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Appropriate probability distribution functions for geotechnical data

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ABSTRACT

A key task in any geotechnical analysis is the selection of the design value. The characteristic design value is a cautious estimate of this parameter and, a normal distribution is typically applied when statistics are used. While the normal distribution is the default probability distribution function (PDF), it has limitations in applying to process geotechnical data. Case studies are used to show the requirement for the appropriate PDF to be applied, and when the normal PDF should not be used to avoid statistical irregularities. One case study is for a road project where the subgrade design CBR index values were required for a pavement design. Another case study is for a bridge project where the rock socket design is based on the rock strength, derived from point load index values. Using goodness of fit tests various PDFs are compared and ranked. Where large Coefficient of Variations (COVs) occur the best fit PDF is consistently not the Normal PDF, and such assumptions of normality should be avoided in the assessment of parameters. The Log Normal PDF generally provides an appropriate PDF, although not necessarily the number 1 ranked. A piling case study shows when low COV occurs then the normal PDF applies.

Keywords: Statistics, Normal Distribution, Characteristic Value, Probability Distribution Function

1 INTRODUCTION

1.1 Characteristic value

A key task in any analysis is the selection of the design value. This selection is often subjective based on experience and personal judgment. The use of statistics provides both transparency and accountability as judgment often varies between practitioners. However, for statistics to be useful, a sufficient number of test results should be obtained and does not completely exclude judgment being applied. Traditional analysis uses the assessment of material parameters to produce a design value which should be "moderately conservative". Depending on the type of project, models adopted, existing knowledge and assessor, this design value can be:

- The lowest test or lowest credible value
- The average (arithmetic typically used – although geometric mean sometimes also used)
- The mode (most frequent value), median or quartile values
- Best estimate or representative value; Nominal values

Look and Campbell (2013) showed that when presented with the same data and 6 choices of design values, then 4 out of 10 geotechnical engineers select one value, while the other 60% selected one of the other listed values. And this disturbingly small agreement ratio was higher than the wider choices of our Civil Engineering colleagues. If the lowest value was selected then, logically one should apply a different partial factor as compared to if a typical or average value was adopted.

Thus the moderately conservative parameter is not a precisely defined value, and needs to be considered in the context of data, reliability and sufficiency as what factors of safety are applied in the analytical process. A high (or low) factor of safety, may result from the choice of parameters rather than providing clarity on a "safe" vs. "unsafe" product.

Assuming sufficient tests are available for a defined “homogenous” layer then a statistical approach may be used. This approach also assumes other ground assessment factors (such as time dependent properties, ground water, stress dependency or strain softening) have been given due consideration.

With the introduction of limit state codes, Characteristic Values were introduced, where the design value is derived from multiplying the characteristic value by a material factor. Characteristic values are sometimes quoted as 95% confidence level i.e. only 5% of values less than this value, and is also referred to this fractile for the characteristic value. A normal distribution is typically applied to derive this design value. Typically a lower characteristic value (LCV) is adopted, however in some cases such as in avoidance of over compaction in expansive clays an Upper Characteristic Value (UCV) may be used (Look et al, 1994).

Eurocode 7: Geotechnical Design (2004) states that “if statistical methods are used, the characteristic value should be derived such that the calculated probability of a worst value governing the occurrence of the limit state under consideration is not greater than 5%”. Frank et. al. (2004), illustrates statistical schemes by applying the student t-distribution, which accounts for the number of samples. Bond (2010), and Orr and Breyesse (2008) provide further discussion on the code applications and reliability in design. Austroads (1992) defined the characteristic value as the value with a probability less than 5% of a worse case occurring, while AS 5100 (2004) and AS 2159 (2009) with its predecessors use a “conservative” value and the latter reserves the term “characteristic value” to when referring to the pile elements.

