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# The role of static shear stress on forms of cyclic liquefaction

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## ABSTRACT

There are divergent opinions on the effect of static shear stress on resistance to cyclic liquefaction of granular soils. For sand with fines, these issues are even more complicated as it relates to use of an 'equivalent' initial state parameter in capturing the effects of fines. To address these issues, the behaviour of sand and sand with fines during cyclic liquefaction was examined with an extensive testing program. Relevant published literature was also re-synthesised. It is noted that cyclic liquefaction can occur in two forms. For cyclic instability, liquefaction resistance is largely governed by the location of the applied peak or trough deviatoric stress state relative to instability stress ratio line. On the other hand, for cyclic mobility, the ratio of trough to peak deviator stress largely governs the liquefaction resistance. The effects of static shear stress on these two parameters are very different. A simple methodology of predicting the form of liquefaction is outlined.

*Keywords:* sand, liquefaction, cyclic instability, cyclic mobility, static shear stress, state parameter

## 1 INTRODUCTION

Experimental research into liquefaction of sandy soils arguably stems from the pioneering studies of earthquake induced liquefaction (Seed and Lee 1966; Seed and Idriss 1967). These laboratory studies revealed that undrained cyclic loading can lead a state of transient zero effective stress and the development of large cyclic strain. Extensive empirical studies have been made in establishing the influence of various factors on the liquefaction resistance of sandy soils. However, liquefaction failures under non-cyclic loading, referred to as static liquefaction have also been reported. In static liquefaction, the effective stress may not reduce to zero but it is in a state of instability that leads to a flow-type failure and therefore the soil appears to be liquefied.

Liquefaction induced by cyclic loading is intrinsically more complicated as cyclic loading needs to be characterised by at least two parameters. The conventional approach is to decompose the applied cyclic deviator stress into a static (also referred to as initial) component,  $q_0$ , and a cyclic component  $q_{cyc}$ . This intrinsically assumes the soil is consolidated under  $q_0$ , and drainage cannot occur under  $q_{cyc}$ . The influence of  $q_0$  and  $q_{cyc}$  cyclic loading on initiation of cyclic liquefaction was studied by a number of researchers, and some of these works are discussed below.

Extensive studies have been made to examine the influence of  $q_0$  in liquefaction resistance. Vaid and Chern (1983) demonstrated that cyclic resistance of Ottawa sand can be increased or reduced by a non-zero value of  $q_0$ , depending on the relative density ( $Dr$ ) of soil and magnitude of  $q_0$ . Their testing program covered  $Dr = 36\%$  to  $76\%$ . Hyodo, et al. (2002) demonstrated that cyclic resistance of Aio sand (at  $Dr=80\%$ ) increased with increase in  $q_0$  when tested at  $Dr = 80\%$  and initial effective confining stress,  $p'_0 = 100\text{kPa}$ , but an opposite trend was observed when tested at  $Dr = 80\%$  and  $p'_0$  of  $3000\text{kPa}$  and  $5000\text{kPa}$ . Vaid and Chern (1985) mentioned two different distinct forms of cyclic liquefaction as "liquefaction" (as a result of strain softening) and cyclic mobility, CM. Further, they commented that influence of  $p'_0$  and  $q_0$  on cyclic strength should be considered separately in regions of liquefaction and CM. Yang and Sze (2011) showed that cyclic resistance of Toyoura sand tends to increase and then decrease with increasing values of  $q_0$  (as expressed in terms of  $\alpha = q_0/2\sigma'_{nc}$ ; where  $\sigma'_{nc}$  is the normal effective stress) for loose sand specimens ( $Dr = 10\%$  and  $20\%$ ), but it continues to increase with  $\alpha$  in medium dense ( $Dr = 50\%$ ) and dense sand samples ( $Dr = 70\%$ ). They categorised failure mechanisms of granular soil under undrained cyclic loading in to three distinct types: flow-type failure,

cyclic mobility and plastic strain accumulation. Further they demonstrated that a threshold value of  $q_0$  exists for both cases of flow-type failure and CM. When  $q_0$  reaches this threshold value, cyclic resistance tends to reduce with further increase in  $q_0$ . Their testing program covered  $q_0$  up to 400 kPa. After testing undisturbed Gioia Tauro sand specimens in anisotropic triaxial tests, Ghionna and Porcino (2006) reported that cyclic resistance of a soil heavily dependent on shear stress reversal condition.

