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# Modelling of ballasted railway track under train moving loads

Md. A. Sayeed[1] and Mohamed A. Shahin[2]

[1]PhD Candidate, Department of Civil Engineering, Curtin University, GPO Box U1987, Perth WA 6845, Australia; email: sayeed.ce00@yahoo.com
[2]Associate Professor, Department of Civil Engineering, Curtin University, GPO Box U1987, Perth WA 6845, Australia; email: m.shahin@curtin.edu.au

## ABSTRACT

Recent congestion of highways in many countries around the world has led railways to become the most popular means of public transportation, which increased the demand for heavier and faster trains. This requires an investigation into the impact of various design parameters affecting the overall railway track performance. Such an investigation is very important for railway geotechnical engineers to arrive at an optimum plan for track design and maintenance. In this paper, a sophisticated three-dimensional (3D) finite element (FE) modelling is developed to investigate the dynamic response of ballasted railway track subjected to train moving loads. All components of the ballasted railway track system are represented including the rail, sleepers, ballast, sub-ballast and subgrade. The viscous quiet boundaries are used to represent infinite boundary conditions so as to absorb the train induced vibration waves at the boundaries. The FE modelling is validated using field measurement data. A detailed parametric study is carried out to investigate the effects of train speed, and modulus and thickness of ballast, sub-ballast and subgrade on track performance. The track response in terms of rail displacement, ballast, subballast and subgrade surface vertical stresses, surface subgrade strain and track stiffness are presented and their practical implications in relation to track design are discussed.

*Keywords:* finite elements, numerical modelling, train moving load, ballasted railway track

## 1 INTRODUCTION

Railway tracks are conventionally founded on compacted granular media of ballast and subballast, laid on subgrade (formation) soil. The key functions of ballast are: distributing the induced train wheel loads through the rail and sleeper, damping the induced train dynamic loads, increasing sideways resistance and providing rapid drainage. Subballast, on the other hand, plays two major roles. Firstly, it is an economical way to increase the granular layer thickness and thus decreasing the stress on top of the subgrade surface, because subballast is usually composed of locally available materials (e.g., sand). Secondly, it provides an effective drainage and serves as a filtering media, hence, ensures safety against the mud pumping (Li 1994). The subgrade is composed of a naturally deposited soil, fill material or a combination of both.

Ballasted railway track design requires an accurate estimation of the granular layer thickness that provides protection against subgrade failure caused by induced train cyclic loading. Without an accurate railway track design, the infrastructure sustains more damage and becomes more outmoded, the performance which might have been achieved is relinquished. Therefore, track maintenance is often invoked, and a large portion of maintenance cost is usually due to geotechnical problems. To reduce this cost and achieve better performance, several factors affecting track response need to be investigated such as the magnitude of wheel loads, number of load cycles, train speed, aggregate type and shape of particles, density and grading of aggregate, modulus and thickness of ballast, sub-ballast and subgrade.

Over the years, railway track analysis and design have evolved from approximate theoretical calculations to sophisticated numerical solutions. However, with the advancement in computer technology the use of numerical modelling for accurate prediction of railway track response is becoming more popular. For example, Shahin and Indraratna (2006) and Shahu et al. (1999) used the finite element (FE) method for investigating the track response considering some of the abovementioned factors. However, most available studies overlooked the true train moving loads along the track and rather provided oversimplified solutions based on a factored static wheel load.

Therefore, in the current study, an advanced three-dimensional (3D) FE modelling is developed to simulate the true dynamic response of railway track induced by train moving loads along the track. The developed FE model is calibrated and validated using field data. Finally, the model is used to provide preliminary track design guidelines via investigating the railway track response to various factors affecting track performance.