Geotechnical material strength parameters typically have a large coefficient of variation (COV) as compared with man-made materials such as steel or concrete. A COV of 20% for concrete is considered poor (Phoon, 2008) while in soil and rock materials such a result would be considered unusual and values of 40% typical. COVs of 90% are common for strength index tests such as CBR (Look, 2009) or Point Load index tests (Look and Wijeyakulasuriya, V., 2009). These 2 projects are used to illustrate an assessment of subgrade strength for pavement design, and an assessment of rock strength for a bridge rock socket design. The assumed distribution can affect the results considerably. For example the probability of failure of a slope can vary by a factor of 10 if a normally distributed or log normal distribution is used. Goodness of fit tests are required to determine the appropriate probability distribution function (PDF).

1.2 Distribution functions

The normal distribution is the taught fundamental distribution, in basic statistics in engineering courses, as it is the simplest distribution to understand, but is not always directly relevant to soils and rocks. The Probability distribution function (PDF) is the relative likelihood a random variable will assume a particular value. The area under a PDF is unity. Cumulative distribution function (CDF) is the probability a value will have a value less than or equal to a particular value. CDF is the integral of the corresponding PDF and an alternative way of presenting the PDF. The most known distribution function is the Normal Distribution (the bell curve). This uses a mean and standard deviation as its arguments. The mean describes the value around which the bell curve is centred. The standard deviation describes the spread of the results. The coefficient of variation (COV) is the ratio of the standard deviation and mean value.

The Normal distribution generally adopted for material modelling suffers from allowing negative values and the resulting error is exacerbated with higher coefficient of variation (COV) values (Fenton & Griffiths, 2008). Thus the appropriateness of alternative distribution models needs to be assessed and goodness-of-fit tests are commonly used for this purpose to discern differences between a hypothesized and the observed distribution. Three widely used tests are the Chi-Square, Kolmogorov-Smirnov (K-S) and the Anderson-Darling (A-D); the A-D test is very similar to the K-S test, but unlike both former tests, places more emphasis on tail values owing to the logarithmic nature of its test statistic and has better discerning ability.

There are over 35 distribution functions with the “best fit” distribution function assessed through goodness-of- fit test to the data. PDFs such as Pearson VI, Lognormal, Gamma, Weibull, or Beta avoid the negative values that sometimes occur when a normal distribution is applied, such as strength indices (Point Load index and CBR) where a large range of results occur in the test data even for a “homogeneous” material. An Exponential PDF is a negative (decreasing) distribution, and applies mainly for the time needed to wait before an event occurs such as earthquakes or useful for a constant probability per unit distance such as lengths of joints.

2 CASE STUDIES

2.1 General description

Case studies are used to illustrate the issues associated with using a PDF with the point load index [$I_s(50)$] for various sedimentary rock types in south east Queensland. A consistent picture begins to emerge on the use of various PDFs for these rock types. The best fit distribution from goodness of fit tests, normal and log normal distribution functions are compared.

Road construction often has a significant number of California Bearing Ratio (CBR) tests to assess the subgrade for pavement design. If the pavement is at grade and of the same geological material over the alignment then statistics should be used to assess the subgrade. A case study is discussed for CBR tests over a 13.2 km alignment where 96 tests results were obtained in the preliminary design phase (Look, 2009). The normal and log normal distribution functions are also compared.

While case studies data from only 5 bridge sites, 24 deep rock socketed piles at one site, 4 segments of a roadway and 1 major pile test site are described in this paper, many more projects were used in obtaining the conclusions on the applicability of appropriate PDFs.

2.2 Pacific Motorway bridge Sites

Table 1 provides the summary statistics for 5 sites for bridge piled foundation on Argillite / Greywacke meta-sedimentary rocks in south east Queensland, Australia, and obtained during the widening of the Pacific Motorway. A normal distribution was adopted for simplicity. Design values were quoted in the geotechnical reports in terms of weathering of the rock. This shows that 2 of 5 sites have negative values at the 5% LCV. This was the first time this author questioned the use of a normal PDF in design.

The COVs shown in Table 1 are higher than the COV of 20% for "poor" man-made materials. These large COVs account for the negative values calculated for the 5% fractile if the normal PDF is used.