The challenge on influence of  $q_0$  is further increased when dealing with sand with fines as only limited published work can be located. The study by Corral and Verdugo (2011) on Torito Dam tailings (sand with 18% non-plastic fines) reported that the cyclic resistance gradually increases as  $q_0$  increases for both loose ( $Dr=45\%$ ) and dense specimens ( $Dr=75\%$ ). Thus, the influence of  $q_0$  on liquefaction resistance is still not fully understood although the above literature review suggested that its influence is dependent on  $Dr$  and  $p'_0$ .

As summarised above, no clear consensus have been reached regarding the role of  $q_0$  in different modes of cyclic liquefaction or cyclic resistance in general. The objective of this paper is to get new insights into how  $q_0$  can affect the liquefaction behaviour by synthesising experimental data from the authors and published literature. It is pertinent to note that cyclic loading can also be characterized by its peak and trough deviator stresses,  $q_{peak}$  and  $q_{trough}$ . Sometimes, it may be more appropriate to analysis cyclic liquefaction factor in terms of  $q_{peak}$  and  $q_{trough}$ , noting that  $q_{cyc} = (q_{peak} + q_{trough})/2$ .

## 2 FORMS OF CYCLIC LIQUEFACTION

Cyclic liquefaction may occur in two different forms. The first form is illustrated in figure 1 which showed a transient near-zero effective stress state in a load cycle. When this occurs, the cyclic stress-strain loop also changes from an almond shape to a butterfly shape. This change in the shape of the stress-strain loop leads to development of large cyclic strain. Thus, some researchers identify, experimentally, the initiation of liquefaction under cyclic loading by a double amplitude strain of 5% for triaxial testing (Ishihara 1993). Its occurrence does not necessarily infer a flow deformation after cyclic loading ceases, i.e. the deformation may stabilise (Ishihara 1993). Even after onset of cyclic mobility, the specimen will be able to support a non-cyclic stress held at a magnitude lightly less that the peak value (as indicated by the filled-square of Figure 1). Most researchers (Vaid and Chern 1985) use the term CM for this form of cyclic liquefaction.