## 2 NUMERICAL MODELLING OF RAILWAY TRACK FOUNDATION

### 2.1 Finite element analysis

In this study, a ballasted railway track is numerically modelled using the 3D finite element package GTS-NX (MIDAS IT. Co. Ltd. 2013). The FE model used is simulating a field case study obtained from the literature where the track material properties and field measurements are available. The case study considered is for a rail track between Brussels and Paris, near Ath, 55 km south of Brussels. Figure 1 shows the FE model of the simulated track, which is composed of layers of ballast and subballast as well as a capping layer founded on natural subgrade soil. The length of the modelled track in *X*-, *Y*- and *Z*-directions are 22.5m, 18m and 11m, respectively. The rail is modelled with one dimensional (1D) beam element along 18m in *Y*-direction with a rectangular cross section. The moment of inertia of the beam section is considered in such a way that it has the same properties as an *I*-profile. All other components of the track are modelled using 3D solid elements. A set of 30 sleepers is placed to support the rails at a spacing of 60cm apart. The rail and sleepers are modelled as elastic material, whereas the ballast and subballast are modelled as elastoplastic Mohr-Coulomb material. Due to the lack of information about the plasticity characteristics of the subgrade soil, it is assumed to be elastic. This assumption is reasonable as the thickness of the granular media (i.e., ballast and subballast) is usually selected so that the level of stress on the track subgrade soils is low, hence, no (or only small) zones of plastic yielding can be developed. The material properties of the track components are summarized in Table 1.

*Table 1: Materials Parameters of HST Track (Degrande and Schillemans 2001)*

| Materials | Rail | Sleeper | Ballast | Subballst | Capping layer | Soil 1 | Soil 2 | Soil 3 |
|---|---|---|---|---|---|---|---|---|
| Modulus of Ellasticity, *E* (MPa) | 210000 | 30000 | 200 | 300 | 200 | 48 | 85 | 250 |
| Poissons Ratio, $\nu$ | 0.3 | 0.2 | 0.1 | 0.2 | 0.2 | 0.3 | 0.3 | 0.3 |
| Density, $\gamma$ (kN/m$^3$) | 76.50 | 20.15 | 17.66 | 21.58 | 21.58 | 18.2 | 18.2 | 18.2 |
| Cohesion, *c* (kPa) | – | – | 0 | 0 | 0 | – | – | – |
| Friction Angle (Degree) | – | – | 50 | 40 | 36 | – | – | – |
| Shear Wave Velocity, $C_s$ (m/s) | – | – | – | – | – | 100 | 133 | 266 |
| Thickness, *t* (m) | – | 0.2 | 0.3 | 0.2 | 0.5 | 1.4 | 1.9 | 6.0 |
| Spacing (m) | – | 0.6 | – | – | – | – | – | – |
| Damping Ratio | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 |

Note: Dash (–) indicates “not applicable”

The reliability of any numerical modelling depends largely on the accuracy of the input data and the choice of an appropriate underlying theory. To ensure accuracy of results, the time step, element size and model boundaries have to be selected carefully. In simulation of moving loads, ‘dispersion’ is another feature which needs to be reduced in the FE model by choosing a proper mesh discretization and mass matrix (Galavi and Brinkgreve 2014). Dispersion is a phenomenon that occurs when the waves with different wavelength travel at different speeds. The above issues are discussed in some detail below.

In the current FE model, the element size is estimated based on the smallest wave length so that high frequency motions can be modelled properly. The element size of the sleeper is selected to be 0.167m x 0.138m x 0.20m, whereas the element size of the granular layers (i.e., ballast and subballast) is used to be 0.167m x 0.158m x 0.20m. The whole railway track model is consisted of 137,000 elements. Viscous boundaries (Kouroussis et al. 2011; Lyster and Kuhlemeyer 1969) are used in the *X*- and *Y*- directions, to represent infinite boundary conditions that absorb the *S* and *P* waves. The nodes at the bottom boundary are fixed in every direction to simulate bedrock.

