# Strength and Deformation Behaviour of Sand under General Stress System

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SUMMARY Drained shear tests have been conducted on fully saturated Ottawa sand specimens under axisymmetric compression and extension and general stress system. All the tests have been conducted in Universal Triaxial Apparatus. The specimens have been prepared under different inclinations to study the effect of structural anisotropy. The various factors which affect the strength and deformation properties under general stress system are discussed. The effect of intermediate principal stress on peak strength, linear and volumetric strains in discussed.

## 1 TESTING OF SOIL UNDER GENERAL STRESS SYSTEM

During the past four decades several apparatuses have been developed for subjecting a cuboidal specimens to general stress system. When the pressure was applied through rigid platens on all the six faces, the edge interference at the junctions posed problems. To avoid this, when undersized plates were used the soil squeezed into the gaps. In case of an apparatus using flexible bags on all six faces, edge interference again posed problems and the magnitude of pressure that could be applied was limited. Compared to the devices employing exclusively either rigid platens or flexible bags, a combination of these two poses lesser mechanical problems. The Universal Triaxial Apparatus developed by Ramamurthy (1970) and Rawat and Ramamurthy (1978) belongs to this category of mixed boundary conditions. In this the lateral pressures are applied through flexible bags and the vertical pressure through enlarged and lubricated rigid platens. The problem confronting the flexible bags of the possibility of applying a limited stress level has largely been overcome by confining the rubber bags in metallic guides. The edge interference has been overcome by providing side vanes to the metallic guides and reinforcing the vertical edges or bags applying higher lateral pressure with prismatic sponge pieces.

## 2 STRENGTH OF SOIL UNDER GENERAL STRESS SYSTEM

In theory of plasticity the effect of intermediate principal stress is accounted for by Lode's stress parameter $\mu$ given by

$$\mu = \frac{(\sigma_2' - \sigma_1') + (\sigma_2' - \sigma_3')}{(\sigma_1' - \sigma_3')} \qquad (1)$$

Where $\sigma_1'$, $\sigma_2'$ and $\sigma_3'$ are the major, intermediate and minor principal stress respectively. Habib (1953) introduced a parameter b, which is the ratio of the deviator stresses, that is

$$b = \frac{\sigma_2' - \sigma_3'}{\sigma_1' - \sigma_3'} \qquad (2)$$

It may be observed that $\mu = (2b-1)$. In axisymmetric compression test, $(\sigma_2' = \sigma_3')$, $\mu = -1$, $b = 0$. In axisymmetric extension test $(\sigma_1' = \sigma_2')$, $\mu = 1$, $b = 1$. These being the two extreme cases betwen which the intermediate principal stress can vary, the range of $\mu$ is from −1 to 1 and that of b is from zero to one.

Most of the investigators agree that peak strength of a soil increases with increase in the value of b upto a particular point (possibly upto plane strain compression). These results mostly belong to compression zone. But most of the controversy centres around the peak strength in extension tests. Reported results show that the values of peak strength beyond plane strain compression may increase, remain constant or decrease. Attempts have been made to attribute these divergencies to equipment defects disregarding many other factors which effect the results. These are briefly discussed here.

### 2.1 Material Properties

The response of a soil to various stress paths depends upon the properties of the material. Pearce (1970) reported that $\sigma_2'$ has no influence on the strength of clay he has tested. Shankariah (1977), investigated the strength and deformation behaviour of two cohesionless soils, namely, (i) Ottawa sand which is uniform material with hard, well rounded, nearly spherical and smooth textured grains and (ii) crushed stone consisting of hard, disc shaped subangular and rough textured particles. The $\phi'$ Vs b(or $\mu$) curves were found to be strickingly different in two materials. Figure-1 is reproduced from Lomise et al (1969). These results are obtained from tests on various soils tested under similar conditions in a single apparatus developed by them. It may be observed that these results incorporate many type of divergencies. Since many of the factors affecting the results are invariant in these tests, a large part of the divergence cannot but be attributed to the differences in the properties of the materials.

