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# Updating system reliability of pile group by load tests

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## ABSTRACT

Proof load tests measure the capacity of piles and are commonly used to confirm the design capacity of the piles has been achieved. The Australian Standard allows a lower factor of safety to be used in the design stage depending on how many proof load tests will be conducted. The statistical reliability of the pile foundations can also be updated based on these test results. Research has been previously conducted on updating the reliability of single piles. This paper develops new methods for updating the system reliability of a pile group. The method updates the reliability of the tested pile group directly. It provides at the same time the updated population parameters, which can be used to predict the reliability of another pile group in the same site.

*Keywords:* Bayesian updating, system reliability, pile groups, load tests

## 1 INTRODUCTION

Generally speaking, geotechnical engineers use two methods to assess the capacity of pile foundations, namely, indirect estimation and direct load tests. There are uncertainties that contribute to both the predicted and tested pile capacity, for example, the inherent in-situ uncertain conditions and the uncertainties involved in construction. Some uncertainties are involved however, only in estimation. The first one is the accuracy of prediction methods. Prediction methods range from simple prediction methods (e.g., static and dynamic methods) to more advanced finite element methods. Although predictions based on static and dynamic formulas are approximate, they are still commonly used in practice because they are quick and cheap. Another uncertainty in the prediction lies in how much site investigation and laboratory test information can be obtained. Due to constraints on budget, engineers usually need to do prediction based on limited amounts of information, and this introduces additional uncertainty in the predictions. Results from load tests not only suggest a more realistic pile capacity value, but also greatly reduce the uncertainty of the pile capacity since the error associated with load test measurements is much smaller than that associated with predictions. For these reasons, the ultimate pile capacity determined from load testing are usually used to confirm that the design load be adequately supported by the planned pile foundation.

Although direct load tests can provide a wealth of information for design and construction of pile foundations and are the most accurate method of determining pile capacity, the pile load test itself involves some degree of uncertainty too. For example, although static load tests are believed to be more accurate than dynamic load tests, there are still uncertainties associated with the interpretation of load-displacement relationships (e.g., Paikowsky 2004). In addition, pile capacity obtained from a static load test cannot be accepted as a unique standard because the static load test yields the pile capacity at the time of test only, due to the consolidation phenomenon.

Nevertheless, confirmation of pile-soil capacity through static load testing is considerably more reliable than capacity estimates from static capacity analyses and dynamic formulas (e.g., Poulos and Davis 1980). By using direct load tests as part of the design process, an improved knowledge of pile-soil behaviour is obtained that may allow a reduction in pile lengths or an increase in the pile design load, either of which may result in potential savings in foundation costs. With the improved knowledge of pile-soil behaviour, a lower factor of safety (FS) may be used on the pile design load.

Calculations of pile and pile group reliability based on load test results have been reported by Kay (1978), Baecher and Rackwitz (1982), Zhang et al. (2001), Zhang (2004), Zhang et al. (2005) and Zhang et al. (2006). The reliability of large pile groups has been shown to be greater than the reliability

of individual piles due to redundancy in the system (e.g., Huang et al. 2013; Zhang et al. 2001). Zhang et al. (2001) used single pile analyses to assess group reliability by adopting a redundancy factor. Paikowsky (2004) suggested that the target reliability for a pile group can be reduced to 2.0-2.5 compared to 3.0 for single piles. Pile groups may be able to support the design load when one or more piles are defective (Poulos 1997). If the reliability of the pile group with defective piles can be quantified, it may be possible to use the defective pile rather than install a replacement pile and increase the size of the pile cap.

In this study we propose a rigorous framework for updating the reliability of single piles and pile groups. In contrast to previous work, we assume that load tests are subject to uncertainty. The proposed framework can be used to assess the reliability of single piles and pile groups subject to general loadings and stratigraphic conditions. The method can be used to assess the reliability of groups where pile tests have been conducted to the ultimate capacity, to below the ultimate capacity but exceeding specified capacity, and where pile tests fail to achieve the specified capacity (or are unknown). In the latter case, the method allows decisions to be made as to whether the reliability of the entire pile group is satisfactory or whether additional piles need to be installed.

