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The effect of subsidence on liquefaction vulnerability following the 2010 – 2011 Canterbury Earthquake Sequence

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ABSTRACT

During the 2010 – 2011 Canterbury Earthquake Sequence (CES), the June 2011 earthquake event caused relatively moderate levels of ground shaking. However, in this event, the incidence and severity of liquefaction-induced land damage was significantly greater in some areas than was anticipated relative to the severity of land damage caused by earlier events in the CES. It was observed that the increased incidence and severity of this type of land damage was spatially correlated with the occurrence of ground subsidence from earlier events due to volumetric densification, liquefaction ejecta, lateral spreading and tectonic movement. These observations formed the basis of a hypothesis that the reduced depth to groundwater as a result of ground surface subsidence effectively reduces the thickness of the non-liquefying crust and that the reduced crust thickness is less able to suppress liquefaction effects at the ground surface resulting in increased vulnerability to liquefaction-induced land damage. This paper illustrates the occurrence of increased liquefaction vulnerability with reference to both to qualitative and quantitative data collected as part of an extensive assessment of the effects of ground surface subsidence of the land in Canterbury. Analysis using two liquefaction vulnerability parameters, namely the Ishihara (1985) criteria and the Liquefaction Severity Number (LSN), quantifies the increase in liquefaction vulnerability caused by the ground surface subsidence.

Keywords: Liquefaction, Subsidence, Settlement, Vulnerability, Ishihara Boundary Curves, Liquefaction Severity Number

1 INTRODUCTION

From 4 September 2010 until early 2012 the Canterbury region of New Zealand including the City of Christchurch was affected by a sequence of earthquakes and aftershocks. The most significant earthquakes during this period were: 4 September 2010 (M_w 7.1), 22 February 2011 (M_w 6.2), 13 June 2011 (M_w 6.0), and 23 December 2011 (M_w 5.9). The highest peak ground accelerations (PGA) in western and northern parts of Christchurch were experienced during the September 2010 event, whereas the highest PGA in central, eastern and southern Christchurch were experienced during the February 2011 and December 2011 events. Detail about the ground motions experienced during the Canterbury Earthquake Sequence (CES) can be found in Bradley and Hughes (2012) and Bradley et al. (2014).

The earthquake shaking from the CES triggered minor-to-severe liquefaction-induced ground surface deformations resulting in land damage throughout the City of Christchurch and surrounding areas (Cubrinovski et al. 2011, T&T 2013 and van Ballegooy et al. 2014b & 2015a). The land damage included, liquefaction ejecta, liquefaction-induced differential settlement, and lateral spreading resulting in extensive residential building damage. It was observed that the majority of the areas affected by severe liquefaction-induced land damage coincided with lower lying areas where the groundwater surface is close to the ground surface. Conversely, areas with less liquefaction-induced land damage was observed in areas with higher elevation and deeper groundwater levels, indicating a negative correlation between liquefaction damage and the depth to the groundwater surface and hence the non-liquefying crust thickness.

It was observed that the extent of liquefaction-induced land damage in the June 2011 event was almost the same as the February 2011 event even though the PGA in June 2011 were significantly lower than those recorded in February 2011 (T&T, 2013). Furthermore, areas that generally experienced minor-to-moderate land damage in the February 2011 event generally experienced less damage in the June 2011 event, whereas areas which experienced moderate-to-severe land damage in the February 2011 event generally experienced more severe damage in the June 2011 event (T&T, 2013). Comparison of LiDAR survey information taken before and after the CES showed significant ground subsidence occurred as a result of the CES due to liquefaction-induced volumetric densification, liquefaction ejecta, lateral spreading and tectonic subsidence (T&T 2013; van Ballegooy et. al. 2014b & 2015a). Of the 140,000 flat land residential properties in Christchurch, approximately 70,000 subsided more than 200mm, 12,000 subsided by more than 500mm and 500 subsided by more than 1m.

