The paper was published in the proceedings of the 12th Australia New Zealand Conference on Geomechanics and was edited by Graham Ramsey. The conference was held in Wellington, New Zealand, 22-25 February 2015.

Exact analytical solution for one-dimensional consolidation of unsaturated soil stratum subjected to damped sine wave loading

L. H. Ho[1], B. Fatahi[2] and H. Khabbaz[3].

[1]PhD Candidate (BEng(hons)), School of Civil and Environmental Engineering, University of Technology Sydney (UTS), Sydney, Australia; email: Liem.Ho@uts.edu.au
[2]Senior Lecturer of Geotechnical Engineering (PhD, CPEng), School of Civil and Environmental Engineering, University of Technology Sydney (UTS), Sydney, Australia; email: Behzad.Fatahi@uts.edu.au
[3]Associate Professor of Geotechnical Engineering (PhD), School of Civil and Environmental Engineering, University of Technology Sydney (UTS), Sydney, Australia; email: Hadi.Khabbaz@uts.edu.au

ABSTRACT

A considerable surcharge exerted on an unsaturated soil stratum leads to the emergence of excess pore pressures. During the consolidation process, these pressures tend to dissipate towards permeable boundary surfaces, resulting in a reduction of the soil volume. Such phenomenon can be mathematically described by inhomogeneous governing equations of flow based on Fick's law (with respect to air phase) and Darcy's law (with respect to water phase). This paper discusses the dissipation of excess pore-air and pore-water pressures and settlement of an unsaturated soil stratum subjected to an external damped sine wave loading. An analytical solution is derived from the governing equations of flow using eigenfunction expansion and Laplace transformation methods. Eigenfunctions and eigenvalues are parts of the general solution and can be obtained based on one-way drainage boundary condition. On the other hand, the damped sine wave loading is mathematically simulated and incorporated in the solution. Once the time variable (t) in partial differential equations is transformed into the Laplace complex argument (s), generalised Fourier coefficients can be computed by taking a Laplace inverse, and then the final solution can be obtained. In this study, the air to water permeability ratio ($\mathrm{k_a/k_w}$), influencing changes in dissipation rates of excess pore pressures and settlement are investigated and discussed. It is observed that the increasing permeability ratio has a significant effect on the change in the pore pressures.

Keywords: Unsaturated Soil, Analytical Solution, One-Dimensional Consolidation, Settlement, Excess Pore Pressures, Damped Sine Wave Loading

1 INTRODUCTION

Consolidation is a process of reducing the soil volume due to the dissipation of excess pore pressures. In early 1940s, Terzaghi (1943) introduced a prevalent theory estimating the one-dimensional (1-D) consolidation for fully saturated soils based on the heat transfer concept. This original yet relatively remarkable theory has been a solid platform for several significant soil mechanics frameworks. However, during engineering developments, key factors such as earthworks, climatic changes, and surface vegetation may result in further creation of unsaturated soils, whose physical properties are much more complicated than those of saturated soils (Fatahi et al. 2009, 2010 and 2014). The past five decades have witnessed a noticeable growth in unsaturated soil studies, particularly consolidation concept, through several investigations and experimental research. The study of consolidation of unsaturated soils was initiated in early 1960s. Some original research in this field was conducted by Blight (1961), Scott (1963) and Barden (1965) and numerous others. In late 1970s, Fredlund and Hasan (1979) proposed inhomogeneous governing equations that contributed significantly to the state of the art research on consolidation field. These partial differential equations (PDEs) were derived from Fick's and Darcy's laws, describing continuous flows of air and water within a soil element, respectively. Unlike saturated soil mechanics, the inclusion of the air phase in the equations has been a major challenge in estimating the dissipation of excess pore pressures and the final settlement.

For over the past two decades, the flexible computational tools have enabled geotechnical engineers to estimate the time-dependent deformation of unsaturated soils. In addition to noticeable numerical methods conducted by Lloret and Alonso (1980), Wong et al. (1998), Conte (2004), and Zhou and Zhao (2013) to name a few, analytical methods have also been frequently updated based on the governing equations given by Fredlund and Hasan (1979). Ho et al. (2014a) adopted the expansion of eigenfunctions and Laplace transform methods to predict the 1-D consolidation behaviour of unsaturated soils subjected to a constant surcharge. Moreover, Qin et al. (2010) proposed an analytical method to determine the dissipation rates of excess pore pressures induced by a time-dependent exponential loading. The inclusion of the exponential loading function generates new consolidation parameters, contributing to more cumbersome calculations for the analytical procedure. Shan et al. (2012) and Zhou et al. (2013), on the other hand, converted the nonlinear governing equations into traditional heat diffusion equations similar to that of saturated soils and proposed alternative solutions for 1-D consolidation of an unsaturated soil stratum subjected to an exponential loading (e.g. Shan et al. 2012; Zhou et al. 2013) and a sinusoidal loading (e.g. Shan et al. 2012). Although the mentioned analytical methods provide several constructive ideas in determining consolidation characteristics of unsaturated soils, these methods are relatively impractical for use due to the complex mathematical computation and the lack consideration of realistic time-dependent loadings exerted on the ground surface. In engineering practice, soil deposits beneath the ground may be subjected to damped and cyclic loadings as the result of traffic loads. A damped wave function can be a good representative to describe such external loadings. Since there has been limited research on soil consolidation process under a damped wave loading, it is essential to address the effects of this complex load on the consolidation behaviour of unsaturated soil deposits.