## 2 RELIABILITY OF PILE GROUPS PRIOR TO TESTING

Suppose a group has  $N$  piles. The capacity of individual piles ( $y$ ) is assumed to be a lognormally distributed random variable with mean  $\mu$  and standard deviation  $\sigma$ . The correlation coefficient between the capacities of piles is  $\rho$ . The joint probability density function of  $N$  piles is

$$f_Y(y_1, \dots, y_N) = \frac{1}{\sqrt{(2\pi)^N |C| \prod_{i=1}^N y_i}} \exp\left(-\frac{1}{2}(y_i - \mu_{\ln y})^T C^{-1}(y_i - \mu_{\ln y})\right) \quad (1)$$

where  $C$  is an  $N$  by  $N$  covariance matrix with diagonal terms equal to  $\sigma_{\ln y}^2$  and off-diagonal terms equal to  $\rho\sigma_{\ln y}^2$ .

A rigorous reliability analysis for the pile group may be performed using a 3D finite-element method. However, we use the concept of group efficiency for simplicity. We restrict our analysis to a vertically loaded group of piles identically constructed. Using a coefficient of group efficiency, the capacity of the pile group is assumed to be

$$q = \eta \sum_{i=1}^N y_i \quad (2)$$

where  $\eta$  is the coefficient of group efficiency.

A limit state function for the capacity of the pile group can be defined as:

$$g = \eta \sum_{i=1}^N y_i - \frac{N\mu}{FS} \quad (3)$$

where  $FS$  is the factor of safety.

Suppose the mean pile capacity ( $\mu$ ) is 1.0. The standard deviation of pile capacity is 0.5. The applied load on the pile is 0.5. The factor of safety of a single pile is thus 2.0. The probability of failure of single pile is

$$p_f = P[y < 0.5] = \Phi\left(\frac{\ln 0.5 - \mu_{\ln y}}{\sigma_{\ln y}}\right) = 0.109 \quad (4)$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution.

The probability of failure of a group of piles cannot be obtained analytically. In this study, Monte-Carlo simulations were conducted to calculate the probability of failure of pile groups. The correlation

coefficient of individual pile capacities is varied over the range of (0.0, 0.25, 0.5, 0.75, 1.0). The number of piles in the group is varied in the range of 2-9. The calculated probability of failure is shown in Figure 1. It can be seen from Figure 1 that the probability of failure of the pile group increases as the correlation between the capacities of individual piles increases. When there is no correlation between the capacities of individual piles, weak piles may be compensated by strong piles.