Due to liquefaction-induced ground subsidence and seasonal increases in groundwater levels, the depth to groundwater during the June 2011 event was closer to the ground surface than it had been in both the September 2010 and February 2011 events. It is important to note that the ground subsidence generally did not induce a change in the groundwater elevation (van Ballegooy et al., 2014a), it was the ground surface elevation that has lowered and hence become closer to the groundwater elevation. This formed the basis of a hypothesis that, in areas where the upper soil layers are potentially liquefiable, reduced depth to the groundwater surface due to ground surface subsidence was effectively reducing the thickness of the non-liquefying crust and that this reduction in non-liquefying crust thickness resulted in increased vulnerability to liquefaction-induced land damage.

This paper provides a comparison of the mapped land damage observations following the September 2010 and June 2011 earthquake events and an analysis of increased liquefaction vulnerability due to the reduction in the thickness of the non-liquefying crust using two liquefaction vulnerability parameters; namely the Ishihara (1985) criteria and the Liquefaction Severity Number (LSN) described in T&T 2013 and van Ballegooy et al. 2014b, 2015a & 2015b. The February 2011 event is not included in this analysis because the September 2010 event did not cause sufficient subsidence to investigate the resulting increased liquefaction vulnerability. Similarly, the December 2011 event is not included in this analysis because detailed land damage mapping was not undertaken for this event.

2 BACKGROUND

In New Zealand, land is insured for natural disaster damage under the 1993 Earthquake Commission (EQC) Act. As a result, following each of the main CES events, extensive mapping was undertaken as well as a significant geotechnical investigation programme in an attempt to characterise the liquefaction-induced land damage and correlate it with the subsurface soil conditions. The observations and analyses presented in this paper have come out of a body of work undertaken over the past 3 to 4 years by Tonkin & Taylor (T&T) on behalf of EQC to assist in the resolution of their land damage claims.

The extensive volume of collected data (summarised in T&T 2013, van Ballegooy et al. 2014, 2014b & 2015a) includes:

- Qualitative land damage and foundation damage mapping following the main events
- An extensive geotechnical site investigation program including 15,000 Cone Penetration Tests (CPT) and 3,000 Boreholes (T&T, 2013) and 900 shallow groundwater monitoring wells;
- LiDAR survey data following each of the main earthquake events allowing estimation of the ground surface subsidence; and;
- An extensive program of groundwater monitoring enabling the development of groundwater models for each of the major earthquake events (van Ballegooy et al, 2014a).

This represents the most extensive collection of information ever undertaken for a sequence of earthquakes. The information listed above is available through the Canterbury Geotechnical Database (CGD): https://canterburygeotechnicaldatabase.projectorbit.com.

The effectiveness and suitability of several CPT-based liquefaction vulnerability parameters including one dimensional volumetric densification settlement and the Liquefaction Potential Index (LPI) were assessed by T&T (2013) and van Ballegooy et al. (2014b & 2015c). These regional studies indicated that while these parameters were predicting higher vulnerability in areas of observed severe liquefaction related damage in some parts of Canterbury, the parameters were also predicting higher vulnerability in areas where little to no damage observations were recorded. These analyses indicated that the existing liquefaction vulnerability parameters were not able to fully capture the consequences of liquefaction. Tonkin and Taylor (2013), van Ballegooy et al. (2014b, 2015a & 2015b), have shown that a new liquefaction vulnerability parameter, LSN, provides a better correlation with the land and foundation damage observations recorded in Canterbury. Likely ranges of LSN values for each of the land damage observation categories in Figure 1 are provided as follows; no liquefaction - 0 to 15, minor-to-moderate liquefaction - 16 to 25, moderate-to-severe liquefaction - 26+. We note that these LSN ranges differ slightly from similar previously published ranges in T&T (2013). This is due to the continued evolution of inputs to the LSN triggering methodology as discussed in van Ballegooy et al. (2015c).

3 LAND DAMAGE OBSERVATIONS

As noted previously, the comparison of observations following each of the major events indicated that the liquefaction-induced land damage following the June 2011 event was significantly greater than anticipated. This can be illustrated by comparing the mapped liquefaction land damage from the September 2010 and June 2011 earthquake events (refer to the first row in Figure 1). The Bradley and Hughes (2012) PGA contours show that the PGA values in southern and eastern parts of Christchurch during the June 2011 earthquake event were higher than the September 2010 earthquake event. This correlates with liquefaction severity observations as there is a higher incidence of liquefaction-induced land damage in southern and eastern areas following the June 2011 earthquake event. To the north of the Central Business District (CBD) the PGA level for both events was approximately 0.2g and as such it would be reasonable to expect similar land performance. However, the liquefaction severity observations indicate worse land performance in this area following the June 2011 event.