The settlement consolidation calculations of soil layers underlying foundations or embankments are usually restricted to saturated soils under monotonic loads. Foundation designers generally neglect the significant effect of dynamic loads in particular the impact loads (e.g. loads due to severe breaking or collision), when applying the traditional deterministic ultimate limit state analysis. Soil under these dynamic loads will produce a reaction, which exceeds its static capacity. This paper highlights the significance of damped impact loads on foundation settlement, while taking into account the behaviour of unsaturated soil lies beneath. Based on the governing equations originally proposed by Fredlund and Hasan (1979), this paper adopts the eigenfunction expansion and Laplace transformation techniques to determine the 1-D consolidation behaviour of an unsaturated soil stratum, in terms of excess pore pressure dissipation and settlement, subjected to a damped sine wave loading. Due to the limitation of paper length, the analytical procedure only adopts a typical one-way drainage boundary condition and uniform initial conditions (i.e. uniform distribution of initial excess pore pressures). The proposed solutions are graphically demonstrated in a subsequent section.

2 ANALYTICAL DEVELOPMENT

The actual soil properties influencing deformation of the ground are relatively ambiguous due to the complicated texture assemblage and the lack of homogeneity of the soil. These may lead to the difficulty in predicting the consolidation characteristics. Therefore, some assumptions helping to obtain solutions must be addressed as follows:

(1) The entire soil strata are assumed to be homogeneous;
(2) The flows of air and water phases are assumed to be continuous and independent;
(3) Solid skeleton and water phase are incompressible;
(4) Effects of environmental factors such as air diffusion and temperature change are ignored;
(5) The deformation only happens along the vertical direction (z-direction); and
(6) Consolidation parameters with respect to air phase (C_a, c_v^a and c_σ^a) and water phase (C_w, c_v^w and c_σ^w) are assumed to be constant during the loading process.

2.1 Governing equations of flow for 1-D consolidation of unsaturated soils

Fredlund and Hasan (1979) proposed the set of governing equations for the 1-D consolidation of an unsaturated soil stratum using the Cartesian coordinate system. These equations are employed to estimate the dissipation process of excess pore-air and pore-water pressures; and can be presented as follows:

$$u_{a,t} + c_v^a u_{a,zz} + C_a u_{w,t} - c_\sigma^a \sigma_{,t} = 0; \quad (1a)$$

$$u_{w,t} + c_v^w u_{w,zz} + C_w u_{a,t} - c_\sigma^w \sigma_{,t} = 0; \quad (1b)$$

where σ is the total stress; u_a and u_w are excess pore-air and pore-water pressures, respectively; $u_{a,t}$ and $u_{w,t}$ are the first order PDEs of pore-air and pore-water pressures with respect to time, respectively; $u_{a,zz}$ and $u_{w,zz}$ are the second order PDEs of pore-air and pore-water pressures with respect to depth, respectively; and $\sigma_{,t}$ are the first order PDE of the total stress with respect to time.

In addition, the consolidation coefficients are:

$$C_a = \frac{1}{\left[\left(\frac{m_1^a}{m_2^a}-1\right)-\frac{n(1-S_r)}{m_2^a(u_a^0+u_{atm})}\right]}; \qquad c_v^a = \frac{k_a R\Theta}{gM}\frac{1}{\left[m_2^a(u_a^0+u_{atm})\left(\frac{m_1^a}{m_2^a}-1\right)-n(1-S_r)\right]}; \qquad c_\sigma^a = \frac{1}{\left[\left(1-\frac{m_2^a}{m_1^a}\right)-\frac{n(1-S_r)}{m_1^a(u_a^0+u_{atm})}\right]};$$

$$C_w = \left(\frac{m_1^w}{m_2^w}-1\right); \qquad c_v^w = \frac{1}{m_2^w}\left(\frac{k_w}{\gamma_w}\right);\ \text{and} \qquad c_\sigma^w = \frac{m_1^w}{m_2^w}; \tag{2}$$

where m_1^a and m_1^w are the coefficients of air and water volume change with respect to the change of net stress $(\sigma - u_a)$, respectively; m_2^a and m_2^w are the coefficients of air and water volume change with respect to the change of suction $(u_a - u_w)$, respectively; k_a and k_w are the air and water permeability coefficients $(m.s^{-1})$, respectively; g is the gravitational acceleration $(\sim 9.8m.s^{-2})$; u_a^0 and u_w^0 are the initial pore-air and pore-water pressures (kPa), respectively; u_{atm} is the atmospheric pressure (kPa); R is the universal air constant $(\sim 8.3J.mol^{-1}K^{-1})$; $\Theta = (\theta^{\circ} + 273)$, is the absolute temperature (K); M is the molecular mass of air phase $(\sim 0.029kg.mol^{-1})$; n is the porosity during consolidation process; S_r is the degree of saturation during consolidation process; and γ_w is the unit weight of water $(\sim 9.8kN.m^{-3})$.