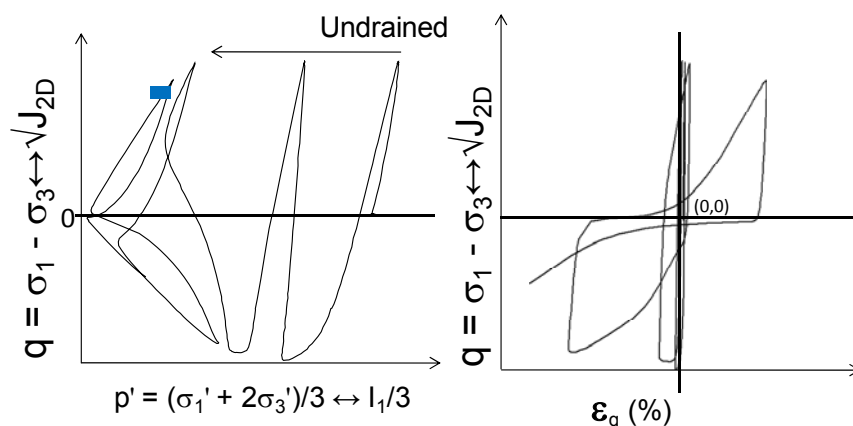


Figure 1. Cyclic mobility

Cyclic undrained loading may also induced a strain-softening behaviour as illustrated in figure 2. In the extreme case, the “residual” resistance may be near-zero. When it is triggered in the field, a flow-like deformation will occur provided that the static shear stress is higher than the residual resistance, i.e. there is no need for a zero effective stress. However, if the cyclic loading is two-way, then the residual strength will reduce to near-zero a few cycles after its triggering. This type of behaviour is a form of instability triggered under cyclic loading, and we will refer it to as cyclic instability (CI). Its manifestation in a load-controlled cyclic triaxial (or simple shear) testing is a run-off in deformation unless a special leading system is used to enable the observation of the strain-softening response. Therefore the criterion of DA strain exceeding 5% can also be used to identify experimentally the onset of CI.

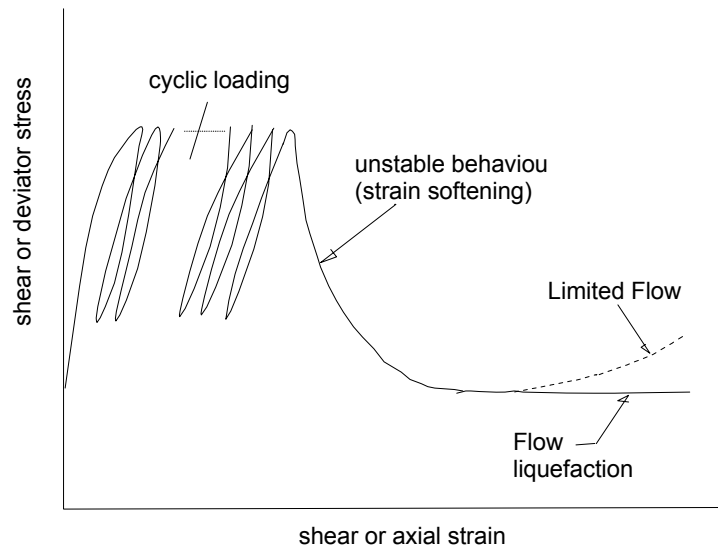


Figure 2. Cyclic liquefaction with deviatoric strain-softening

### 3 ANALYSIS OF CYCLIC INSTABILITY

#### 3.1 Triggering of cyclic instability

Experimental studies on clean sand demonstrate that there is a linkage between cyclic instability and static liquefaction (Gennaro et al. 2004; Hyodo et al. 1994; Vaid and Chern 1983). Lo and co-workers compared monotonic and cyclic liquefaction behaviour using replicate test-pairs (Baki et al. 2012; Lo et al. 2008; Lo et al. 2010a). Replicate specimens have near-identical void ratio,  $e_0$  and  $p'_0$  at start of shearing. The test results unambiguously showed that CI was triggered shortly after the effective stress path, ESP crossed the instability line defined by  $\eta_{IS}$ , the instability stress ratio that defines the onset of static liquefaction. Note that  $\eta_{IS}$  is less than  $M$ , the effective stress ratio at critical state.

Baki (2011) investigated whether cyclic instability can be predicted from monotonic behaviour of an equivalent specimen. An equivalent sample is one with the same equivalent granular state parameter,  $\psi^*$ , but do not have to have identical  $e_0$ ,  $p'_0$ . The concept and definition of  $\psi^*$  was discussed in earlier publications (Mizanur and Lo 2012; Rahman and Lo 2014; Rahman et al. 2008; 2011) and briefly summarised in Appendix A for sake of completeness.