Table 1: I_s (50) Values for Bridge sites on Meta-sedimentary rocks (Look, 2001).

Bridge Site	Highly / Moderately Weathered Rock		Slightly Weathered Rock	
	Mean (MPa) / COV (%)	LCV (MPa)	Mean (MPa) / COV (%)	LCV (MPa)
A	2.03 / 121%	0.23	2.05 / 79%	1.48
B	0.57 / 123%	(-)	1.94 / 54%	1.42
C	0.98 / 30%	0.49	Insufficient data	
D	2.20 / 58%	1.50	2.60 / 36%	1.96
E	1.50 / 135%	(-)	2.57 / 71%	(-)

2.3 Gateway Bridge

2.3.1 Project description

The Gateway Upgrade Project (GUP) was the largest road and bridge infrastructure project ever undertaken in Queensland, Australia. The six lane bridge structure spans 1.6 km between abutments, with a central river span ~260 m. The river span of the bridge is founded upon 1.5 metre diameter bored piers socketed into sedimentary rock. River piers consisted of 24 piles that extended to a depth of over 50m below the river level. Pier 6 (only) located in the river is discussed further.

A statistical analysis of intact rock strength properties of the sub-horizontally interbedded sandstone stratum underlying one of the main river span (Pier 6) of the bridge was carried out. Boreholes at each of the 24 socket location at this one pier showed that signification variation of ground conditions could occur locally between piles, even for the same pier location and for piles approximately 3.5m apart.

2.3.2 Point load index values

Point Load strength Index results (330 No) in a distinctly weathered (highly to moderately) interbedded sandstone-siltstone were carried out. The UCS / $I_s(50)$ ratio for both the axial and diametral direction was established and highlighted the need to account for strength anisotropy for socket design in view of the radial normal stresses on the socket wall (Look and Wijeyakulasuriya, 2009).

For that bridge pier the COV was 91% for all $I_s(50)$ test results, but varied from approximately 31% to 156% for the individual pile location. The statistical analysis of Pier 6 construction phase $I_s(50)$ (diametral) data was carried out using @Risk software. This identified the best fitting distribution in each case from a set of 35 PDFs. However, the best fit distributions identified depend on the significance level assumed and the type of goodness of fitness test used, and hence may not be unique. However, the overall position varies by only a few places. Table 2 shows the best fit distributions and the ranking of Normal and the log-Normal at each of the piles as well as for the combined diametral data (330 Nos.) at Pier 6. The Anderson – Darling goodness of fit statistical test was applied to rank the various PDFs.

Table 2: Best Fit Distribution models at the 24 pile locations at Pier 6.

Pier 6	A – D Test (5% significance level)		
Pile #	Rank 1	Normal – Rank	Log Normal - Rank
P6-1	No Data	--	--
P6-2	No Data	--	--
P6-3	Log Logistic	8	3
P6-4	Normal	1	4
P6-5	Expon	6	13 (n/a)
P6-6	Log Logistic	7	3
P6-7	Weibull	7	5
P6-8	InvGauss	8	7
P6-9	Log Normal	8	1
P6-10	Log Normal	9	1
P6-11	Log Logistic	9	4
P6-12	Log Logistic	10	3
P6-13	Inv Gauss	8	7
P6-14	Logistic	6	7
P6-15	Expon	7	2
P6-16	Log Logistic	7	10
P6-17	Normal	1	4
P6-18	Pearson V	8	13
P6-19	Pearson V	8	3
P6-20	Pearson V	8	3
P6-21	Inv Gauss	5	3
P6-22	LogLogistic	8	4
P6-23	Normal	1	5
P6-24	LogLogistic	8	4
P6-ALL	LogLogistic	9	3

The loglogistic distribution is overall the best, and the Pearson V ranks second for this best fit statistical model at this site. The lognormal ranks 3rd overall as compared to the 9th ranked Normal PDF. Given the greater familiarity of the logNormal over the loglogistic among engineers, and the difficulties associated with the Normal, the logNormal distribution is proposed for use despite it not being necessarily the best ranking candidate. The Normal PDF did rank 1st in 3 cases above, but overall represents a low ranked PDF. The COVs in those 3 cases varied from low values of 40% to 55% as compared to the average COV of 91%.