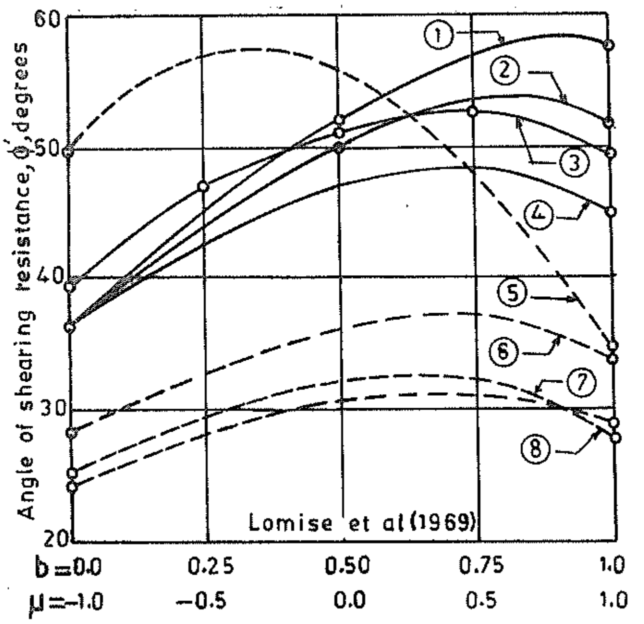

Figure 1 Effect of Material Properties on Peak Strength

2.2 Shape of the Specimen

Reades (1972) and Rawat (1976) among others reported practically identical results in cylindrical and cubical samples under axisymmetric compression. But higher peak strengths in cuboidal specimen in axisymmetric extension have been reported by Green (1971) and Reades (1972). It was suspected that this higher strength in cuboidal specimens could be due to corner effects. But when the strains were monitored accurately using X-ray method, no nonuniform strain conditions due to corner effects were noticed in a cuboidal specimen in extension. Therefore higher strengths in cuboidal specimens cannot be attributed definitely to corner effects. Thus, it cannot, at present, be said how the shape of the specimen affects the results.

2.3 Consolidation Pressure

Results of Marachi et al (1969) indicate that the increase in the value of peak strength from axisymmetric compression to plane strain compression decreases with increase in cell pressure ($\sigma_3'$) at which the samples have been tested. They reported that the difference in the peak value of $\phi'$ between axisymmetric compression and plane strain compression was 7° in dense specimens at $\sigma_3' = 68.95$ KN/m$^2$, 4.5° at $\sigma_3' = 551.6$ KN/m$^2$ and zero at $\sigma_3' = 3447.5$ KN/m$^2$. This reduced difference in $\phi'$ could be due to restricted movement of particles at high cell pressures and/or due to grain crushing.

2.4 Stresspath Followed During Shearing

A given set of stress conditions at failure can be reached by following different stresspaths. The value of b (or $\mu$) at failure depends on the values of three principal stresses, which may be attained by proceeding in several ways. The stresspath followed during shearing may effect the results.

2.5 Mean Normal Stress at Failure

Generally it is a normal practice to choose the consolidation pressures in extension tests such that the value of either $\sigma_{1f}'$ or $\sigma_{3f}'$ are comparable to those in compression. In either case the mean normal stress at failure will be higher in extension tests than those in compression, since in axisymmetric compression $\sigma_1' >, \sigma_2', = \sigma_3'$, and in axisymmetric extension $\sigma_1 = \sigma_2 > \sigma_3$. It is fairly well established fact that principal stress ratio $\sigma_1' / \sigma_3'$ of a given material decreases with increase in mean normal stress at failure ($\sigma_{mf}'$).

2.6 Porosity - Basis of Compression

Whatever may be the basis on which the consolidation pressures are chosen (keeping constant either $\sigma_{1f}'$ or $\sigma_{3f}'$ or $\sigma_{mf}'$) they are normally higher in extension tests. Therefore for a given initial porosity, the post consolidation porosity is lower in extension tests than that in compression tests. The results of extension and compression tests are compared based on initial porosities by some investigators and post consolidation porosities by others. This factor should be kept in view while comparing the results of different investigators.

2.7 Equipment Defects

There is no ideal equipment which can subject specimens of all types of soils, to all types of stress paths and stress levels. Equipment defects include, among other things, end restraints, undersized or oversized platens or bags, corner interferences, limited stress levels and so on. Hither to there has been a tendency to attribute most of the divergencies in the results exclusively to equipment defects. Though this may be unfair, many times the equipment defects contribute a major part in the divergencies of test results.

2.8 Type of Deposition and Compaction

The structural anisotropy of a specimen depends upon the method of deposition and subsequent compaction. The structural anisotropy of the specimen may lead to nonuniform deformation and directional variation of strength. However, not much is known about the directional variation of strength due to structural anistoropy of the material under various stress paths. The present investigation is an attempt in this direction.