Non-symmetrical two-way cyclic loading with  $q_{peak} = 112$  kPa and  $q_{trough} = -39$  kPa, was applied to a specimen with 30% fines and an initial condition (prior to shearing) of  $\psi^*(0) = +0.052$  and  $p'_0 = 350$  kPa. The test results are shown in figure 3. The  $\eta_{IS}$  value as determined from static liquefaction response of a replicate specimen is denoted by a solid line and the pair of dotted lines indicates the uncertainties in  $\eta_{IS}$ . The ESP moved leftward with loading cycles. At the 5<sup>th</sup> load cycles, the ESP just touched the  $\eta_{IS}$  zone and the prescribed  $q_{peak}$  and  $q_{trough}$  could still be developed. The peak of the ESP in 6<sup>th</sup> cycle just crossed the  $\eta_{IS}$  zone when both leftward movement of the ESP and axial strain development began to accelerate. After this, the prescribed  $q_{peak}$  value cannot be developed and the cyclic axial strain exceeded 5%. It is also interesting to note that, despite instability was triggered on the compression side, the deviatoric resistance in the extension was also lost.

#### 3.2 Influence of $q_0$

The criterion for triggering CI has two implications:

- (i) The replicate or equivalent specimen must be adequately loose for static liquefaction to occur.
- (ii) The proximity of the first  $q_{peak}$  stress point (or  $q_{trough}$  stress point if CI occurs in extension) from the  $\eta_{IS}$ -line has a controlling influence on number of cycles to liquefaction.

The rationale for the first implication is evident:  $\eta_{IS}$  can only be defined for when static instability can occur. The second implication is a reasonable hypothesis that can be evaluated by test results.

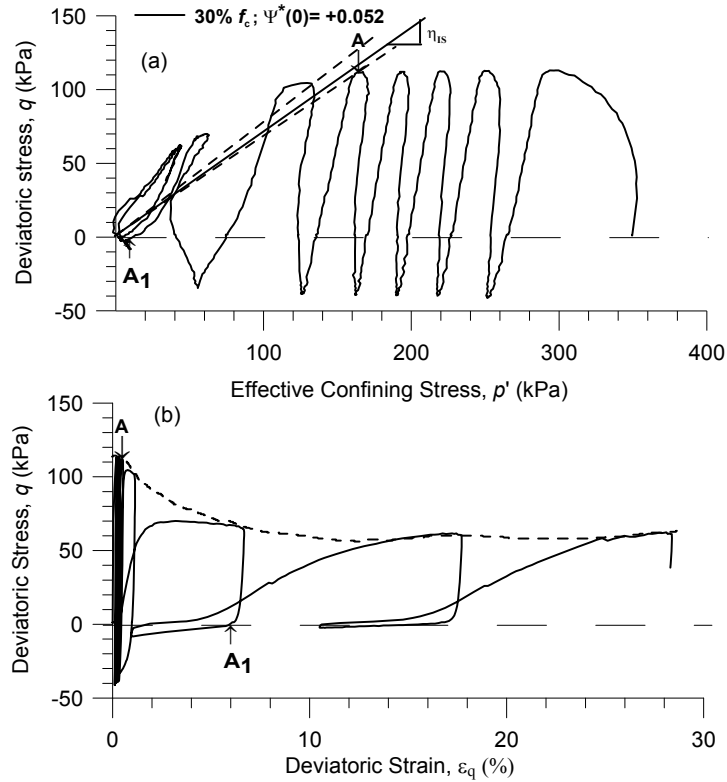


Figure 3. Instability triggered by under 2-way cyclic loading (a) ESP, (b)  $q$ - $\epsilon_q$  plot

Cyclic triaxial tests that manifested CI were extracted from Baki (2011). The material tested is a sand with fines. The host sand is a uniform size quartz sand (SP) called Sydney sand whereas the fines is well-graded low-plasticity fines (PI=27, LL=54) were used in this study. Fines content,  $f_c$ , in this study was in the range of 0-30% by dry weight. Details of the tested material can be found in Baki et al. (2014). The loading system is specially designed (Lo et al. 2010b) so that the strain softening response can be measured despite in a load-controlled cyclic triaxial test. Test results for instability triggered on compression side is synthesised in figure 4, which shows a correlation between  $(q_{peak}/p'_0)/\eta_{IS}$  and  $N_L$ . The parameter  $(q_{peak}/p'_0)/\eta_{IS}$  represents the proximity of the  $q_{peak}$  stress point from the  $\eta_{IS}$ -line.  $q_{peak}$  is normalised by  $p'_0$  to factor-in the influence of initial effective mean stress, whereas a further normalisation by  $\eta_{IS}$  measures the proximity relative to  $\eta_{IS}$  (because our data involves a range of  $\eta_{IS}$ ). It is evident that increase in  $(q_{peak}/p'_0)/\eta_{IS}$  reduces  $N_L$ . Since  $q_{peak} = q_0 + q_{cyc}$ , an increase in  $q_0$ , for the same  $q_{cyc}$ , and other factors being the same, increases  $q_{peak}$ , which in turn reduces  $N_L$ .