This is illustrated further by comparison of the LSN maps in the second row in Figure 1. The LSN maps are contour plots generated from the calculated LSN values at each CPT location. The maps are created by analysing the top 10m of each CPT using the Boulanger and Idriss (2014) simplified liquefaction evaluation method discussed in van Ballegooy (2014b). The depth to groundwater used in the analysis for each event is the depth to the groundwater surface immediately prior to the CES and do not take into account changes in the depth to the groundwater surface due to subsidence and seasonal variation (refer to T&T, 2013). As expected the LSN map is predicting more severe liquefaction in eastern and southern parts of Christchurch in the June 2011 event (due to higher PGA) when compared to the September 2010 event. However, comparison with the area to the north of the CBD shows the LSN map predicting similar liquefaction land damage in this area for both the September 2011 and June 2011 earthquake events. As noted previously, the liquefaction severity observations in this area showed an increase in liquefaction severity following the June 2011 event.

The LSN frequency histograms in the third row in Figure 1 show the distribution of the calculated LSN for observations of no liquefaction, minor-to-moderate liquefaction and moderate-to-severe liquefaction at the ground surface for the September 2010 and June 2011 earthquake events. Due to the magnitude scaling inherent in the Boulanger and Idriss (2014) simplified liquefaction triggering method and because each of the categories of land damage observations on the LSN frequency histograms is normalised to a unit area of one (i.e. 100%), earthquakes of different magnitude and PGA should result in the same distribution of land damage. This means the LSN frequency histogram should be independent of the earthquake event size and hence, if the LSN model were appropriately predicting liquefaction vulnerability, similar distributions of calculated LSN for each of the land damage categories would be expected for the September 2010 and June 2011 events being considered in this paper.

For both events there is a correlation between the calculated LSN and observed liquefaction severity observations. The observations of no liquefaction damage are associated with lower calculated LSN values and the observations of severe liquefaction damage are associated with higher calculated LSN values. However, the distribution of calculated LSN values for the moderate-to-severe liquefaction observations for the September 2010 event is higher than for the June 2011 event. This is particularly evident with reference to the median calculated LSN values for the September 2010 and June 2011 events which range between 20 to 25 and 15 to 20 respectively. As discussed in subsequent sections, these LSN distributions become similar once the reduction in non-liquefying crust thickness due to ground surface subsidence is taken into account for the June 2011 event. Likewise, similar observations are made for the February 2011 event analyses (these analyses are not presented in this paper due to space constraints).

4 ISHIHARA AND LSN LIQUEFACTION VULNERABILITY CRITERIA

Ishihara (1985) published observations on the protective effect of an upper layer of non-liquefied material against the effects of liquefaction at the ground surface. Ishihara plotted observations of the expression of liquefied material at the ground surface using the thickness of the overlying non-liquefying surface layer (H_1) or "crust" and the thickness of the underlying liquefied material (H_2), and defined boundary curves that separated those sites where liquefied material was expressed at the ground surface and sites that did not. The Ishihara boundary curve for a M_w 7.5, PGA 0.3g earthquake event is presented in Figure 2. It is important to note that, for points plotted to the left of the boundary curve, the further they plot to the left of the boundary curve the greater the severity of liquefaction-induced ground damage.