2.2 Boundary and initial conditions

Figure 1 illustrates a referential profile of a typical one-way drainage unsaturated soil system with an infinite width and a finite thickness H. This system consists of a permeable top surface and an impervious base. Soon after the application of external loads, excess pore-air and pore-water pressures move towards the permeable top surface but cannot dissipate through the impervious base of the soil stratum. At $t \geq 0$, the one-way drainage boundary condition can be simulated mathematically as shown in (3a) and (3b):

$$u_a(0,t) = u_w(0,t) = 0; \tag{3a}$$

$$u_{a,z}(z,t)\big|_{z=H} = u_{w,z}(z,t)\big|_{z=H} = 0. \tag{3b}$$

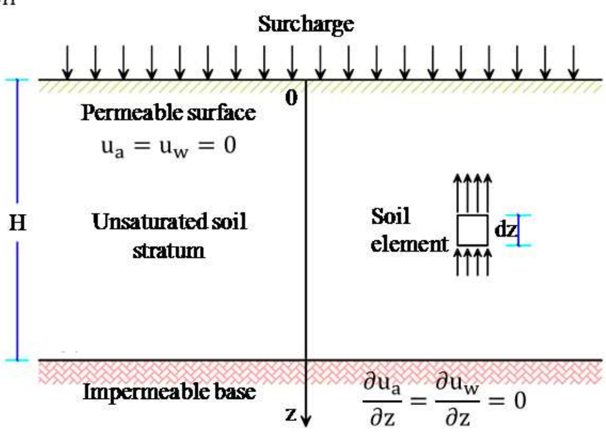

Figure 1. Referential profile of one-way drainage boundary system

In this study, an application of load on the soil surface is assumed to immediately generate uniformly distributed initial excess pore pressures. The initial pore pressures in the domain $z \in (0, H)$ can be presented as below:

$$u_a(z,0) = u_a^0; \tag{4a}$$

$$u_w(z,0) = u_w^0. \tag{4b}$$

2.3 Eigenfunction expansions and Laplace transformation methods

The eigenfunction expansion method can be applied to inhomogeneous differential problems involved with piecewise smooth functions (Haberman 2012). This method uses homogeneous forms for general solutions of $u_a(z,t)$ and $u_w(z,t)$, as follows:

$$u_a(z,t) = \sum_{k=0}^{\infty} Z_a(z).T_a(t); \quad (5a)$$

$$u_w(z,t) = \sum_{k=0}^{\infty} Z_w(z).T_w(t); \quad (5b)$$

where $Z_a(z)$ and $Z_w(z)$ are eigenfunctions with respect to the depth z; and $T_a(t)$ and $T_w(t)$ are generalised Fourier coefficients varying with time t.

According to Haberman (2012), the eigenfunction expansion method is employed to solve the inhomogeneous PDEs (i.e. (1a) and (1b)) by expanding the general solutions in a series of the eigenfunctions of the related homogeneous problem. In this study, the eigenfunctions can be expressed in an ordinary Fourier sine function. Additionally, the eigenvalues corresponding to particular sine functions, denoted by λ, is found to be $\lambda_k = [(2k+1)\pi/(2H)]^2$ ($k = 0, 1, 2 ...$) for the one-way drainage system, indicating that the eigenfunctions are $Z(z) = \sin[(2k+1)\pi z/(2H)]$. Thus,

$$u_a(z,t) = \sum_{k=0}^{\infty} T_a(t)\sin\left(\sqrt{\lambda_k}z\right); \quad (6a)$$

$$u_w(z,t) = \sum_{k=0}^{\infty} T_w(t)\sin\left(\sqrt{\lambda_k}z\right). \quad (6b)$$

The term-by-term differentiations with respect to time t in (6a) and (6b) will result in:

$$u_{a,t}(z,t) = \sum_{k=0}^{\infty} T_{a,t}(t)\sin\left(\sqrt{\lambda_k}z\right); \quad (7a)$$

$$u_{w,t}(z,t) = \sum_{k=0}^{\infty} T_{w,t}(t)\sin\left(\sqrt{\lambda_k}z\right). \quad (7b)$$

Combining (1a) and (1b) with (7a) and (7b) leads to:

$$\sum_{k=0}^{\infty} T_{a,t}(t)\sin\left(\sqrt{\lambda_k}z\right) = \left[c_v^a\lambda_k T_a(t) - C_a T_{w,t}(t)\right]\sin\left(\sqrt{\lambda_k}z\right) + c_\sigma^a\sigma_{,t}(t) \quad (8a)$$

$$\sum_{k=0}^{\infty} T_{w,t}(t)\sin\left(\sqrt{\lambda_k}z\right) = \left[c_v^w\lambda_k T_w(t) - C_w T_{a,t}(t)\right]\sin\left(\sqrt{\lambda_k}z\right) + c_\sigma^w\sigma_{,t}(t) \quad (8b)$$