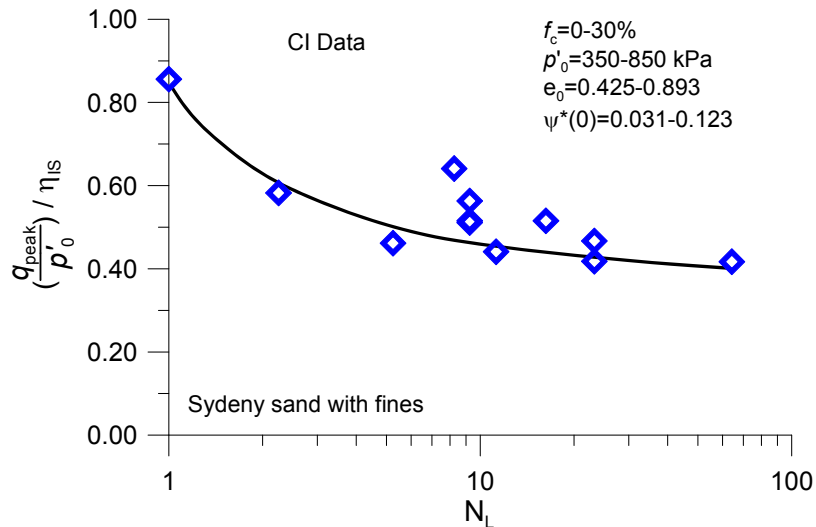


Figure 4. Analysis of CI data for Sydney sand with fines (data extracted from Baki 2011)

#### 4 ANALYSIS OF CYCLIC MOBILITY

CM is characterised by leftward movement of the ESP towards the pair of failure lines (one in compression and other in extension). Indeed where CM is triggered, the ESP traced “upward and downward” along a pair of straight lines with slopes slightly higher those of critical state failure. This suggests triggering CM is strongly influenced by growth of pore water pressure, pwp, with load cycles. It is well established that pwp increases significant with load reversal. Therefore, it is reasonable to hypothesised that, for cyclic liquefaction in the form of CM, load reversal as measured by  $|q_{trough}/q_{peak}|$  will reduce  $N_L$ . This hypothesis is evaluated by examining the CM data of Hyodo et al. (2002) and Sze (2010). The CM data points covered two sands (Toyoura and Aio sand) tested at two different Dr and  $p'_0$  in the range of 86.70 kPa to 5000 kPa,  $q_0$  from 20 kPa to 2000 kPa. This yield 7 data series, with each series plotted with a different symbol. Figure 5 shows that for each test series, a clear relationship between  $|q_{trough}/q_{peak}|$  and  $N_L$  is manifested, with  $N_L$  reduces with increase in  $|q_{trough}/q_{peak}|$ . Since an increase in  $q_0$ , for the same  $q_{cyc}$ , will reduce  $|q_{trough}/q_{peak}|$ , this in turn increases  $N_L$  (by “slowing down” the generation of pore water pressure with load cycles).