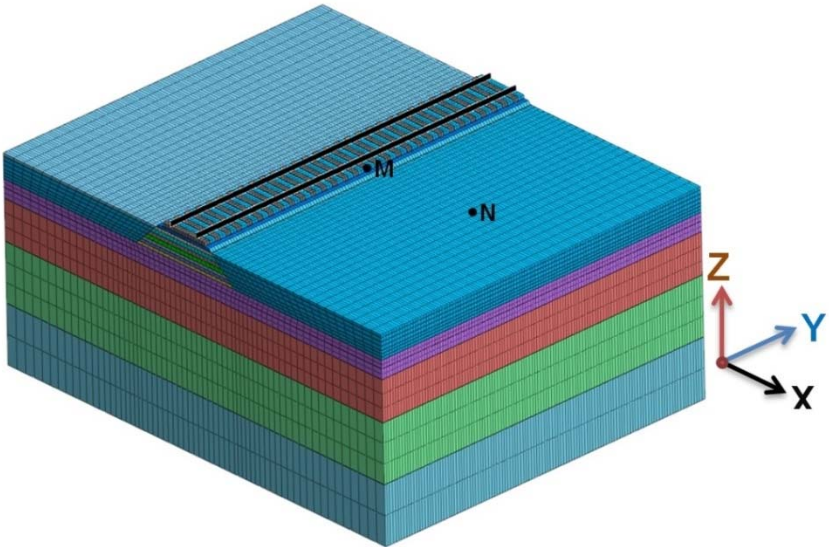



*Figure 1. FE railway track model*

Rayleigh damping coefficients (i.e., mass proportional, $\alpha$, and stiffness proportional, $\beta$) are also used to absorb the incident Rayleigh waves. In order to establish $\alpha$ and $\beta$, it is necessary to adequately relate these parameters with the hysteretic damping coefficient ($\xi$) for representation of soil damping. For this purpose, eigenvalue analysis is performed to obtain the first two frequencies $\omega_i$ of the full model. Using the obtained two frequencies, the corresponding $\alpha$ and $\beta$ are achieved, as follows:

$$\xi_i = \frac{1}{2}\left(\frac{\alpha}{\omega_i} + \beta\omega_i\right) \quad (1)$$

During simulation of the moving loads, the proper time step is defined based on the well-known Courant number (Galavi and Brinkgreve 2014) and its value is always considered to be less than one.

### 2.2 Simulation of moving loads

To simulate the moving loads along the railway track, the FE is modelled in accordance with Araujo (2011) in which each FE node of the rail is subjected to a wheel load whose value changes in time. As shown in Figure 2, the value of the wheel point load, *F*, at one certain node *N*+1 increases once the wheel leaves node *N*, reaching its peak value when the wheel is directly above that node and finally decreasing back to zero when the wheel reaches the next node *N*+2. In this fashion, a series of train wheels can be considered to move along the track. For example, for train speed of 86km/h, the wheel point load will pass the distance between two consecutive FE rail nodes (which is equal to 60cm in accordance with the sleepers spacing) in 0.025 sec.

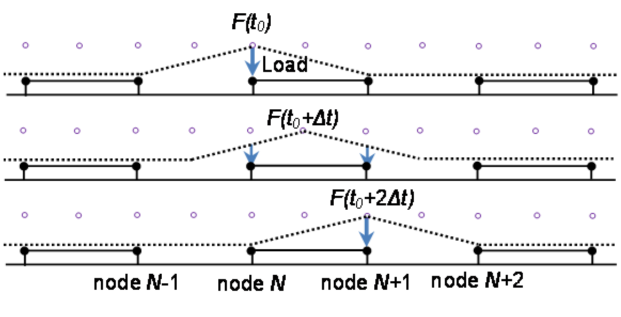



*Figure 2. Simulation of moving loads (Araujo, 2011)*

## 2.3 Validation of FE model

To validate the FE model, the vibration made (i.e., the time history response of the track) during the passage of the Thalys High Speed Train (HST) at 314 km/h is predicted at two observation points and the results are compared with the field measurements reported by Cunha and Correia (2012). One point of measurement is located at the sleeper, next to the rail (i.e., Point M), and the other is located on the ground at a horizontal distance equal to 7.25m from the rail (i.e., Point N), as shown in Figure 1 earlier. The geometry of the Thalys HST is shown in Figure 3 and its characteristics including the carriage length ($L_t$), distance between two bogies ($L_b$), distance between axles ($L_a$) and total axle load ($W_t$) of each carriage are summarized in Table 2.

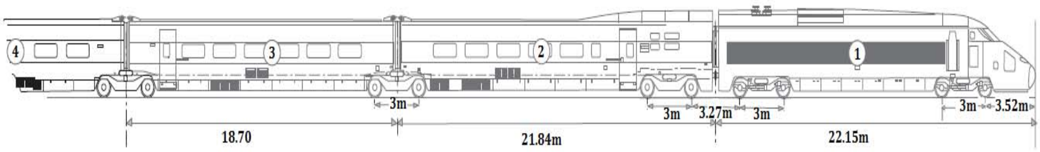
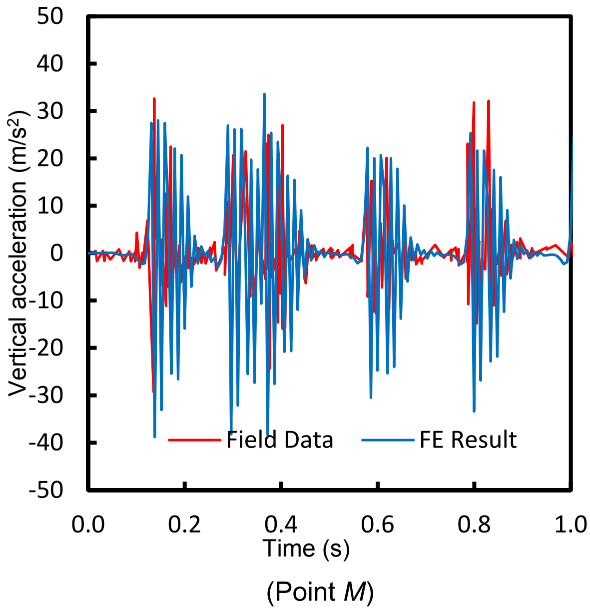