By computing orthogonality of sine functions, (8a) and (8b) become:

$$T_{a,t}(t) = c_v^a\lambda_k T_a(t) - C_a T_{w,t}(t) + \mu_k c_\sigma^a\sigma_{,t}(t) \quad (9a)$$

$$T_{w,t}(t) = c_v^w\lambda_k T_w(t) - C_w T_{a,t}(t) + \mu_k c_\sigma^w\sigma_{,t}(t) \quad (9b)$$

where $\mu_k = 2/\left(H\sqrt{\lambda_k}\right)$. (10)

To solve the first order derivatives, (9a) and (9b) can be converted into Laplace transformed equations with a complex variable s. It is assumed that the coefficients of permeability and the volume change coefficients with respect to air and water phases are constant during the consolidation process, thus,

$$[s\overline{T}_a(s) - T_a(0)] = c_v^a\lambda_k\,\overline{T}_a(s) - C_a\,[s\overline{T}_w(s) - T_w(0)] + \mu_k c_\sigma^a[s\overline{\sigma}(s) - \sigma(0)] \quad (11a)$$

$$[s\overline{T}_w(s) - T_w(0)] = c_v^w\lambda_k\,\overline{T}_w(s) - C_w\,[s\overline{T}_a(s) - T_a(0)] + \mu_k c_\sigma^w[s\overline{\sigma}(s) - \sigma(0)] \quad (11b)$$

where $\overline{T}_a(s)$, $\overline{T}_w(s)$, and $\overline{\sigma}(s)$ are Laplace transformed functions with complex argument s;

$T_a(0) = \frac{\int_0^H [u_a(z,0)\sin(\sqrt{\lambda_k}z)]dz}{\int_0^H [\sin^2(\sqrt{\lambda_k}z)]dz} = \mu_k u_a^0$; and

$$T_w(0) = \frac{\int_0^H [u_w(z,0)\sin(\sqrt{\lambda_k}z)]dz}{\int_0^H [\sin^2(\sqrt{\lambda_k}z)]dz} = \mu_k u_w^0. \quad (12)$$

On the other hand, the damped sine wave loading is described as a sinusoidal function with its amplitude approaching zero when time increases. Figure 2 presents a typical damped sine wave function. The general damped sine wave loading can be simulated as below:

$$q(t) = q_0[Ce^{-ct}\sin(\vartheta t) + 1] \quad (13)$$

where C is the dimensionless parameter influencing the loading amplitude; c is the loading parameter controlling damping rate (s^{-1}); and ϑ is the angular frequency ($rad.s^{-1}$). Hence,

$$s\overline{\sigma}(s) - \sigma(0) = \frac{Cq_0\vartheta s}{(c+s)^2+\vartheta^2} \tag{14}$$

Substituting (12) and (14) into (11a) and (11b) and then simultaneously solving for $\overline{T}_a(s)$ and $\overline{T}_w(s)$ will lead to:

$$\overline{T}_a(s) = \mu_k\left\{\frac{u_a^0(C_wC_a-1)s+c_v^w(C_au_w^0+u_a^0)\lambda_k}{(C_wC_a-1)s^2+(c_v^w+c_v^a)\lambda_k s-c_v^w c_v^a(\lambda_k)^2} + \frac{Cq_0\vartheta s[(C_ac_\sigma^w-c_\sigma^a)s+c_v^w c_\sigma^a\lambda_k]}{[(c+s)^2+\vartheta^2][(C_wC_a-1)s^2+(c_v^w+c_v^a)\lambda_k s-c_v^w c_v^a(\lambda_k)^2]}\right\}; \tag{15a}$$

$$\overline{T}_w(s) = \mu_k\left\{\frac{u_w^0(C_wC_a-1)s+c_v^a(C_wu_a^0+u_w^0)\lambda_k}{(C_wC_a-1)s^2+(c_v^w+c_v^a)\lambda_k s-c_v^w c_v^a(\lambda_k)^2} + \frac{Cq_0\vartheta s[(C_wc_\sigma^a-c_\sigma^w)s+c_v^a c_\sigma^w\lambda_k]}{[(c+s)^2+\vartheta^2][(C_wC_a-1)s^2+(c_v^w+c_v^a)\lambda_k s-c_v^w c_v^a(\lambda_k)^2]}\right\}. \tag{15b}$$