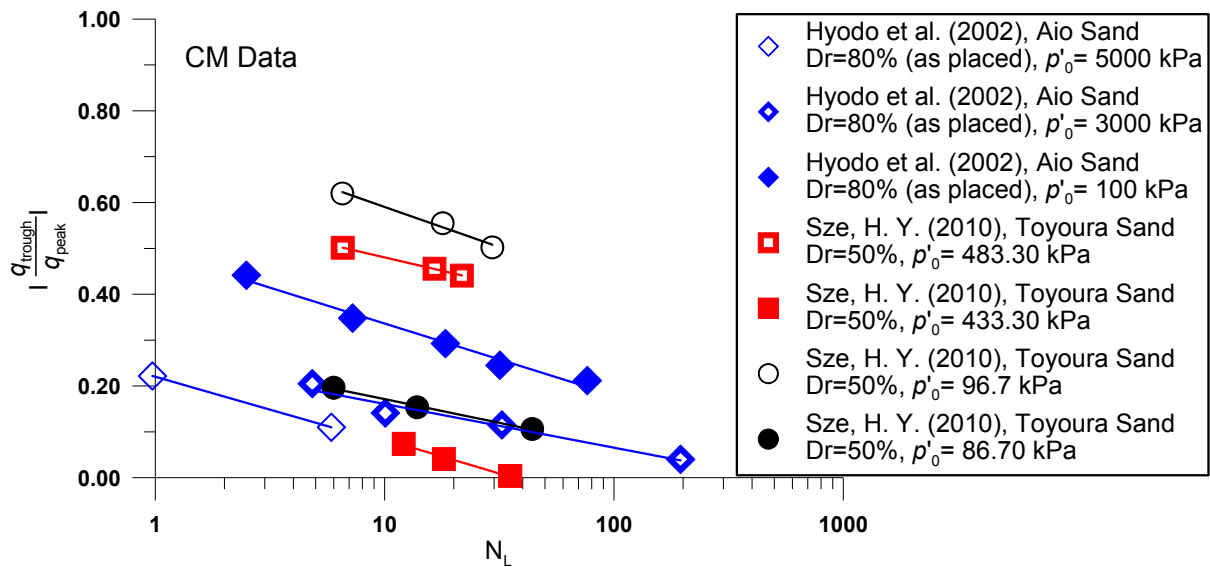


Figure 5. Analysis of CM data

#### 5 PREDICTING INFLUENCE OF $q_0$

From the above two sections, one can infer that the influence of  $q_0$  depends on the form of cyclic liquefaction. For CI, an increase in  $q_0$  will increase  $q_{peak}$  which has a dominating effect on reducing  $N_L$ . For CM, an increase in  $q_0$  will reduce reduces  $|q_{trough}/q_{peak}|$  which has a dominating effect on increasing  $N_L$ . However, there remains the question: If cyclic liquefaction occurs, what form will it take? A review of published data shows that for dense sand, cyclic liquefaction occurred in the form of CM, whereas CI was observed for very loose sand. This leaves a wide “density gap”. Furthermore, the influence of  $p'_0$  and  $f_c$  (for sand with fines) have not been addressed.

Recently, Rahman et al.(2014) demonstrated that different forms of cyclic liquefaction can be predicted based on the initial state of the soil and fines content, as defined by the value of  $\psi^*$ . Their testing program covered a wide range of initial testing conditions:  $f_c$  up to 30%,  $p'_0$  from 100 kPa to 1250 kPa and  $\psi^*(0)$  from +0.123 to -0.203. They showed that tests with  $\psi^*(0) > 0$  showed cyclic instability behaviour. On the other hand, cyclic mobility behaviour was observed for the samples tested with initial conditions  $\psi^*(0) < 0$ . However, specimens tested with  $\psi^*(0) \approx \pm 0.030$ , the form of cyclic liquefaction cannot be predicted with certainty. Thus, the range of  $\psi^*(0)$  is an index by which different forms of cyclic liquefaction behaviour can be predicted capturing the influence of  $f_c$ ,  $p'_0$  and  $q_0$ . Thus, by showing that  $\psi^*$ , will largely determine the form of cyclic liquefaction, one in turn can determine the influence of  $q_0$  on  $N_L$  (or liquefaction resistance).