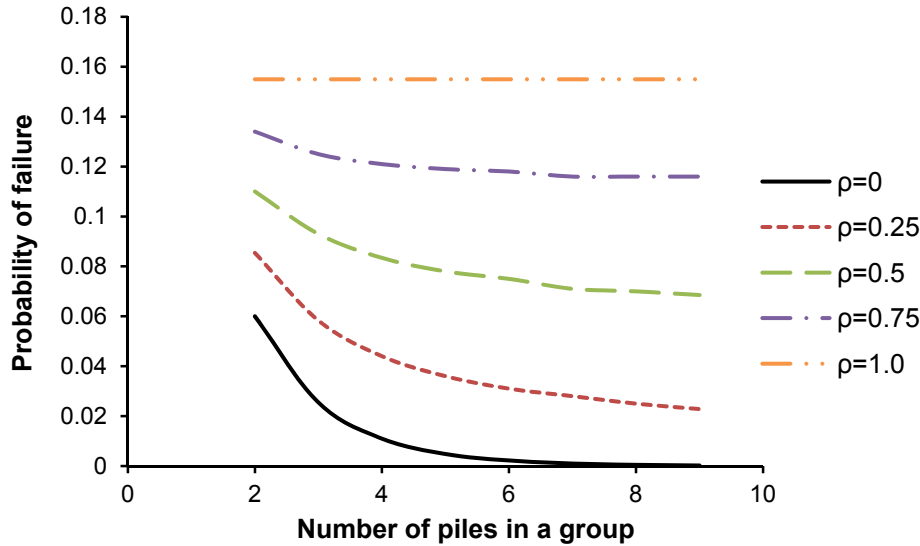


Figure 1. Probability of failure of pile groups

The system redundancy factor is defined as the ratio of the probability of single pile failure to the probability of system failure. For the pile groups considered, the system redundancy factors are shown in Table 1. It can be seen from Table 1 that system redundancy increases as the number of piles increases, and decreases as the correlation of the capacities of individual piles increases.

Table 1: System redundancy factor of pile groups

N	$\rho=0$	$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1.0$
2	1.82	1.28	0.99	0.81	0.70
3	4.29	1.87	1.17	0.87	0.70
4	9.91	2.48	1.31	0.90	0.70
5	22.71	3.03	1.40	0.92	0.70
6	49.55	3.52	1.45	0.92	0.70
7	109.00	3.89	1.54	0.94	0.70
8	235.93	4.36	1.56	0.94	0.70
9	539.60	4.78	1.59	0.94	0.70

### 3 RELIABILITY OF PILE GROUPS UPDATED BY LOAD TESTS

Suppose a group has  $N$  piles. To be general, suppose  $n$  out of  $N$  piles are tested up to load  $T$ ,  $m$  piles have failed to achieve  $T$  with known capacities  $y_i$ ,  $i = 1, \dots, m$ . The Bayesian posterior distribution of the pile capacity ( $y$ ) is

$$f''(y|\mathbf{y}) \propto \exp\left(-\frac{\sum_{i=1}^m (y_i - y - \mu_{err})^2}{2\sigma_{err}^2}\right) \left(\Phi\left(\frac{(y-T) - \mu_{err}}{\sigma_{err}}\right)\right)^{n-m} \frac{1}{\prod_{i=1}^N y_i} \exp\left(-\frac{1}{2}(y_i - \mu_{lny})^T C^{-1}(y_i - \mu_{lny})\right)$$

(5)

If the capacities of failed piles are not known, the Bayesian posterior distribution is

$$f''(y|\mathbf{y}) \propto \left(\Phi\left(\frac{\mu_{err} - (y-T)}{\sigma_{err}}\right)\right)^m \left(\Phi\left(\frac{(y-T) - \mu_{err}}{\sigma_{err}}\right)\right)^{n-m} \frac{1}{\prod_{i=1}^N y_i} \exp\left(-\frac{1}{2}(y_i - \mu_{lny})^T C^{-1}(y_i - \mu_{lny})\right)$$

(6)

The posterior distribution cannot be obtained analytically. One method of determining the posterior distribution is to sample it many times. The Markov Chain Monte Carlo (MCMC) method is commonly used to sample the posterior distribution. A Markov chain is a series of samples that current sample only depends on the previous sample. A Markov chain is formulated so that it converges to an invariant distribution which is the posterior distribution. The chain is also formulated so that it is irreducible and aperiodic to guarantee convergence. One advantage of the MCMC method is that the normalizing constant in Bayes' formula for the posterior distribution is not required. The basic idea goes back to Metropolis et al. (1953) and was extended by Hastings (1970).