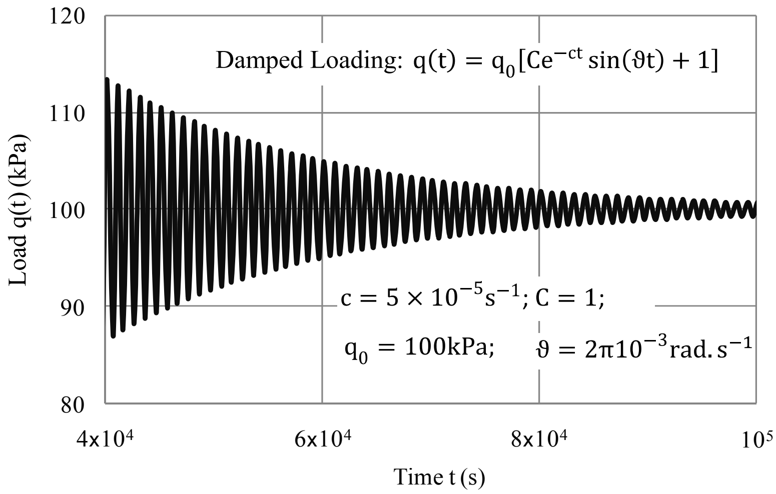

Figure 2. Damped sine wave loading varying with time

Taking the Laplace inverse of (15a) and (15b) to find the generalised Fourier coefficients $T_a(t)$ and $T_w(t)$, and combining these results with (6a) and (6b) will yield in:

$$u_a(z,t) = \sum_{k=0}^{\infty}\mu_k\sin\left(\sqrt{\lambda_k}z\right)\left\{\frac{\omega\left(e^{\alpha_1\lambda_k t}-e^{\alpha_2\lambda_k t}\right)+\psi\left(e^{\alpha_1\lambda_k t}+e^{\alpha_2\lambda_k t}\right)}{2\eta} + \frac{\Upsilon e^{\alpha_1\lambda_k t}\alpha_1\lambda_k(\alpha_1+\beta)}{[(\alpha_1\lambda_k+c)^2+\vartheta^2](\alpha_1-\alpha_2)} + \frac{\Upsilon e^{\alpha_2\lambda_k t}\alpha_2\lambda_k(\alpha_2+\beta)}{[(\alpha_2\lambda_k+c)^2+\vartheta^2](\alpha_2-\alpha_1)} + \frac{\Upsilon e^{-ct}[\varrho\cos(\vartheta t)+\varsigma\sin(\vartheta t)]}{\vartheta[(\alpha_1\lambda_k+c)^2+\vartheta^2][(\alpha_2\lambda_k+c)^2+\vartheta^2]}\right\} \tag{16a}$$

$$u_w(z,t) = \sum_{k=0}^{\infty}\mu_k\sin\left(\sqrt{\lambda_k}z\right)\left\{\frac{\omega'\left(e^{\alpha_1\lambda_k t}-e^{\alpha_2\lambda_k t}\right)+\psi'\left(e^{\alpha_1\lambda_k t}+e^{\alpha_2\lambda_k t}\right)}{2\eta} + \frac{\Upsilon' e^{\alpha_1\lambda_k t}\alpha_1\lambda_k(\alpha_1+\beta')}{[(\alpha_1\lambda_k+c)^2+\vartheta^2](\alpha_1-\alpha_2)} + \frac{\Upsilon' e^{\alpha_2\lambda_k t}\alpha_2\lambda_k(\alpha_2+\beta')}{[(\alpha_2\lambda_k+c)^2+\vartheta^2](\alpha_2-\alpha_1)} + \frac{\Upsilon' e^{-ct}[\varrho'\cos(\vartheta t)+\varsigma'\sin(\vartheta t)]}{\vartheta[(\alpha_1\lambda_k+c)^2+\vartheta^2][(\alpha_2\lambda_k+c)^2+\vartheta^2]}\right\} \tag{16b}$$

where $\eta = [(c_v^w - c_v^a)^2 + 4c_v^w c_v^a C_w C_a]^{\frac{1}{2}}$;

$\omega = (c_v^a - c_v^w)u_a^0 - 2c_v^w C_a u_w^0$; $\psi = \eta u_a^0$;

$\omega' = (c_v^w - c_v^a)u_w^0 - 2c_v^a C_w u_a^0$; $\psi' = \eta u_w^0$;

$\alpha_1 = \frac{1}{2}\left(\frac{c_v^w+c_v^a+\eta}{1-C_wC_a}\right)$; $\alpha_2 = \frac{1}{2}\left(\frac{c_v^w+c_v^a-\eta}{1-C_wC_a}\right)$;

$\beta = \frac{c_\sigma^a c_v^w}{C_a c_\sigma^w - c_\sigma^a}$; $\beta' = \frac{c_\sigma^w c_v^a}{C_w c_\sigma^a - c_\sigma^w}$;

$\Upsilon = \frac{Cq_0\vartheta(C_a c_\sigma^w - c_\sigma^a)}{1-C_aC_w}$; $\Upsilon' = \frac{Cq_0\vartheta(C_w c_\sigma^a - c_\sigma^w)}{1-C_aC_w}$;

$\varrho = \vartheta\lambda_k[\alpha_1\alpha_2\beta(\lambda_k)^2 - 2c\alpha_1\alpha_2\lambda_k - (c^2+\vartheta^2)(\alpha_1+\alpha_2+\beta)]$;