3 EXPERIMENTAL WORK

The tests were conducted on standard Ottawa sand (18-52 B.S. sieves) consisting of hard, well rounded, nearly spherical and smooth textured particles. Fully drained tests were conducted on freshly boiled, saturated cubical specimens of nominal size 76 mm specimens were prepared at three densities by vibrating in a sample former, the relative density varying from 50 to 100 percent. To study the directional variation of strength, samples were prepared at eight different angles of deposition $\theta$ namely 0, 15, 30, 45, 60, 70, 80 and 90 degrees, where $\theta$ is the angle made by the direction of deposition with the vertical axis of the specimen (Figure-2). The method of specimen preparation is explained in detail elsewhere (Shankariah, 1977).

All the specimens were consolidated isotropically. The consolidation pressures in other stress paths were so chosen that, for a given porosity, the mean normal stress at failure would be comparable to that in axisymmetric compression. This procedure was adopted to eliminate the possible effect of variation in mean normal stress at failure on the values of peak strengths obtained in different stress paths.

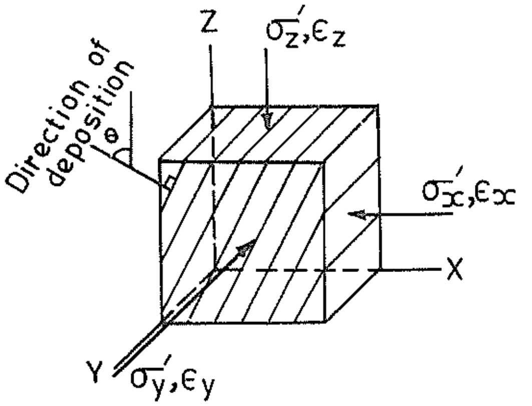

Figure 2 Reference Directions

The samples were sheared following five different stress paths

i) Axisymmetric compression ($\sigma_z' > \sigma_x' = \sigma_y'$); the lateral pressures were kept constant and the axial stress was increased during shearing.

ii) General compression ($\sigma_z' > \sigma_x' > \sigma_y'$): $\sigma_y'$ was kept constant during shearing and both $\sigma_x'$ and $\sigma_z'$ was increased to failure.

iii) Plane strain compression ($\sigma_z' > \sigma_x' > \sigma_y'$ and $\epsilon_x \approx 0$): Samples were sheared by keeping $\sigma_y'$, constant and increasing both $\sigma_x'$ and $\sigma_z'$. $\sigma_x'$ was increased continuously at such a uniform rate so as to prevent lateral deformation on those faces ($\epsilon_x \approx 0$).

iv) General Extension ($\sigma_x' > \sigma_y' > \sigma_z'$): During shearing $\sigma_y'$ was kept constant, $\sigma_x'$ was increased and $\sigma_z'$ was reduced. Tests were repeated with three different values of $\sigma_x'$ at failure so that each test will provide a different point on $\phi'$ vs b (or $\mu$) diagram.

v) Axisymmetric extension ($\sigma_x' = \sigma_y' > \sigma_z'$) Samples were sheared by keeping the lateral pressures constant and reducing the vertical stress.

## 4 PRESENTATION AND DISCUSSION OF RESULTS

### 4.1 The Angle of Shearing Resistance

The angle of shearing resistance is found to be constant in the regions $0 < \theta < 30°$ and $60° < \theta < 90°$, there being a discontinuity in the region $30° < \theta < 60°$. This phenomenon was observed at all the three porosities of the specimens. Therefore the results of tests with $\theta = 0°$ and $60°$ which represent two distinct groups are presented here.

Figure-3 shows the variation of maximum angle of shearing resistance with b (or $\mu$) It may be observed that, with increase in the value of b, the peak strength increases up to a value of b equal to about 0.3, which, corresponds to plane strain condition and then decreases continuously upto