### 3.1 Unknown pile capacity

A pile group presented in Section 2 is used again to illustrate how the probability of failure is updated by load tests. The group has 5 piles with mean individual pile capacity of 1.0 and individual standard deviation of 0.5. The applied load to the group is 2.5 so that the factor of safety is 2.0. It is assumed that the individual pile load tests are conducted to mean capacity, i.e.,  $T = \mu = 1.0$ . It is further assumed that the mean test error  $\mu_{err} = 0.0$  and the standard deviation of test error  $\sigma_{err} = 0.1$ . The capacities of piles that failed to achieve  $T$  are assumed to be unknown and, in this case, Eq. (6) is applicable. The updated probability of failure will depend on the number of tests and the number of tests that failed to achieve  $T$ . The number of tests is varied in the range of 1-5. The MCMC method was used and one million simulations were conducted. The results are shown in Figure 2 and Figure 3. It can be seen from Figure 2 and Figure 3 that the probability of failure of pile groups increases as the number of failed tests increases. It is noted however, that the probability of failure is larger than the prior probability of failure only when all tested piles fail regardless how many tests are conducted. This is because that the piles that pass the test increase the mean posterior capacity. Comparing Figure 2 and Figure 3, it can be seen that the  $p_f$  of the pile group is significantly higher if the capacities of piles are positively correlated.

The results in Figure 2 and Figure 3 are useful for assessing the reliability of pile groups with defective piles. There are usually two options when there are defective piles, one is to replace the piles and another is to do more tests. For the group shown in Figure 2 and  $\rho = 0.5$ , suppose the target reliability index is at least 3 ( $p_f < 0.0014$ ). If the first load test failed, according to Figure 2,  $p_f = 0.1279$ , which is unacceptable. If one more test is conducted is positive,  $p_f$  is reduced to 0.002. One more test performed immediately after finding the first test failed, to avoid remobilising a pile driving rig, may be less expensive than replacing the first defective pile.

### 3.2 Known pile capacity

The capacities of the failed piles are usually known to a degree of accuracy. This information can be further utilized to reduce uncertainties. In this case, Eq. (5) is applicable. The updated  $p_f$  of a group will depend on the number of tests, the number of failed piles and the capacities of the failed piles. Suppose 3 out of 5 piles were tested and 3 piles failed. To investigate the effects of the capacities of failed piles on the updated probability of failure of pile group, the mean capacities of the three failed piles are varied from 0.4 to 0.8. For each cases considered, the capacities of the other two failed piles are 0.1 above and below the mean capacity of failed piles. The updated  $p_f$  of the group is shown in Figure 4. The corresponding probability of failure when the capacities of failed piles are not known is also shown in Figure 4. It can be seen that the probability of failure increases as the mean capacity of failed piles decrease. The results also show that by incorporating more information in the updating, more accurate estimation of the performance of pile group will be obtained.

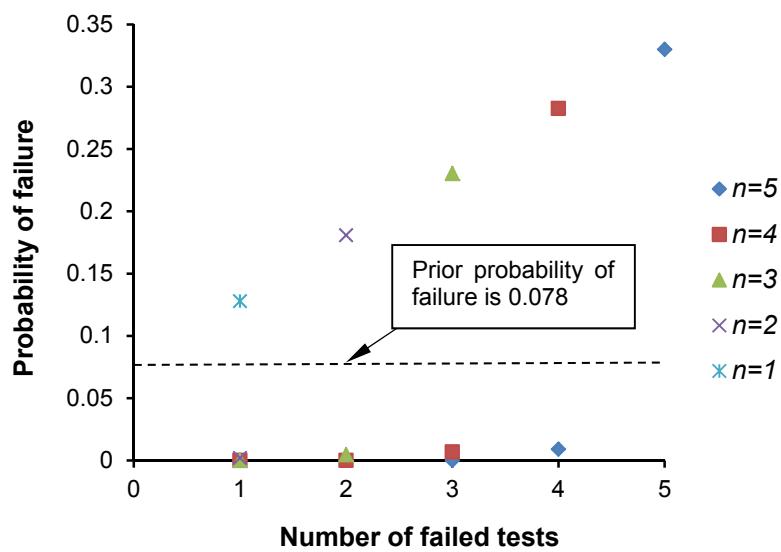


Figure 2. Probability of failure verse the number of failed tests ( $N = 5$ ,  $\rho = 0.5$ , prior probability of failure is 0.078)