*Figure 3. Geometry of Thalys HST (Degrande and Schillemans 2001)*

*Table 2: Geometry and load characteristics of Thalys HST (Degrande and Schillemans 2001)*

| Carriage Name | Carriage Number | Axles per Carriage | $L_t$ (m) | $L_b$ (m) | $L_a$ (m) | $W_t$ (kN) |
|---|---|---|---|---|---|---|
| Locomotive | 2 | 4 | 22.15 | 14.00 | 3.00 | 168 |
| Side Carriage | 2 | 3 | 21.84 | 18.70 | 3.00 | 143 |
| Central Carriage | 6 | 2 | 18.70 | 18.70 | 3.00 | 168 |

Figure 4 shows a comparison between FE predicted values and field measurements of the vertical acceleration over a time of one second at the observation points *M* and *N*. It can be seen that the FE predicted response agrees well with the field measurements. It should be noted that some peak values of the field measurements are higher than the others and this is explained by Cunha and Correia (2012). Irrespective of this, it can be seen that the developed FE model predicts the track response with appreciative accuracy.



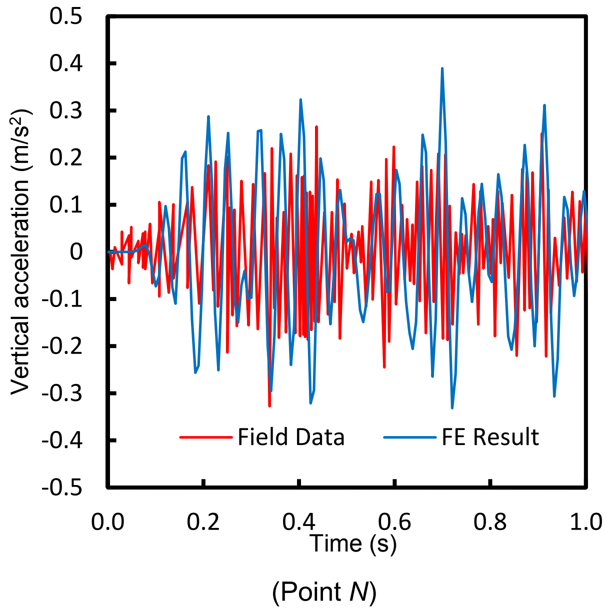




*Figure 4. Comparison of FE predicted versus field measured response at observation points M and N.*

## 3 PARAMETRIC STUDY

This section presents a parametric study using FE analysis on railway track response for several track parameters. A nominal fixed track is adopted and used as a bench mark for the basis of comparison. A practical range of values of different track components are varied and the track behavior is compared with respect to the nominal case. Table 3 shows the nominal values of material properties and Table 4 demonstrates the range of variables considered in the parametric study. When the impact of a certain parameter is investigated within the range shown in Table 4, the other parameters remained fixed at their constant nominal values shown in Table 3. It should be noted that the values in Tables 3 and 4 are based on similar values given by other researchers (e.g., Selig and Li 1994; Shahu et al. 1999).