Figure 1 illustrates the best fit distribution (a LogLogistic in this example) as compared with the lognormal and normal PDFs. Even at the 10% fractile (the statistical term for the LCV), the normal PDF has a very low value and the lognormal would produce a value 7 times higher than the normal distribution. Figure 1 also shows at the 25% fractile the 3 PDFs shown would provide approximately the same value of 0.4 MPa. This highlights the requirement to select the correct PDF in any statistical model especially at the low fractiles. The 5% fractiles associated with the PDFs are:

- Loglogistic (Best fit) – $I_s(50)$ = 0.17 MPa
- Lognormal – $I_s(50)$ = 0.16 MPa
- Normal – $I_s(50)$ = (- 0.18) MPa

Obviously a negative value makes no sense, and many engineers then become sceptical on the use of statistics, when the issue is really the application of the appropriate PDF. Table 3 shows the statistical properties of a few of these piles and the combined results of all pile data. Even for individual piles large COVs, and negative or unrealistically low characteristic values are evident.

Table 3: Results of distribution models at 10% for $I_s(50)$ values at bridge pier 6.

Pier 6	Diametral $I_s(50)$ statistics			10% Characteristic (MPa)	
Pile #	Mean (MPa)	COV	No. of points	Normal	Log Normal
P6-5	0.82	39 %	10	0.46	n/a
P6-6	1.01	151%	10	0.26	0.43
P6-7	0.57	56%	15	0.15	0.19
P6-8	0.74	68%	15	0.12	0.30
P6-21	0.94	37%	16	0.48	0.51
P6-22	0.81	113%	17	(-0.13)	0.20
P6-23	0.81	40%	13	0.40	0.45
P6-24	0.61	87%	18	(-0.12)	0.12
P6-ALL	0.82	91%	330	0.03	0.24

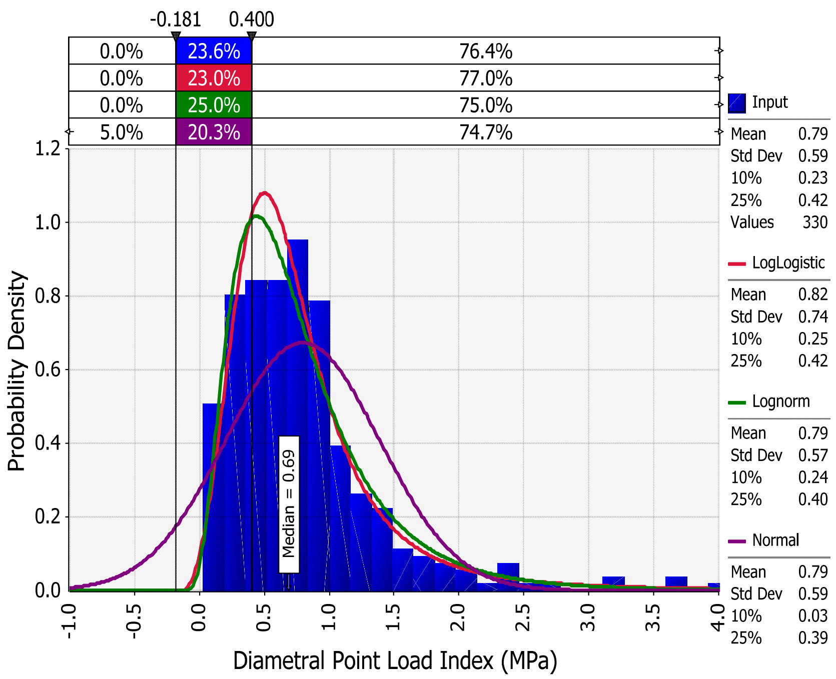

Figure 1. Comparison of distributions for the diametral Is (50) at pier 6

Table 1 also showed the 5% fractile is consistently negative or unrealistically low for the 5 separate bridge sites. This illustrates the 5% to 10% values can be misleading when a normal distribution is applied but approximately comparable at the 20% to 30% value. The consequences of applying various fractiles to commonly used rock socket design equations are discussed in Look and Lacey (2013).