$\varrho' = \vartheta\lambda_k[\alpha_1\alpha_2\beta'(\lambda_k)^2 - 2c\alpha_1\alpha_2\lambda_k - (c^2+\vartheta^2)(\alpha_1+\alpha_2+\beta')]$;

$\varsigma = (c^2+\vartheta^2)^2 + c(\alpha_1+\alpha_2-\beta)(c^2+\vartheta^2)\lambda_k + [(c^2-\vartheta^2)\alpha_1\alpha_2 - \beta(\alpha_1+\alpha_2)(c^2+\vartheta^2)](\lambda_k)^2 - c\alpha_1\alpha_2\beta(\lambda_k)^3$; and

$$\varsigma' = (c^2+\vartheta^2)^2 + c(\alpha_1+\alpha_2-\beta')(c^2+\vartheta^2)\lambda_k + [(c^2-\vartheta^2)\alpha_1\alpha_2 - \beta'(\alpha_1+\alpha_2)(c^2+\vartheta^2)](\lambda_k)^2 - c\alpha_1\alpha_2\beta'(\lambda_k)^3. \quad (17)$$

The dissipation of excess pore-air and pore-water pressures along vertical direction can be predicted based on (16a) and (16b). For fully saturated soils ($S_r = 1$), under the constant loading ($\sigma_{,t} = 0$), the coefficient of permeability and the initial pressure with respect to air phase (k_a and u_a^0, respectively) become zero, resulting in the consolidation parameters C_a and c_v^a equal to zero. In addition, the coefficients of volume change of water m_1^w and m_2^w are equal to the conventional volume change m_v. As the result, (16) will be converted to the traditional Terzaghi's consolidation equation.

2.4 Consolidation settlement

The settlement of the unsaturated soil stratum can be determined based on the constitutive relations that link the stress and deformation state variables as below:

$$S(t) = \left|\int_0^H \varepsilon_v(z,t)dz\right| = \left|m_1^s HCq_0 e^{-ct}\sin(\vartheta t) + (m_2^s - m_1^s)\left[\int_0^H u_a(z,t)dz - Hu_a^0\right] - m_2^s\left[\int_0^H u_w(z,t)dz - Hu_w^0\right]\right| \quad (18)$$

where ε_v is the volumetric strain; $m_1^s = m_1^w + m_1^a$, is the coefficient of volume change of the soil element with respect to the change in the net stress $(\sigma - u_a)$; and $m_2^s = m_2^w + m_2^a$, is the coefficient of volume change of the soil element with respect to the change in the matric suction $(u_a - u_w)$.

3 WORKED EXAMPLE

In this section, the changes in normalised pore pressures u_a/u_{atm} and u_w/u_{atm} as well as the normalised settlement ($S^* = S(t)/(m_1^s q_{max} H)$) are investigated against the air to water permeability ratio k_a/k_w. The following properties and parameters are adopted and assumed to remain constant during the loading process:

Loading parameters:	$c = 5\times10^{-4}s^{-1}$;	$C = 1$;	$\vartheta = 2\pi10^{-3}\ rad.s^{-1}$.	(19)
Material properties:	$n = 0.50$;	$S_r = 80\%$;	$k_w = 10^{-10}ms^{-1}$;	
	$u_a^0 = 20kPa$;	$u_w^0 = 40kPa$;	$q_0 = 100kPa$;	
	$u_{atm} = 100kPa$;	$H = 10m$;		
	$m_1^s = -2.5\times10^{-4}kPa^{-1}$;	$m_2^s = 0.4m_1^s$;		
	$m_1^w = 0.2m_1^s$;	$m_2^w = 4m_1^w$.		(20)
Physical properties:	$R = 8.314J.mol^{-1}K^{-1}$;	$M = 0.029kg.mol^{-1}$;		
	$\Theta = (\theta^\circ + 273.16)K$;	$\theta^\circ = 20°C$.		(21)

The application of an external load on the ground surface leads to an instantaneous undrained compression and induces excess pore pressures. Such induced pressures form the initial conditions for the compression process. According to Fredlund and Hasan (1979), under isotropic conditions, an initial loading $q_0 = 100kPa$ will generate an initial excess pore-air pressure $u_a^0 = 20kPa$ and an initial excess pore-water pressure $u_w^0 = 40kPa$, as shown in (20). The excess pore pressure dissipation and consolidation settlement induced by damped sine wave loading are investigated at the depth $z = H/2$ for the one-way drainage boundary system. As illustrated in Figure 2, the typical damped sine wave loading function consists of oscillations with peak to peak amplitude of about $2q_0$ at the very beginning; then, the amplitude exponentially decreases with time and consequently, the external loading becomes constant $q_0 = 100kPa$. Correspondingly, excess pore-air and pore-water pressure dissipation patterns appear to vibrate due to the loading-unloading process at the early stages, and then stabilise and gradually dissipate after $t = 10^5s$ (Figure 3). Figures 3a depicts a group of parallel curves of excess pore-air pressure for different values of k_a/k_w during the later stages. It is noticed that higher values of k_a/k_w result in a higher dissipation rate of excess pore-air pressure. On the other hand, Figures 3b shows the noticeable variations in excess pore-water pressure dissipation after $t = 10^5s$. The rate of water flowing out of the soil during this time is more rapid due to higher k_a/k_w.