*Table 3. Nominal track properties used for the parametric study*

| Parameter | Rail | Sleeper | Ballast | Subballast | Subgrade |
|---|---|---|---|---|---|
| Modulus of Elasticity, $E$ (MPa) | 207000 | 30000 | 300 | 200 | 50 |
| Poisson's Ratio, $\nu$ | 0.30 | 0.20 | 0.30 | 0.35 | 0.35 |
| Density, $\gamma$ (kN/m$^3$) | 76.60 | 20.20 | 17.80 | 22.00 | 18.50 |
| Thickness, $t$ (m) | – | 0.20 | 0.30 | 0.20 | 6.75 |
| Shear Wave Velocity, $C_s$ (m/s) | – | – | – | – | 100 |
| Inertia, $I_x$ (m$^4$) | 3.075 x10$^{-5}$ | – | – | – | – |
| Length (m) x Width (m) | – | 2.5×0.275 | – | – | – |
| Spacing (m) | – | 0.60 | – | – | – |
| Hysteretic Damping Ratio | 0.01 | 0.01 | 0.02 | 0.02 | 0.03 |

*Table 4. Range of variable track properties used for the parametric study*

| Parameter | Lower bound | Nominal | Upper Bound |
|---|---|---|---|
| Ballast Modulus, $E_b$ (MPa) | 150 | 300 | 500 |
| Subballast Modulus, $E_{sb}$ (MPa) | 80 | 200 | 400 |
| Subgrade Modulus, $E_{sg}$ (MPa) | 15 | 50 | 140 |
| Ballast Thickness, $t_b$ (m) | 0.15 | 0.30 | 0.90 |
| Subballast Thickness, $t_{sb}$ (m) | 0.10 | 0.20 | 0.60 |
| Subgrade Thickness, $t_{sg}$ (m) | 1.75 | 6.50 | 9.50 |
| Train speed (km/h) | 86 | 86 | 344 |

### 3.1 Track response

In this section, the track response for the number of parameters given in Table 4 is investigated in terms of the rail deflection, surface vertical stresses of ballast, subballast and subgrade, surface strain of subgrade and track stiffness. It should be noted that the track response of this section is based on Thalys HST moving along the track at a speed of 86km/h, and the results are shown in Figure 5. It should be noted that the horizontal line in each graph of Figure 5 represents the track response for the nominal case, as defined by the properties given in Table 3, whereas the vertical lines represent the upper and lower ranges of predicted track response for the values of parameters given in Table 4. The numbers inside each graph represent the upper and lower bounds of the parameters considered.

Figure 5(a) shows that the subgrade modulus is the most significant factor affecting the rail deflection. A decrease in the subgrade modulus leads to dramatic increase in the rail deflection, and similar trend is observed when the ballast thickness is decreased. In contrast, a decrease in the subgrade thickness results in a decrease in the rail deflection. Figure 5(b) depicts that the ballast modulus and subgrade modulus are found to have the most influential impact on the ballast surface vertical stress. An increase in the ballast modulus causes an increase in the ballast surface vertical stress, while a decrease in the subgrade modulus leads to an increase in the ballast surface vertical stress.

