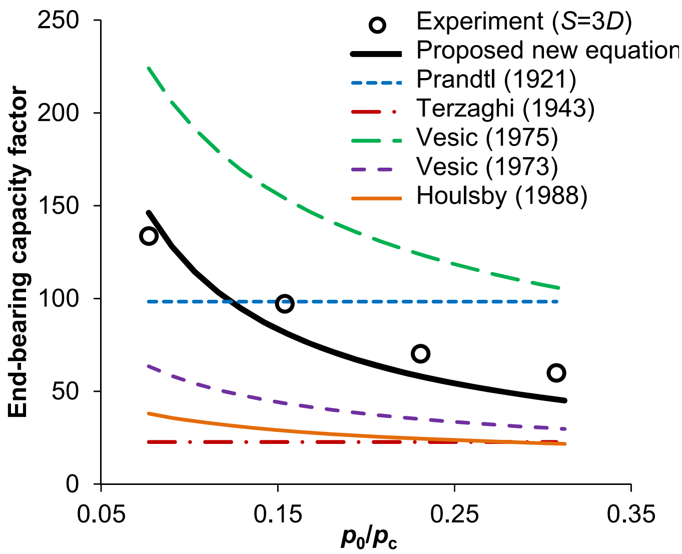

Figure 1. Comparison of various end-bearing capacity factors against initial mean stress normalised by p_c for Chiibishi sand (mainly replotted from Zhang et al. 2013; here with the additional result from Houlsby equation (13))

3 CASE STUDY – DEMONSTRATION OF USING NEW EQUATION

This section illustrates how to quickly and accurately estimate the end-bearing capacity factor using the proposed new equation.

Considering the general relationship: $M = 6\sin\phi/(3-\sin\phi)$ and $(G/K) = (3-6\nu)/(2+2\nu)$, the original equation (14) can be converted to become:

$$N_q^* = \left(\left(\frac{6\sin\phi}{3-\sin\phi}\right)^3 + 7\left(\frac{3-6\nu}{1+\nu}\right)\right)\left(\frac{p_c}{p_0}\right)^{0.84} \tag{15}$$

The equation (15) herein only possesses three physically identifiable mechanical parameters: internal friction angle, ϕ, Poisson's ratio, ν and the critical comminution pressure, p_c. In contrast, the equation (14) requires five parameters as previously explained.

Let us first examine p_c, which is dependent on initial GSD *via* the indicator of crushability ϑ (i.e., $p_c = \sqrt{2KE_c/\vartheta}$). This explains that the various initial GSD's of a given granular soil would lead to different isotropic comminution pressures. For instance, the higher the ϑ (i.e., more uniform grain sizes), the lower the p_c would be, indicating a truth that the soils with more uniform particles are easier to crush than those with more well-graded particles. As long as p_c is appropriately determined in an isotropic compression test, it is herein not necessary to know the exact initial cumulative GSD. It also becomes irrelevant whether the soil consists of a single source or a mixture of granular material sources which may have different individual values of K and E_c since these values will be reflected in the measured value of p_c.

For determining internal friction angle ϕ, the most straightforward way is to carry out a general direct shear test while a ν of 0.3 can be estimated as being appropriate in general engineering practice.

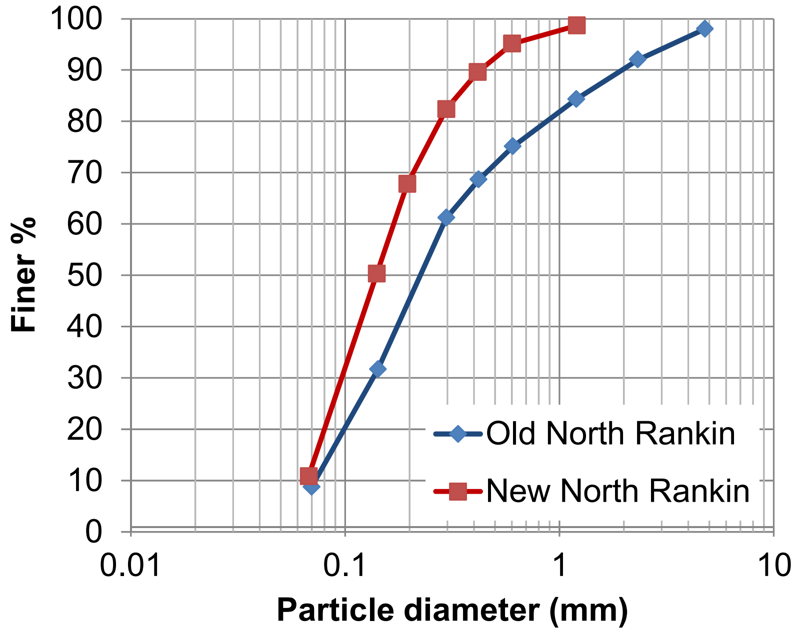

Figure 2. Replot of initial cumulative GSD distributions of two North Rankin sands from Allman (1988)

An example is extracted from Allman (1988) where either old or new North Rankin sand or a mixture of the both sands was used, as plotted in Figure 2. Regardless of the exact source of the sand and initial GSD, an isotropic compression curve was presented in Figure 3. It showed a nonlinear ln*p*-e curve where the abrupt change of the slope is between 200kPa and 370kPa, indicating the communition (yielding) pressure, p_c may be approximately estimated to be an average of 280kPa. A Φ of 35° has been measured while ν, as suggested, is first assumed to be 0.3.