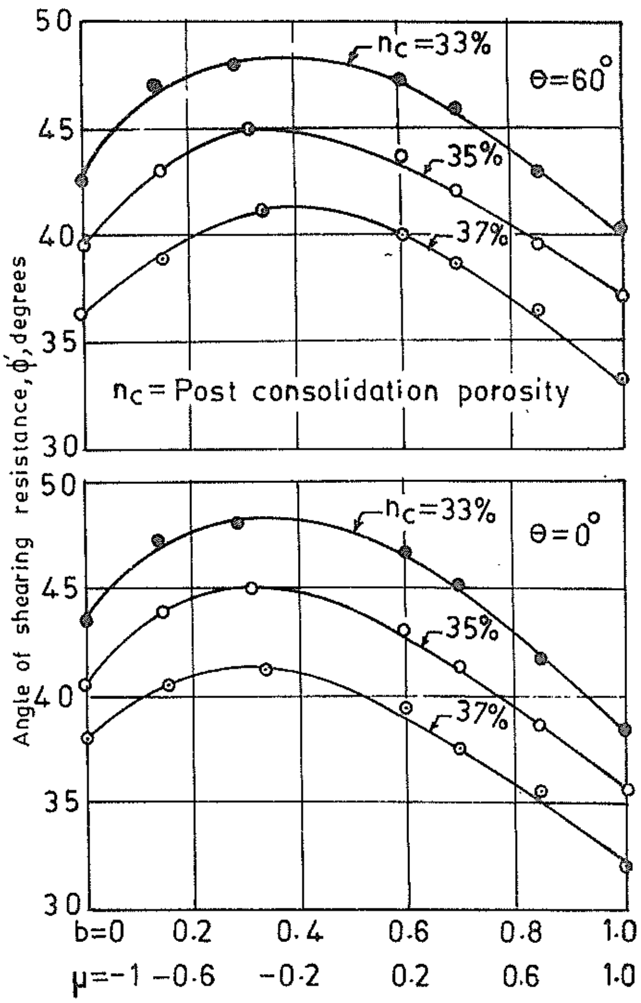

Figure 3 The Angle of Shearing Resistance

b=1 (axisymmetric extension). The curves pertaining to three porosities are nearly parallel.

When $\theta = 0^\circ$, the increase in the value of $\phi'$ from axisymmetric compression to plane strain compression is 4.5° in dense and medium dense state and 3.9° in loose state. In axisymmetric extension the values of $\phi'$ are lower by about 5° than those in axisymmetric compression.

When $\theta = 60^\circ$ the values of $\phi'$ in plane strain compression are higher by about 5.5° than those in axisymmetric compression at all the three porosities. The values of $\phi'$ in axisymmetric extension are lower by 2° to 3° than those in axisymmetric compression.

In axisymmetric compression the angle of shearing resistance is higher when $\theta = 0^\circ$ than when $\theta = 60^\circ$ by about 1.1° to 1.7°. This difference decreases with increase in the value of b and vanishes at plane strain condition. In extension the value of $\phi'$ is lower when $\theta = 0^\circ$ than when $\theta = 60^\circ$ by 1° to 2°. This difference which is zero at plane strain condition, increases with increase in the value of b beyond plane strain condition and attains its peak value under axisymmetric extension.

When $\theta = 0^\circ$ the direction of deposition coincides with the vertical axis of the specimen. The plane of preferred orientation of particles, which is the plane normal to the direction of deposition, is a major principal plane in compression tests where as it is a minor principal plane in extension tests. It may be observed from the above results that when the plane of preferred orientation is normal to the major principal stress (in compression) the material offers maximum resistance, where as it develops lesser resistance when the plane of preferred orientation is normal to the minor principal stress (in extension).

When the value of $\theta$ increases from 60° to 90°, the direction of deposition approaches to the direction of the normal to the vertical axis of the specimen. Under these conditions the plane of preferred orientation is nearly normal to the minor principal stress in compression while it is nearly normal to the major principal stress in extension. Therefore the values of $\phi'$ are lower in compression and higher in extension than the corresponding values when $\theta = 0^\circ$.

It may also be concluded that material enjoys maximum freedom of movement under axisymmetric conditions and hence develops maximum strength anisotropy. As the deviation from axisymmetric conditions increases the freedom of the particles is increasingly restricted and hence the strength anisotropy is restricted more and more, culiminating finally in plane strain condition wherein the particles have least freedom for movement and strength anisotropy is almost completely suppressed. However, because of the restricted movement, the particles develop higher resistance and hence shows maximum strength under plane strain conditions.

The angle of deposition, $\theta$ did not show appreciable influence on the pattern of variation of linear and volumetric strains with b. Therefore the results of only one deposition angle $\theta = 0^\circ$ are presented below.

## 4.2 Lateral Strain $\epsilon_{xf}$

The lateral strain at failure $\epsilon_{xf}$ is nearly zero in plane strain compression. As $\mu$ increases numerically the value of $\epsilon_{xf}$ increases in compression zone, reaching its maximum value in axisymmetric compression (Fig.4). As we move from plane strain compression towards axisymmetric compression, the value of $\sigma_x'$ decreases relatively and hence the lateral strain $\epsilon_{xf}$ increases with numerical increase in the value of $\mu$. In axisymmetric compression $\epsilon_{xf}$ decreases with increase in porosity. In plane strain compression the value of b decreases with decrease in porosity. Therefore the curves in compression cross in the zone of general compression before reaching plane strain condition.