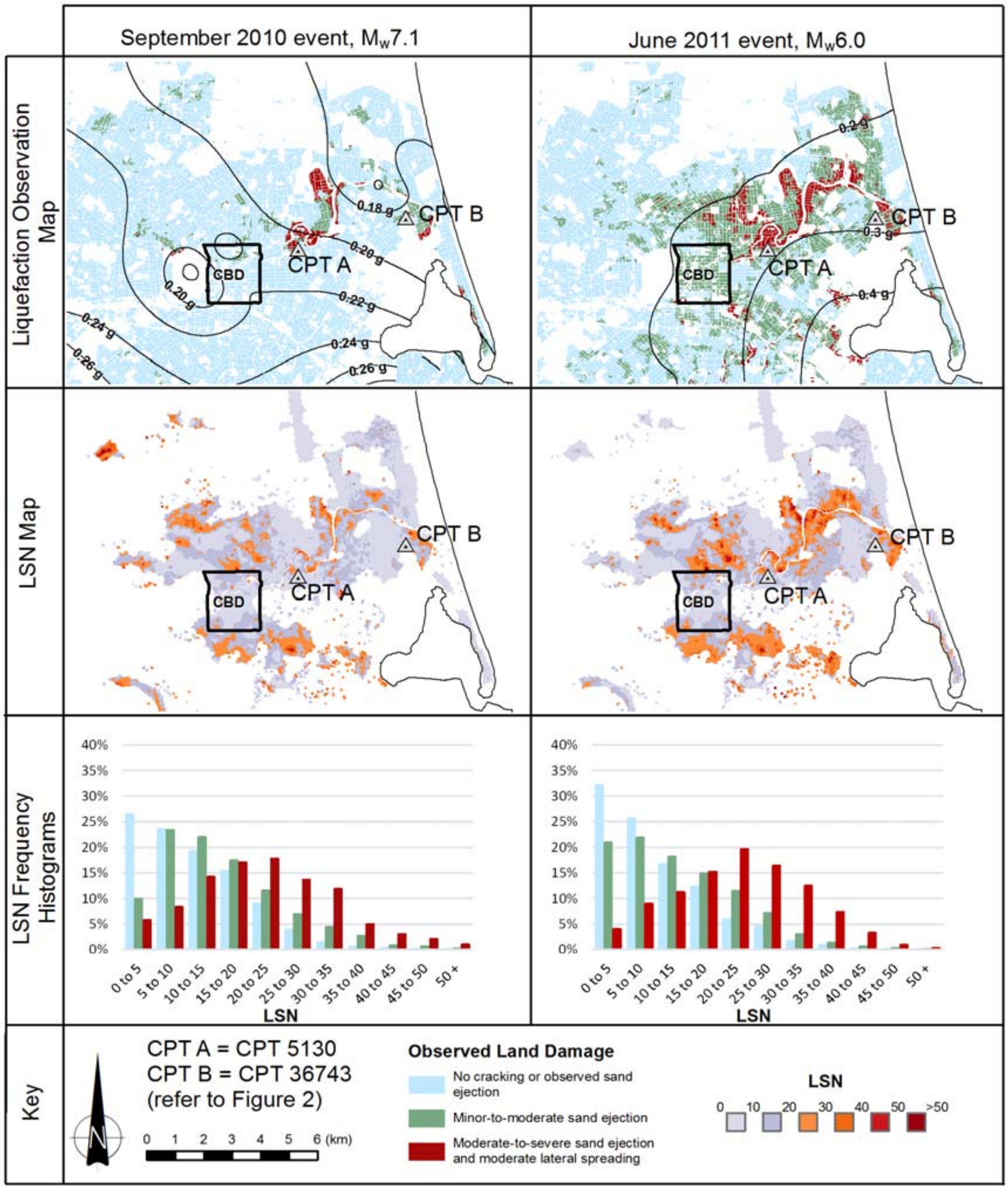

Figure 1. Comparison of September 2010 and June 2011 earthquake events. The top row shows the liquefaction severity observation maps with the Bradley and Hughes (2012) PGA contours overlaid. The middle row shows the calculated LSN over the top 10m for each CPT using the Boulanger and Idriss (2014) simplified liquefaction evaluation method with the pre CES median depth to groundwater model. The bottom row shows LSN frequency histograms for the observations of no liquefaction, minor-to-moderate liquefaction and moderate-to-severe liquefaction manifestation at the ground surface.