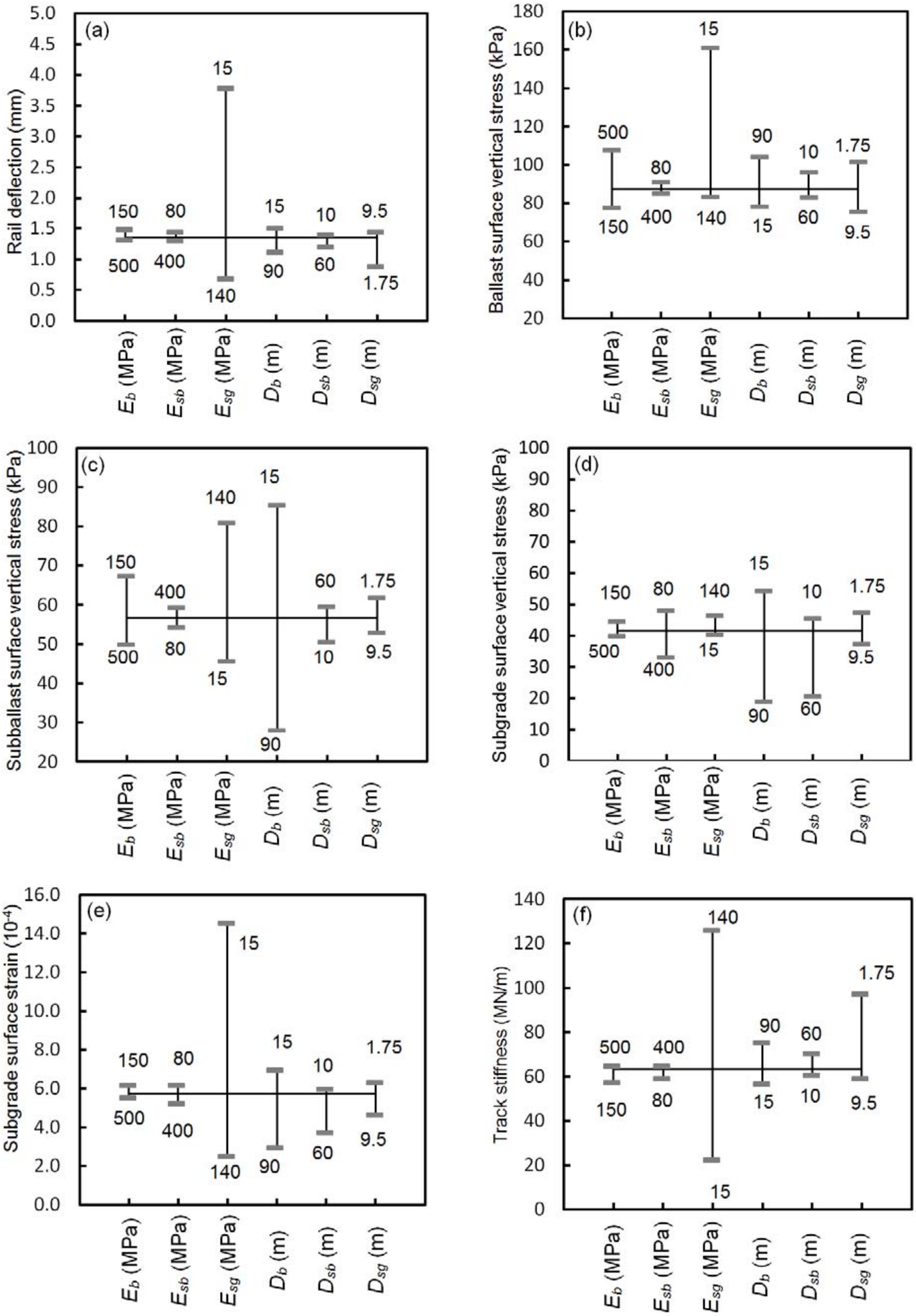



*Figure 5. Effect of track influencing parameters on track performance*

The track response with respect to the subballast and subgrade surface vertical stresses are almost identical, as shown in Figure 5(c)&(d), in which the subgrade modulus, ballast and subballast thicknesses are the most influential factors affecting track performance. An increase in the subgrade modulus leads to an increase in both the subballast and subgrade surface vertical stresses. On the other hand, an increase in the ballast depth causes a reduction in both the subballast and subgrade surface vertical stresses. The subballast thickness increases its own surface vertical stress; however, it reduces the stress on the subgrade soil. In Figure 5(e), it is evident that the subgrade modulus and ballast depth are the most significant factors influencing the subgrade surface strain. With the increase of these two parameters, the subgrade surface strain reduces dramatically.

Finally, the track performance is measured by a parameter called 'track stiffness', which is defined as the force that causes a unit vertical deflection of track (Selig and Li 1994). Figure 5(f) confirms that the most dominating factor influencing track stiffness is the subgrade modulus. It can be seen that less than nine-fold increase in the subgrade modulus from 15 to 140 MPa leads to an increase in track stiffness of approximately five times. It can also be seen that the depth of ballast layer plays an important role in increasing the ballasted track stiffness, whereas a decreases in the subgrade thickness leads to an increases in track stiffness. Overall, it is clearly evident from Figure 5 that the subgrade modulus has the greatest influence on track response.

### 3.2 Effect of train speed

The train speed is another important factor that influences track performance. In this study, the effect of train speed is investigated for the nominal case by considering the Thalys HST moving at three different speeds of 86km/h, 172km/h and 344km/h, and the results are shown in Figure 6. Because the time taken for the train to travel through the track is different, depending on the train speed (i.e., higher speed train takes less time to cross the track than lower speed train), it is convenient to represent the results in terms of the relative time rather than the actual time, and the relative sleeper deflection rather than the actual sleep deflection. The relative deflection ($\delta_r$) and relative time ($t_r$) are those which are related to the deflection and time corresponding to a selected reference train speed of 86km/h, and are calculated as follows:

$$\delta_r = (\delta_v / \delta_{86}) \times 100\% \qquad (2)$$

$$t_r = (t_v \times V) / 86 \qquad (3)$$

where, $\delta_v$ is the sleeper deflection at certain train speed *V*, $\delta_{86}$ is the maximum sleeper deflection at train speed of 86 km/h and $t_v$ is the traveling time at certain train speed *V*.