3 CBR VALUES FOR A ROAD PROJECT

This case study is for the subgrade assessment of a transport alignment using the results of soaked CBR testing in the preliminary design phase. The lower characteristic value (LCV) is compared for various reliability levels and distribution functions. The CBR laboratory model needs to be appropriate to the field CBR – this may involve soaked vs. unsoaked CBR, equilibrium CBR and levels of compaction achievable. The terms percentage defective used in roadways is synonymous to the % fractile or LCV. Variation would also occur both in testing at a specified location and spatially for a given project. Reliability is often discussed in codes and design procedures. For roads, reliability design typically allows defects as follows:

- 90% reliability for major roads: i.e. 10% of test values less than the specified value.
- 70% reliability for minor roads i.e. 30% of results defective

A lumped approach using all test results combined show a COV of 90% over the route. This is very high but even when assessed as design segments a COV of 50% was the lowest calculated value (Figures 2 and 3, Table 4). A CBR of -5.4% and -1.7% applies at the 5% and 10% fractile, respectively. A lognormal or best fit Pearson V PDF, results in a CBR of 3% at the 10% fractile. The PDF of Figure 2 is transformed to the CDF of Figures 3 to show the cumulative distribution and its variation from better fitting statistical distribution models.

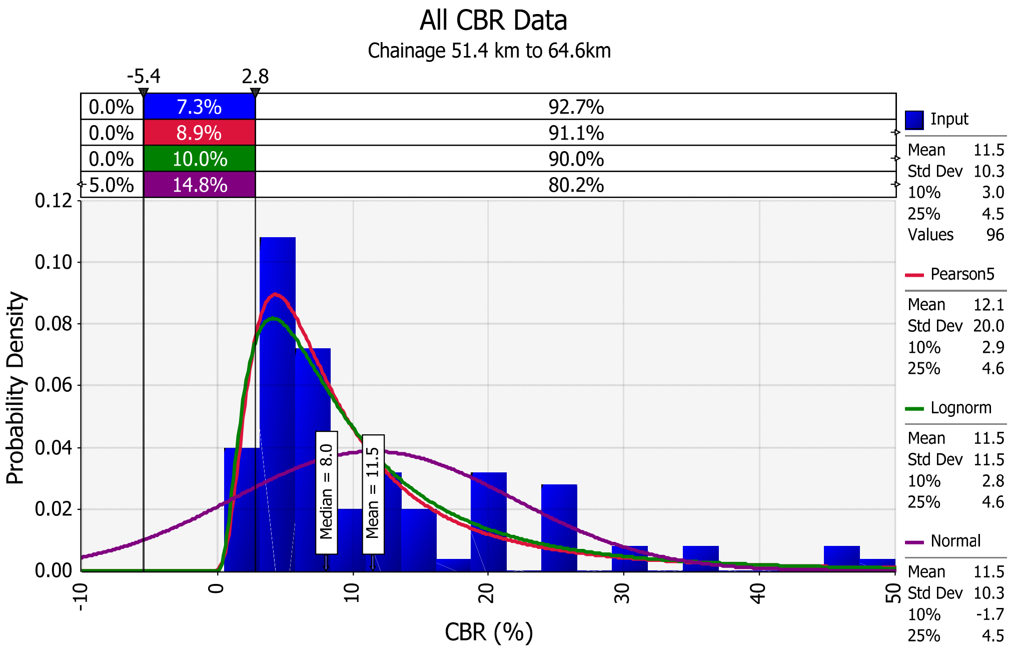

Figure 2. Statistical distribution models for the full 13km length of the site

Table 4 Results of distribution models at 10% and 25% risk for the various CBR zones.