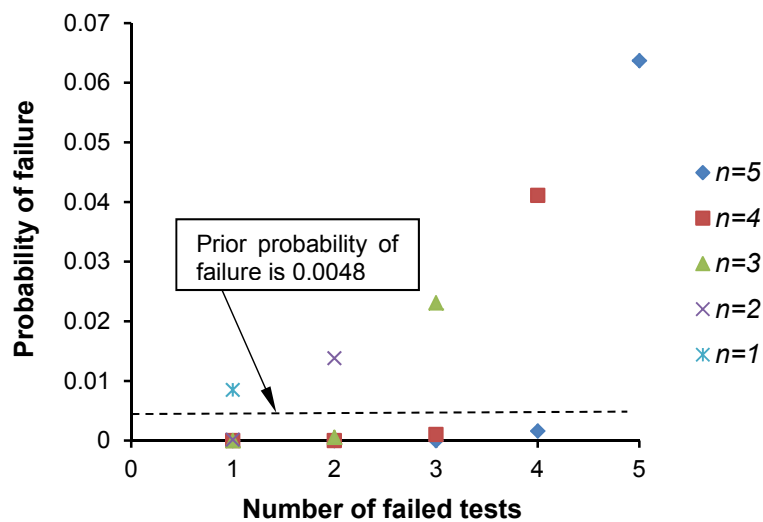


Figure 3. Probability of failure verse the number of failed tests ( $N = 5$ ,  $\rho = 0.0$ , prior probability of failure is 0.0048)

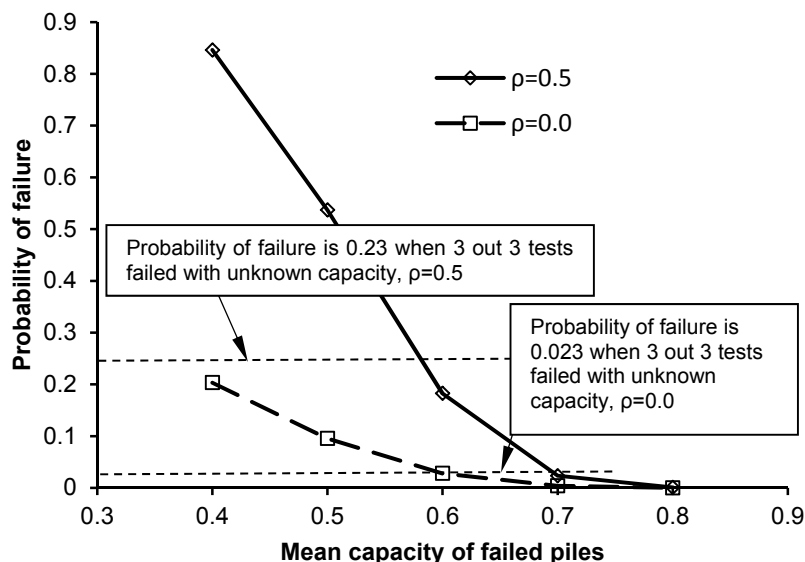


Figure 4. The influence of the capacities of failed piles on the probability of failure

#### 4 CONCLUDING REMARKS

The reliability of pile groups updated by load tests is assessed by Bayesian updating and MCMC. It is shown that the system reliability of a pile group is significantly smaller if the individual capacities are positively correlated rather than uncorrelated. The method presented in this paper can be used to both refine and validate the initial design of a pile group by load tests. In the case where load tests on individual piles are not favorable, additional load tests may be conducted. If these results are favorable, the reliability of the pile group may be sufficient even when defective piles are found. It is also shown that incorporating more information in Bayesian updating, more accurate prediction can be obtained.

#### 5 ACKNOWLEDGEMENTS

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