To demonstrate the correlation between increased liquefaction vulnerability and reduced non-liquefying crust thickness due to ground subsidence with the Ishihara (1985) boundary curves, two example CPTs in Christchurch (located on Figure 1) have been plotted on an H_1 versus H_2 graph in Figure 2. In both cases the ground has subsided by approximately 0.5m. CPT 5130 is representative of ground which experienced a reduction in crust thickness from 3.3m to 2.8m as a result of ground subsidence during the CES, whereas CPT 36743 is representative of ground which experienced a reduction in crust thickness from 1.8m to 1.3m as a result of ground subsidence during the CES. For each CPT location, the pre CES and post CES median depth to groundwater were estimated based on the median depth to groundwater surfaces presented in van Ballegooy et al. (2014a). The change in thickness of H_1 and H_2 was estimated by analysing the thickness of liquefiying material using the Boulanger and Idriss (2014) simplified liquefaction triggering method for both the pre CES and post CES median depth to groundwater conditions. The resulting change in H_1 and H_2 thickness for each CPT is plotted on Figure 2.

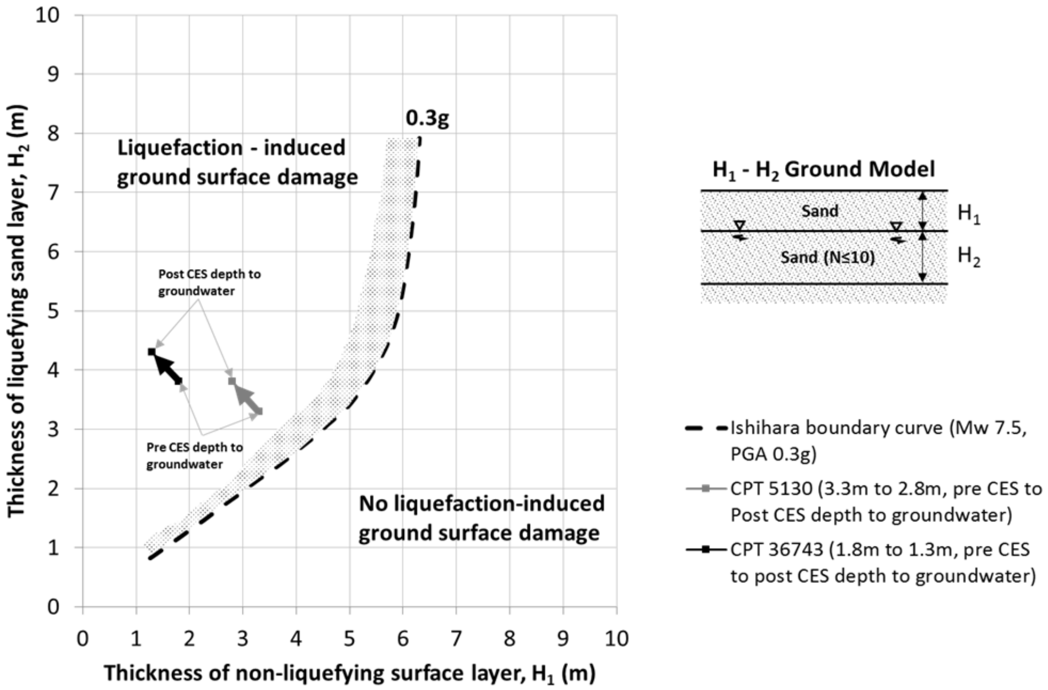

Figure 2. Ishihara (1985) boundary curve for M_w 7.5, PGA 0.3g earthquake event (note the reference numbers used to identify the CPT correspond to the CGD reference numbers).

Each CPT plots to the left of the Ishihara (1985) boundary curve which indicates that, for a M_w 7.5, PGA 0.3g earthquake event, expression of liquefied material would be expected at the ground surface. This is validated by observations of liquefaction ejecta at each CPT location for all four of the main earthquake events. For each CPT the reduction in non-liquefying crust thickness results in a shift to the left on the Ishihara plot indicating that both soil profiles now have an increased vulnerability to liquefaction as a result of ground subsidence caused by the CES. Typically areas in Christchurch where the ground conditions plot on the right hand side of the curve have not significantly subsided due to liquefaction related effects and hence these points have not shifted measurably to the left by comparison, but may have subsided due to tectonic related effects. Therefore, most movement occurred on the left side of the curve, i.e. ground that was previously vulnerable to liquefaction has become more vulnerable. It is noted that there are areas in Christchurch, where the upper soil layers are more silty and do not liquefy. In these areas the thickness of the non-liquefying H_1 soil layer has not reduced as a result of ground surface subsidence and hence the liquefaction vulnerability has not increased.