## 6 CONCLUSIONS

Cyclic liquefaction can occur in two forms: cyclic instability and cyclic mobility. The behaviour patterns of these two forms of cyclic liquefaction are very different.

In cyclic instability  $N_L$  will largely reduce with the proximity of the  $q_{peak}$  stress point from the  $\eta_{IS}$ -line. This implies  $q_0$ , which increases  $q_{peak}$ , will largely reduce  $N_L$ . However, in cyclic mobility,  $q_0$  will reduce  $|q_{trough}/q_{peak}|$  and this in turn increase  $N_L$ . Thus, the trend for CM is opposed that of cyclic instability. This offers a rational explanation for the divergent opinions on the effect of  $q_0$  on resistance to cyclic liquefaction of granular soils. Preliminary experimental evidence supports such a theory.

The form of cyclic liquefaction, if triggered, can be predicted by the equivalent granular state parameter,  $\psi^*$ , at start of undrained cyclic loading.  $\psi^*$  embeds the influence of initial density state, stress state and fines content as long as the soil fabric is of a fines-in-sand matrix.

Combining the above two findings enable one to determine the influence of  $q_0$  on the liquefaction resistance of a sandy soil.

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## APPENDIX

To take into account the presence of fines on density state, one can define an equivalent granular void ratio,  $e^*$ , as an alternative to  $e$ , as proposed by Thevanayagam et al. (2002)

$$e^* = \frac{e + (1-b)f_c}{1 - (1-b)f_c} \quad (1)$$

where  $f_c$  is fines content and  $b$  represents the fraction of fines that are active in force transmission in the soil skeleton. Eqn (1) above requires  $f_c$  is less than a threshold value  $f_{thre}$ , so that the soil fabric is still of a fines-in-sand matrix. To predict  $b$ , Rahman and Lo (2008) proposed a semi-empirical equation expressed as below.

$$b = \left[ 1 - \exp\left(-0.3 \frac{(f_c / f_{thre})}{k}\right) \right] \times \left( r \frac{f_c}{f_{thre}} \right)^r \quad (2)$$

where  $r = d/D$ ,  $k = 1 - r^{0.25}$ , and where  $D$  is the size of sand and  $d$  is the size of fines. Since sand and fines are generally not single-size materials,  $D/d$  was generalized to  $D_{10}/d_{50}$ , where the subscripts denote fractile passing. As an initial approximation,  $f_{thre}$  can be taken as 0.30; but it may be determined more reliably using the following equation developed by Rahman et al. (2009).

$$f_{thre} = 0.40 \left( \frac{1}{1 + e^{\alpha - \beta \chi}} + \frac{1}{\chi} \right) \quad (3)$$

Where  $\alpha = 0.50$  and  $\beta = 0.13$ .

The critical state (or steady state) data points, when plotted in a  $e^*$ - $\log(p')$  space, follows a single relationship irrespective of  $f_c$ . This single relationship is referred to as the equivalent granular steady state line (EG-SSL).  $\psi^*$  is defined as the distance (measured in  $e^*$  direction) between the state point from the EG-SSL (Rahman et al. 2008) as illustrated in Fig. 6 below.

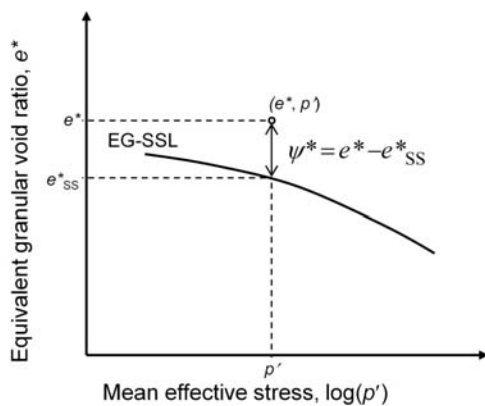


Figure 6. Definition of  $\psi^*$