The excess pore-water pressure patterns with different values of k_a/k_w then converge into a single curve and fully dissipate at the same time.

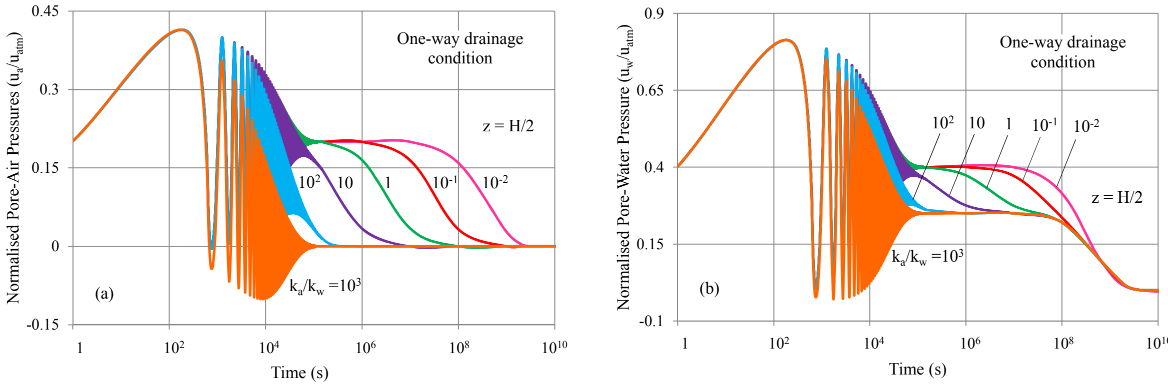

Figure 3. Variations in (a) excess pore-air and (b) excess pore-water pressures with different k_a/k_w due to the damped sine wave loading

Figure 4 shows the normalised settlement of the unsaturated soil stratum due to the damped sine wave loading. The compression patterns initially show oscillations with decreasing amplitude. These oscillations indicate the soil deforms in a way that the air and water are squeezed out during loading and then are absorbed back in the soil during unloading process; hence, the soil volume continuously changes with time at the beginning. As the damped sine wave loading stabilises at $q_0 = 100\text{kPa}$ (the amplitude approaches zero), the soil begins to settle gradually with a relatively slow rate, similar to that induced by the constant loading, as proposed by Ho et al. (2014a). The soil is eventually subjected to no further deformation after $t = 10^{10}\text{s}$. Sufficient details about the consolidation of the unsaturated soil layer under the application of damped sine wave load can be found in Ho et al. (2014b).

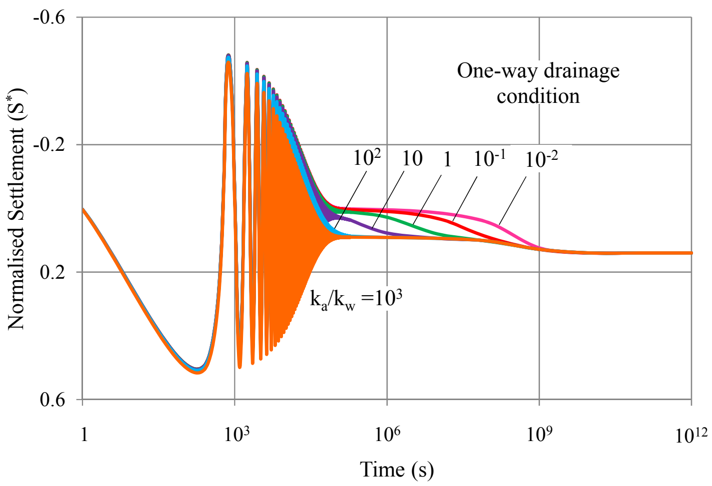

Figure 4. Variations in settlement with different k_a/k_w due to the damped sine wave loading

On the other hand, the studies of cementation effects and elastic viscoplastic model for soft soils have recently been of major interest in geotechnical engineering. These areas have attracted a large number of current studies, conducted by Nguyen et al. (2014), Azari et al. (2014), and Le et al. (2015) to name a few. Having realised great potentials in such areas, further investigations on consolidation in unsaturated soils capturing the mentioned effects will be implemented so as to enhance the reliability for the prediction model.

4 CONCLUSIONS

This paper introduces the application of eigenfunction expansion and Laplace transformation techniques to determine excess pore pressures and 1-D settlement of the unsaturated soil layer subjected to the damped sine wave loading. In this study, the one-way drainage boundary condition and the uniformly distributed initial pore pressures are adopted for the analytical development. The general solution consisting of the eigenfunctions is employed. Once the first order derivatives with respect to time are derived using orthogonality of sine function, exact solutions can be computed using Laplace transform and Laplace inverse methods. When the soil is in saturated state, the proposed final solution can be simplified and transformed into the classical Terzaghi's consolidation equation. This shows that the proposed solution is applicable for different soil states.