The simple ground model that the Ishihara curves were developed from is an over simplification of the complex, highly stratified solid profiles typically encountered in the Canterbury region. Nevertheless, it is a useful model for understanding liquefaction vulnerability and the phenomenon of increasing vulnerability to liquefaction-induced land damage due to a reduction in the thickness of the non-liquefying crust as a result of ground surface subsidence. The two example CPTs used in this analysis have been purposely selected because in an M_w 7.5, PGA 0.3g earthquake event, the liquefaction triggering analysis undertaken indicates behaviour which approximates a simple model of a non-liquefying crust overlying a liquefying soil layer without the interruption of non-liquefying soil layers within the liquefying H_2 layer and is the same as the soil model used to develop the Ishihara (1985) boundary curves.

For this reason CPT based liquefaction vulnerability parameters were developed. Recent work by van Ballegooy et al. (2015b) shows there is a good correlation between the LSN index parameter and the Ishihara (1985) boundary curves. This correlation shows that LSN greater than 20 occur for soil profiles to the left of the Ishihara (1985) boundary curve and LSN values less than 15 occur for soil profiles to the right of the Ishihara (1985) boundary curves. Hence, LSN values between 15 and 20 are representative of soil profiles which plot on the Ishihara (1985) boundary curves and are threshold values which indicate the likely occurrence of the expression of liquefaction ejecta at the ground surface. This range is consistent with the regional liquefaction vulnerability studies based on 15,000 CPT (T&T 2013 and van Ballegooy et al. 2014b & 2015a).

To demonstrate the correlation between increased liquefaction vulnerability due a reduction in non-liquefying crust thickness and the calculated LSN parameter, the LSN values for a M_w 7.5 earthquake event with a range of PGA values were calculated for the two CPTs presented in Figure 2 using the same groundwater conditions. The calculated LSN values are plotted against PGA and are presented in Figures 3a and 3b. For the purposes of comparison with Figure 2 the liquefaction expression threshold range of LSN = 15 to 20 (representing the range of LSN values that are equivalent to the Ishihara (1985) boundary curves) and a line representing PGA = 0.3g (which represents the PGA of the Ishihara boundary curve considered in Figure 2) are shown on Figures 3a & 3b.

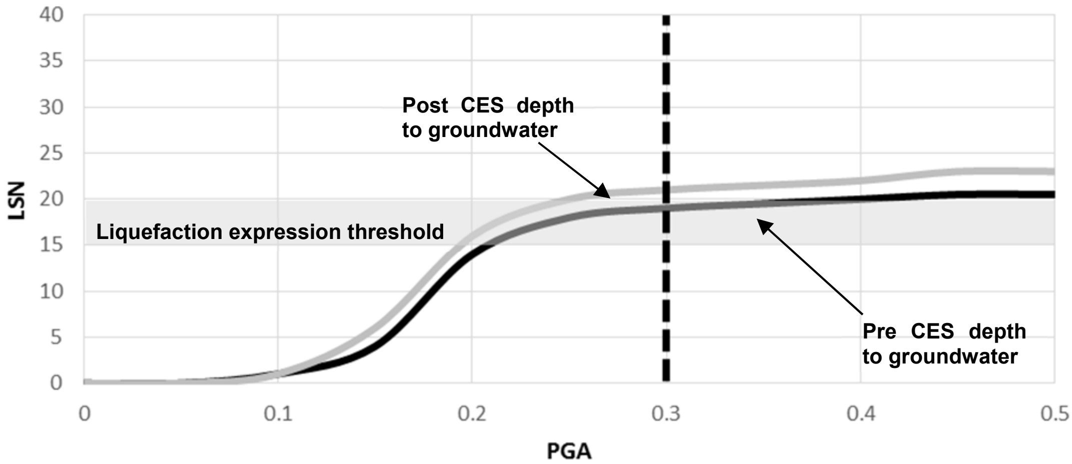

Figure 3a. CPT 5130 LSN vs. PGA for a M_w 7.5 earthquake event for CPT 5130 with a pre and post CES median depth to groundwater of 3.8 to 3.3m