Chainage	All (13km)	3.5km	1.5km	3.5km	4.5km
No. Of values	96	31	6	38	21
COV	90%	72%	50%	101%	71%
LCV and PDF	Soaked CBR (%) values				
10% LCV					
Best fit	2.9	3.0	15.0	2.3	4.0
Log – normal	2.8	3.0	14.8	2.2	4.0
Normal	(-1.7)	0.7	10.6	(-2.8)	1.3

Chainage	All (13km)	3.5km	1.5km	3.5km	4.5km
25% LCV					
Best Fit	4.6	4.2	18.8	3.8	6.5
Log – Normal	4.6	4.2	18.7	3.7	6.5
Normal	4.5	4.4	19.6	3.2	7.1

At the 5 % fractile the CBR value is unrealistically negative if the normal distribution is applied for all data or for design segments, and even in most 10 % fractile. The best fit and log normal distribution provides comparable values. At the 25 percentile defects, the best fit, the log normal and normal are comparable for the LCV in all cases. This observation is comparable to the point load index best fit PDFs previously discussed.

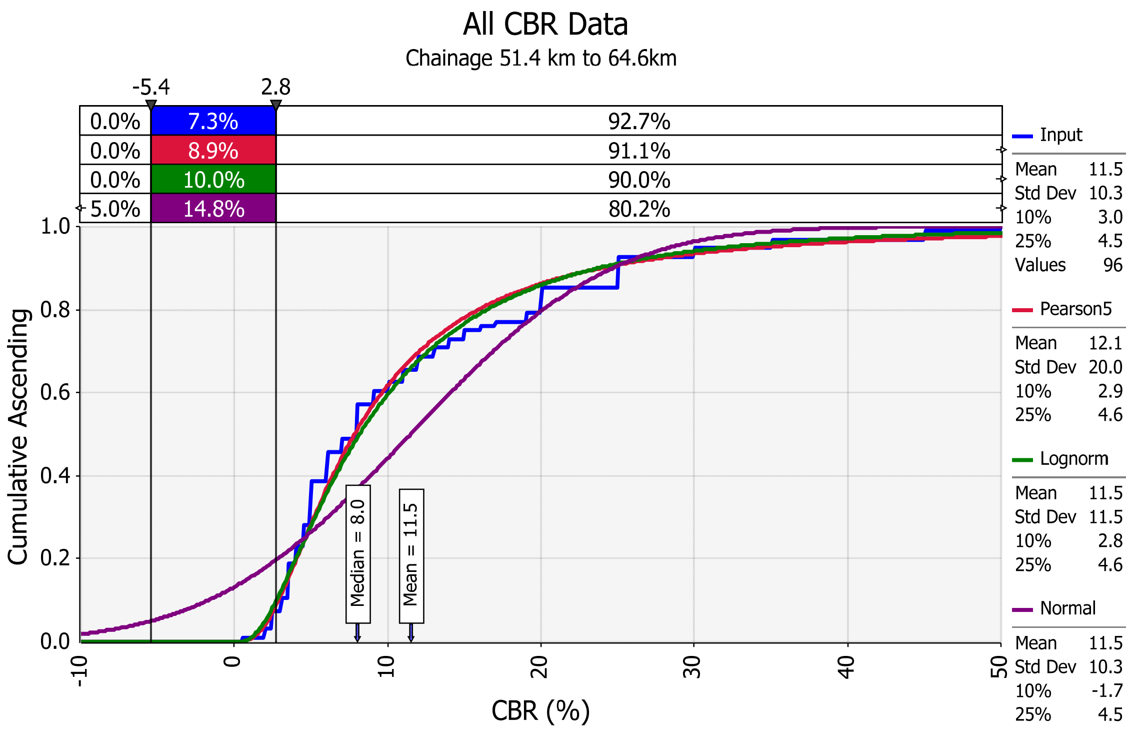

Figure 3. Cumulative distributions for chainage 51.4 to 64.6 km (full length).