The measured ln*p*-e curve can also be calibrated against the prediction of the breakage model (Einav 2007a), as shown in Figure 3, to estimate the elastic bulk modulus K (here 50MPa was determined using the breakage model), for example, or just *via* an elastic calculation to fit the linear part of the experimental curve. This, together with ν, enables an estimate of elastic shear modulus G (about 23MPa) for the purpose of using Vesic's equations (6) and (8).

All current equations (4), (5), (6), (8), (13) and (15) are employed and compared with the measured data from Allman (1988), as shown in Figure 4. It can be seen that the proposed new equation (15) shows the best agreement while Vesic and Houslby equations, although show the pressure dependence of bearing capacity, significantly overestimate the bearing capacity factor.

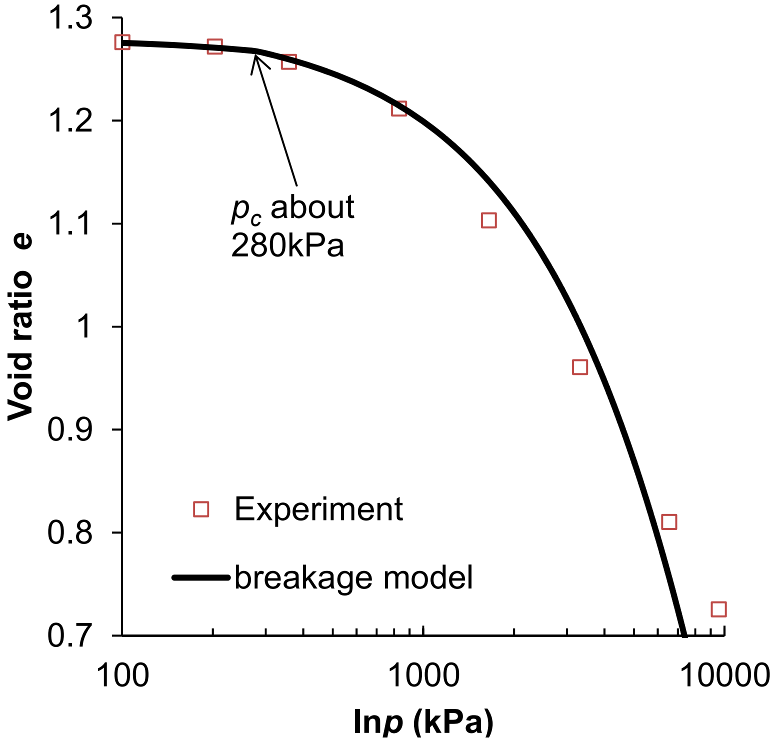

Figure 3. Plot of isotropic compression test curve of Rankin sands from Allman (1988) and prediction from the breakage model Einav (2007a).

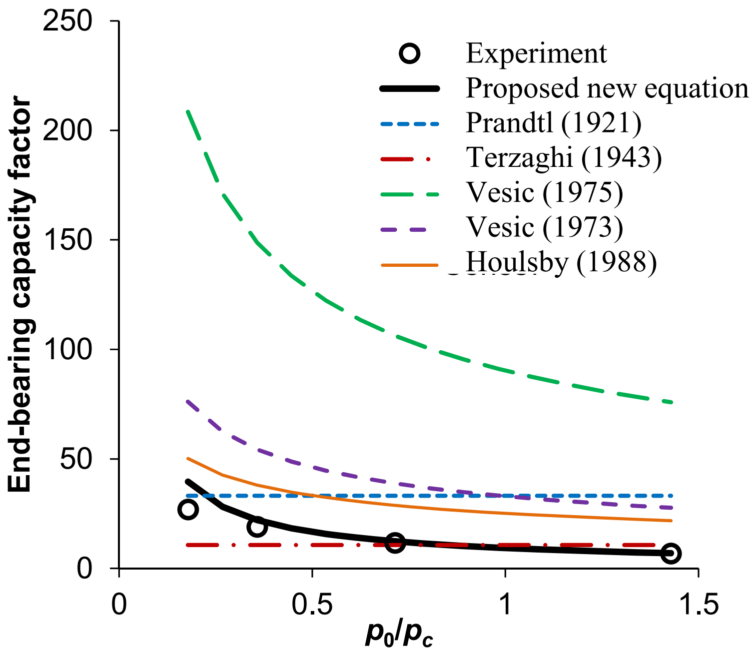

Figure 4. Comparison of various end-bearing capacity factors against initial mean effective stress normalised by p_c for North Rankin sands (experimental data from Allman (1988))

4 CONCLUSION

From the review of, and comparing of, a number of existing equations supplying the end-bearing capacity factor for predicting pile tip resistance, a comparison with the recently proposed new equation, which originated from breakage mechanics, suggests the new theory is particularly able to deal with soil crushability issues, and arguably provides the best assessment of end bearing.

The equation is straightforward to use with only three parameters obtainable from an isotropic compression test and a direct shear test. This feature is of particular interest for engineering purposes to quickly and accurately assess the end-bearing capacity of piles in weak granular soils, such as in calcareous and carbonates soils.

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