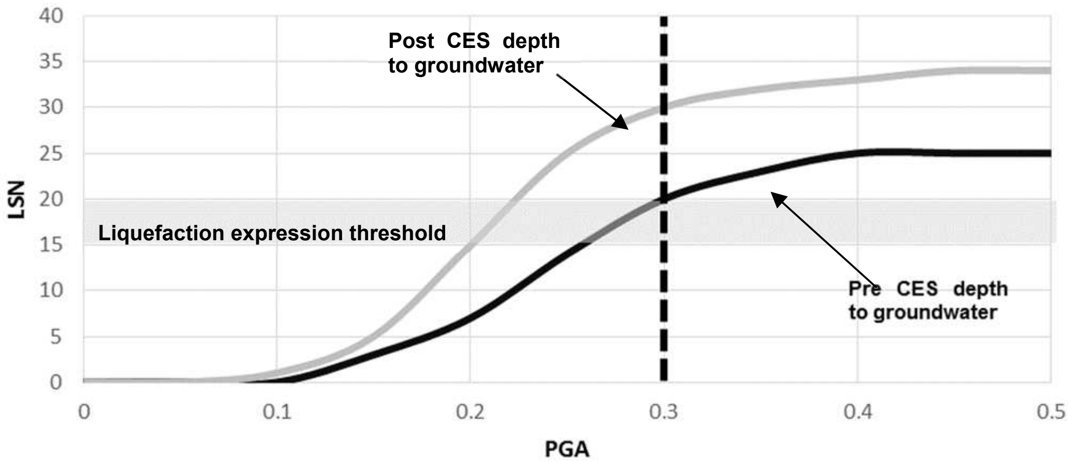

Figure 3b. CPT 36743 LSN vs. PGA for a M_w 7.5 earthquake event for CPT 36743 with a pre and post CES median depth to groundwater of 1.8 to 1.3m

Figures 3a & 3b indicate that with both CPT show increasing LSN values with increasing PGA as a result of liquefaction triggering in more soil layers as the seismic demand increases. The LSN parameter also increases as the depth to groundwater becomes smaller and closer to the ground surface. It is noted that prior to the CES, the soil profiles represented by each of these CPTs had a similar vulnerability to liquefaction at a M_w 7.5, PGA = 0.3g earthquake with LSN values typically around 20. This is near the upper end of the liquefaction expression threshold when compared with the Ishihara (1985) boundary curves and it would be reasonable to anticipate some liquefaction-induced land damage at both of these CPT locations.

However, comparing the post CES LSN values for the two CPT shows a difference in changes in the predicted future site performance. At PGA of 0.3g, the LSN value of CPT 5130 has increased by approximately 2 LSN points and is just above the upper bound of the liquefaction expression threshold of 20 due to a smaller percentage change in the non-liquefying crust thickness. This does not represent a significant change in vulnerability to liquefaction-induced land damage and it would be reasonable to expect the ground to perform in a similar manner to its pre CES condition. However at the same PGA of 0.3g, the LSN value of CPT 36743 with the post CES groundwater conditions has now increased by approximately 10 LSN points to 30 due to a larger percentage change in the non-liquefying crust thickness. This value is well above the liquefaction expression threshold and it would be reasonable to anticipate significantly worse liquefaction-induced land damage at this CPT location with the post CES groundwater conditions.

The effect of ground subsidence on predicted liquefaction vulnerability can be demonstrated on a regional basis by reanalysing the June 2011 event LSN map produced in Figure 1 with an adjustment to the depth to groundwater model to allow for the ground subsidence caused by the September 2010 and February 2011 earthquakes. Comparison of the June 2011 LSN maps from Figure 1 and Figure 4 shows an overall increase in calculated LSN generally in areas where moderate-to-severe land damage was observed, improving the correlation between the LSN parameter and the observed land damage. Figure 4 shows an increase in liquefaction vulnerability in the area to the north of the CBD for the June 2011 event.