4 PILE DATA FROM CENTRAL QUEENSLAND

Figures 4 shows the total compression capacity from pile tests with CAPWAP (Case Pile Wave Analysis Program) used. Dynamic pile load testing using measured field parameters in a wave equation analysis is considered one of the more reliable methods of assessing pile capacities. A COV of 16% represents a smaller spread of results as compared to the previous case studies previously discussed, with COVs greater than 50% producing statistical anomalies, when the normal PDF is used. The normal and best fit PDF (Logistic in this case) shows a reasonable agreement even at the 5% fractile.

This case study shows that for low COVs the normal PDF can then be applied. This also supports the use of higher reduction factors (0.8 for dynamic load testing) compared to lower values (0.7 to 0.4 based on redundancy and risk rating) used during the design phase (AS2159, 2009). Lab and field test values used in the design are more variable, as shown in the CBR and $I_s(50)$ case studies discussed previously.

5 CONCLUSION

The Point load and California Bearing Ratio are indices of strength that are not well modelled by the

Normal distribution. The normal and log normal distribution functions were compared to illustrate the pitfalls in using the normal distribution where large COVs occur. Assuming a normal distribution PDF produces dubious results for the 10% fractile or below.

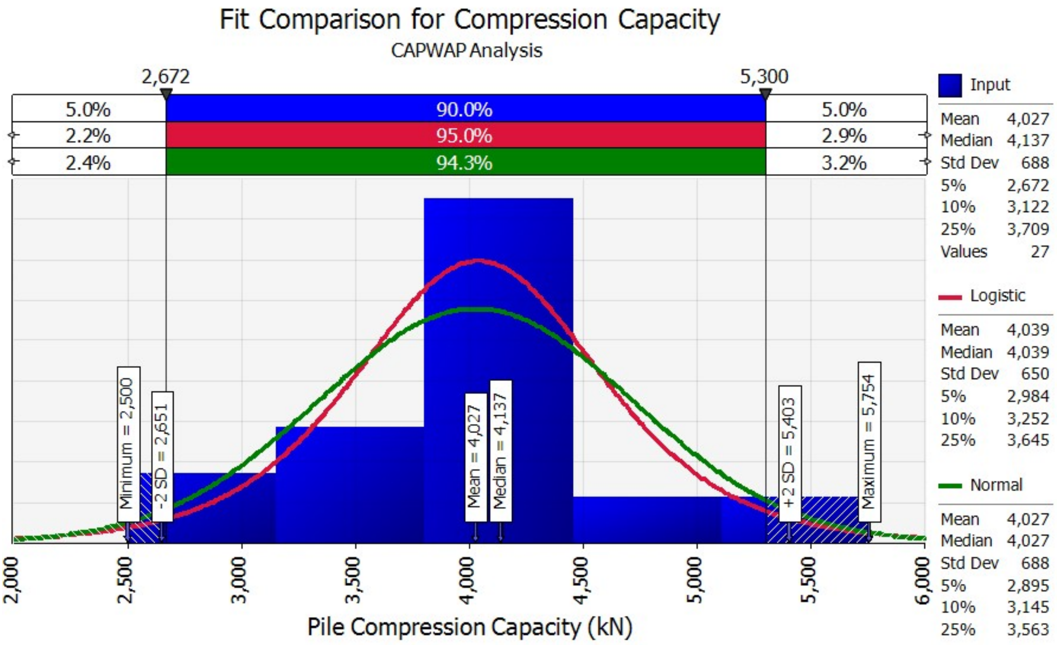

Figure 4. Total Pile Compression Capacity

Characterisation of the strength properties, through goodness-of-fit tests, showed the use of log-normal distributions produced realistic characteristic strengths at the low fractiles, while comparable predictions based on a Normal distribution showed unrealistically low and negative values. The lognormal is not the best fitting distribution, but is consistently close to the best fit PDF. Some limit state code stipulate characteristic design strengths at low percentile values. Yet the normal distribution model is not appropriate at the 10% fractile or below. The normal distribution was comparable with the lognormal and best fit PDF at the 20% to 30% fractile. At low COVs then the normal PDF applies.

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