The changes in excess pore pressures and consolidation settlement under the damped sine wave loading have been investigated against the air to water permeability ratio. The excess pore-air and pore-water pressure and compression patterns initially oscillate due to the loading-unloading process but then tend to stabilise as the amplitude approaches zero. When the loading becomes stable, it is found that higher k_a/k_w results in more rapid dissipation for the pore-air pressures. Meanwhile, the excess pore-water pressures may vary at first but soon converge into a curve and diminish at the same time.

REFERENCES

Azari, B., Fatahi, B., and Khabbaz, H. (2014). "Assessment of the elastic viscoplastic behaviour of soft soils improved with vertical drains capturing reduced shear strength of a disturbed zone". International Journal of Geomechanics, DOI: 10.1061/(ASCE)GM.1943-5622.0000448.

Barden, L. (1965). "Consolidation of compacted and unsaturated clays". Géotechnique, 15 (3), 267-286.

Blight, G. E. (1961). "Strength and consolidation characteristics of compacted soils". Dissertation, University of London, London.

Conte, E. (2004). "Consolidation analysis for unsaturated soils". Canadian Geotechnical Journal, 41 (4), 599-612.

Fatahi, B., Khabbaz, H., and Indraratna, B. (2009). "Parametric studies on bioengineering effects of tree root-based suction on ground behaviour". Ecological Engineering, 35(10), 1415-1426.

Fatahi, B., Khabbaz, H., and Indraratna, B. (2010). "Bioengineering ground improvement considering root water uptake model". Ecological Engineering, 36(2), 222-229.

Fatahi, B., Khabbaz, H., and Indraratna, B. (2014). "Modelling of unsaturated ground behaviour influenced by vegetation transpiration". Geomechanics and Geoengineering: An International Journal, 9(3), 187-207.

Fredlund, D. G., and Hasan, J. U. (1979). "One-dimensional consolidation theory: unsaturated soils". Canadian Geotechnical Journal, 16 (3), 521-531.

Haberman, R. (2012). "Applied partial differential equations: with Fourier series and boundary value problems". Pearson Higher Education, Massachusetts, 784 pp.

Ho, L.H., Fatahi, B., and Khabbaz, H. (2014a). "Analytical solution for one-dimensional consolidation of unsaturated soils using eigenfunction expansion method". International Journal for Numerical and Analytical Methods in Geomechanics, 38(10), 1058-1077.

Ho, L.H., and Fatahi, B. (2014b). "Exact analytical solution for one-dimensional consolidation of unsaturated soils subjected to time-dependent loading". International Journal of Geomechanics (under review).

Le, T.M., Fatahi, B., and Khabbaz, H. (2015). "Numerical optimisation to obtain elastic viscoplastic model parameters for soft clay". International Journal of Plasticity, 65, 1-21.

Lloret, A., and Alonso, E. E. (1980). "Consolidation of unsaturated soils including swelling and collapse behaviour". Géotechnique, 30 (4), 449-477.

Nguyen, L.D., Fatahi, B., and Khabbaz, H. (2014). "A constitutive model for cemented clays capturing cementation degradation". International Journal of Plasticity, 56, 1-18.

Qin, A. F., Chen, G. J., Tan, Y. W., and Sun, D. A. (2008). "Analytical solution to one-dimensional consolidation in unsaturated soils". Applied Mathematics and Mechanics, 29 (10), 1329-1340.

Qin, A. F., Sun, D. A., and Tan, Y. W. (2010). "Analytical solution to one-dimensional consolidation in unsaturated soils under loading varying exponentially with time". Computers and Geotechnics, 37 (1-2), 233-238.

Scott, R. F. (1963). "Principles of Soil Mechanics". Addison Wesley Publishing Company, Massachusetts, 550 pp.

Shan, Z. D., Ling, D. S., and Ding, H. J. (2012). "Exact solutions for one-dimensional consolidation of single-layer unsaturated soil". International Journal for Numerical and Analytical Methods in Geomechanics, 36 (6), 708-722.

Terzaghi, K. (1943). "Theoretical Soil Mechanics". John Wiley and Sons, New York, 510 pp.

Wong, T. T., Fredlund, D. G., and Krahn, J. (1998). "A numerical study of coupled consolidation in unsaturated soils". Canadian Geotechnical Journal, 35 (6), 926-937.

Zhou, W. H., and Zhao, L. S. (2013). "One-dimensional consolidation of unsaturated soil subjected to time-dependent loading with various initial and boundary conditions". International Journal of Geomechanics, 14 (2), 291-301.

Zhou, W. H., Zhao, L. S., Li, X. B. (2014). "A simple analytical solution to one-dimensional consolidation for unsaturated soils". International Journal for Numerical and Analytical Methods in Geomechanics, 38 (8), 794-810.