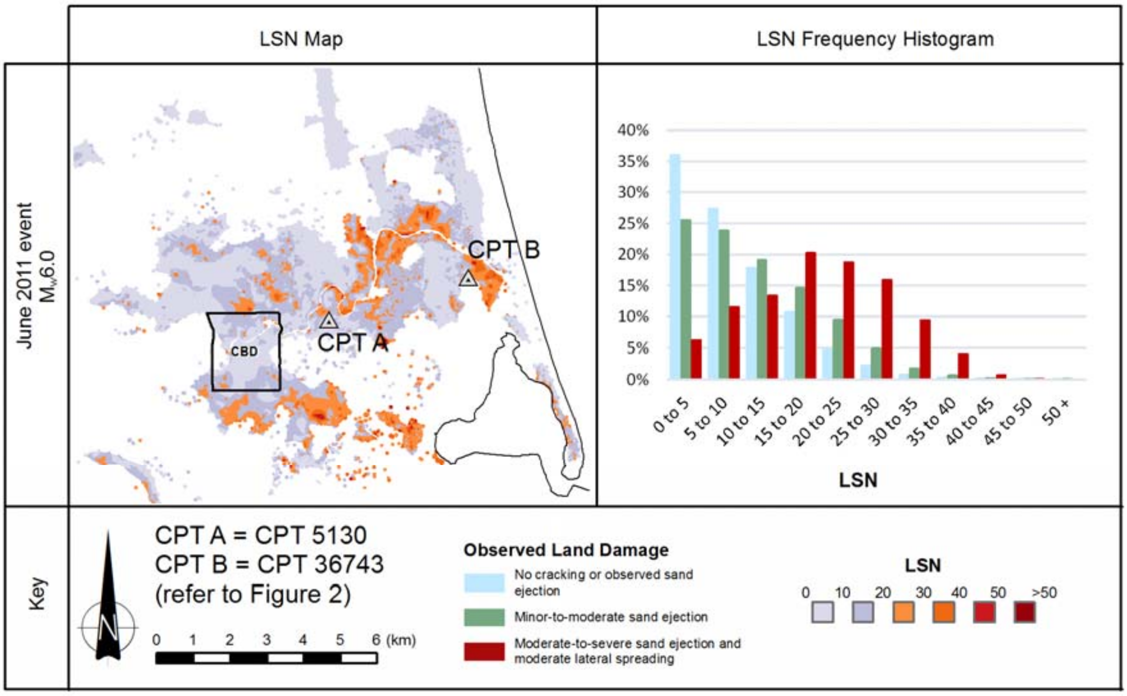

Figure 4. June 2011 earthquake event with adjusted depth to groundwater model. On the left side of the Figure is the LSN map over the top 10m for each CPT using the Boulanger and Idriss (2014) *simplified liquefaction evaluation method. On the right side of the Figure is the LSN frequency histogram for the observations of no liquefaction, minor-to-moderate liquefaction and moderate-to-severe liquefaction manifestation at the ground surface.*

Comparison of the frequency histograms in Figure 1 and Figure 4 also illustrates the improved correlation achieved by adjusting the groundwater model to allow for ground subsidence. The LSN frequency for the September 2010 histogram in Figure 1 has a similar distribution to the LSN frequency histogram for the June 2011 event in Figure 4. This is illustrated by the median LSN value for the areas with severe liquefaction observations which is between 20 and 25 LSN points and is the same value as the September 2010 event median calculated LSN value.

5 CONCLUSION

This paper has shown that for soil profiles with liquefiable soil layers near the ground surface, decreasing the depth to the groundwater surface results in an increase in vulnerability to liquefaction-induced land damage. This change in liquefaction vulnerability does not occur where surficial soil layers do not liquefy. This paper has also demonstrated how the Ishihara (1985) H_1 – H_2 plot and the LSN parameter can be used to examine and quantify the change in liquefaction vulnerability due to change in the depth to the groundwater surface. While occurrence of increased liquefaction vulnerability has been demonstrated with reference to data collected as part of an assessment of the effects of ground surface subsidence of the land in Canterbury as a result of the CES, this is not the only mechanism by which increased liquefaction vulnerability can occur. For example, sea level rise, seasonal variation and subsidence due to natural and anthropogenic processes (e.g. gas extraction) are alternate mechanisms which can also reduce the depth to the groundwater surface and hence result in a change in vulnerability to liquefaction-induced land damage.

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