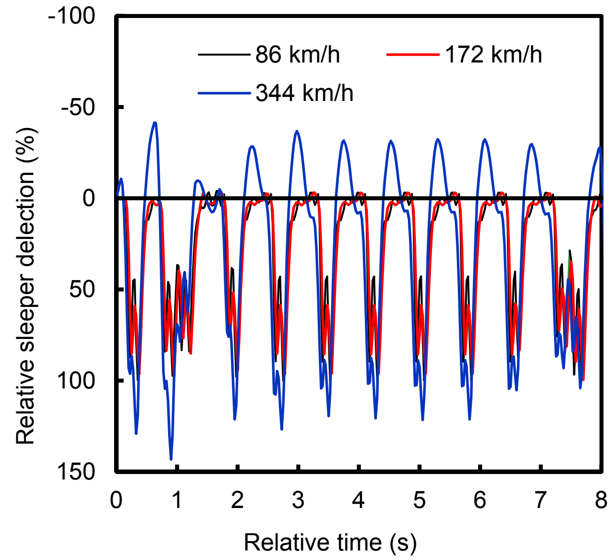



*Figure 6. Time history dynamic response of sleeper deflection for different train speed*

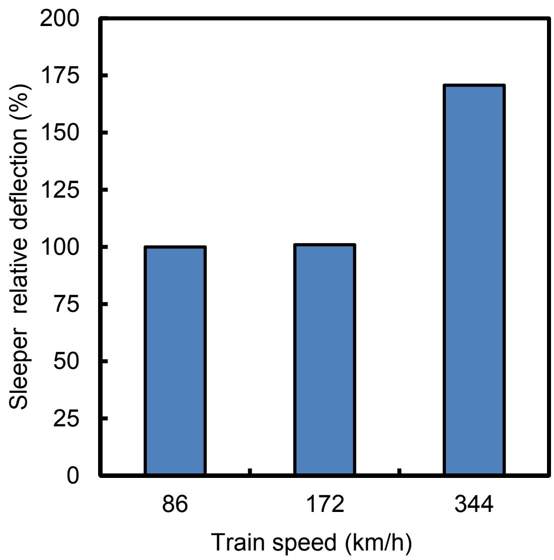



*Figure 7. Effect of speed on rail deformation*

It can be seen from Figure 6 that the relative sleeper deflection (negative values mean upward deflection and positive values mean downward deflection) for both train speed of 86km/h and 172km/h are almost identical, indicating no impact. However, at train speed of 344km/h, the sleeper deformation increased significantly. In addition, an upward sleeper deflection is observed as the train speed is being close to the shear wave velocity of the subgrade soil (i.e., $C_s$ = 100 m/s). This behavior confirms good consistency in the qualitative sense with the results of the boundary element (BE) method reported by Galvín et al. (2010).

Figure 6 also illustrates that the influence of train speed on the sleeper deflection is not significant for train speeds of 86km/h and 172km/h. However, the sleeper relative deflection increases dominantly for train speed of 344km/h, emphasizing the significant importance of train high speed. However, it should be emphasized that the above results are for the nominal case of railway track properties given in Table 3 and should not be generalized for other track properties unless more investigation is conducted, which will be carried out in the subsequent phase of this ongoing research.

## 4 CONCLUSION

In this study, 3D finite element model was developed to evaluate the dynamic response of ballasted railway track subjected to train moving loads. To obtain an accurate prediction of track response, the time step, element size and model boundaries were selected carefully. The computed track response was compared with field measurements available in the literature. It was demonstrated that the FE model can be employed to simulate and understand the dynamic behaviour of ballasted railway track.

Parametric study was also carried out to investigate the track response for a number of track parameters including the train speed, modulus and thickness of ballast, subballast and subgrade. It was found that the subgrade modulus is the most influencing factor on the overall track performance. It was noted that the decrease in subgrade modulus significantly affects track response including the rail deflection, ballast and subballast surface vertical stresses, surface subgrade strain and track stiffness. This clearly indicates that maintenance would be a critical issue for tracks built on soft subgrade. It was also demonstrated that train speed affects track performance; however, more investigation is still needed to confirm the true impact of train speed and the current conclusion in relation to this matter should be considered preliminary for the time being.

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