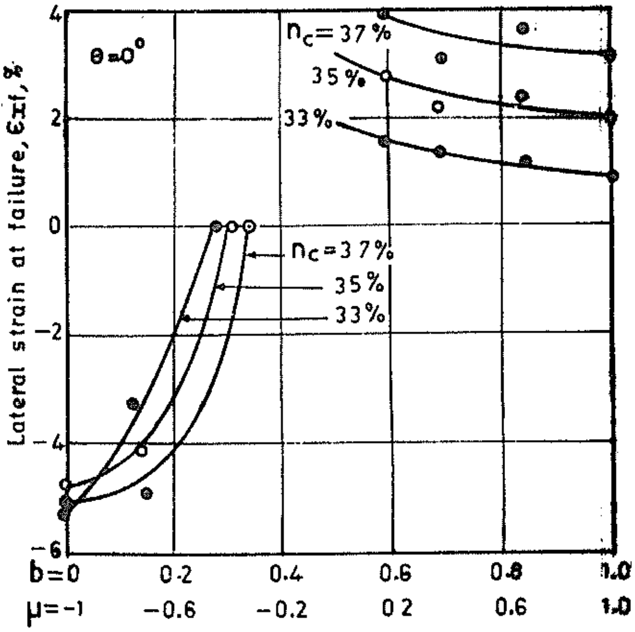

Figure 4 Lateral Strain $\epsilon_{xf}$

In extension, the value of $\epsilon_{xf}$ decreases with increase in $\mu$, the value being least in axisymmetric extension. With increase in the value of $\mu$, $\sigma_x'$ decreases relatively and $\epsilon_{xf}$ (which is compressive) also decreases.

## 4.3 Lateral Strain $\epsilon_{yf}$

Figure-5 shows the variation of lateral strain at failure $\epsilon_{yf}$ with b. Both the lateral strains in conjucture with the axial strains have to respond to the volume changes taking place in the specimen. Therefore, the variation of $\epsilon_{yf}$ has to be considered keeping in view the variations in other strains.

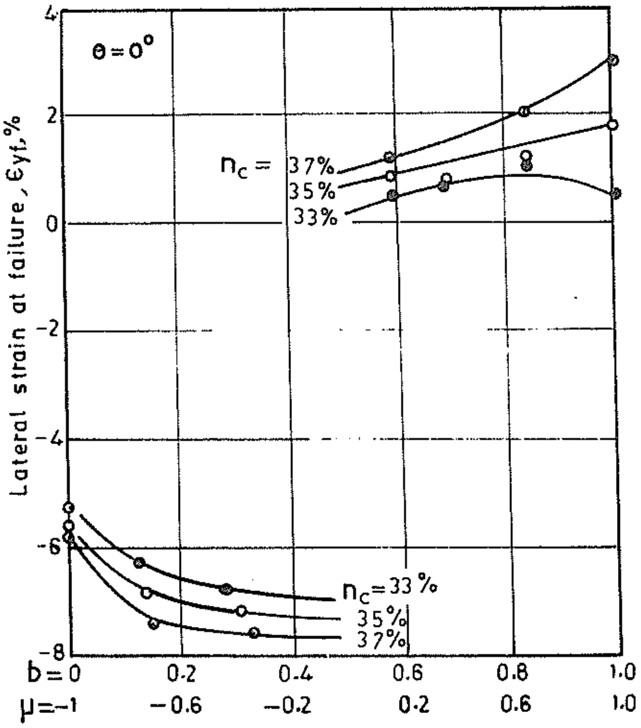

Figure 5 Lateral Strain $\epsilon_{yf}$

It was observed earlier that, in compression, as b increases $\epsilon_{xf}$ decreases tending to attain a zero value in plane strain condition. The axial strain is always compressive. Therefore most part of the dilation is taken care of by $\epsilon_{yf}$. Therefore, as b increases, $\epsilon_{yf}$ increases.

In extension as b decreases, $\epsilon_{yf}$ decreases approaching a zero value, that is a plane strain condition in extension. It will be interesting to compare the results of plane strains in compression and extension tests. As can be seen from Figures 4 and 5 that there exists a discontinuity in the relationship of linear strains Vs b between compression and extension test results. If compression and extension tests are conducted upto plane strain condition and beyond, the relationship of linear strains Vs b will be confirmed in the discontinuous region.

4.4 Axial Strains $\epsilon_{zf}$

The axial strain at failure is maximum under axisymmetric conditions both in extension and compression (Figure 6). With increasing deviation from axisymmetric conditions the axial strain at failure decreases. It may be concluded from this that the brittle nature of failure increases with increasing deviation from axisymmetric conditions.

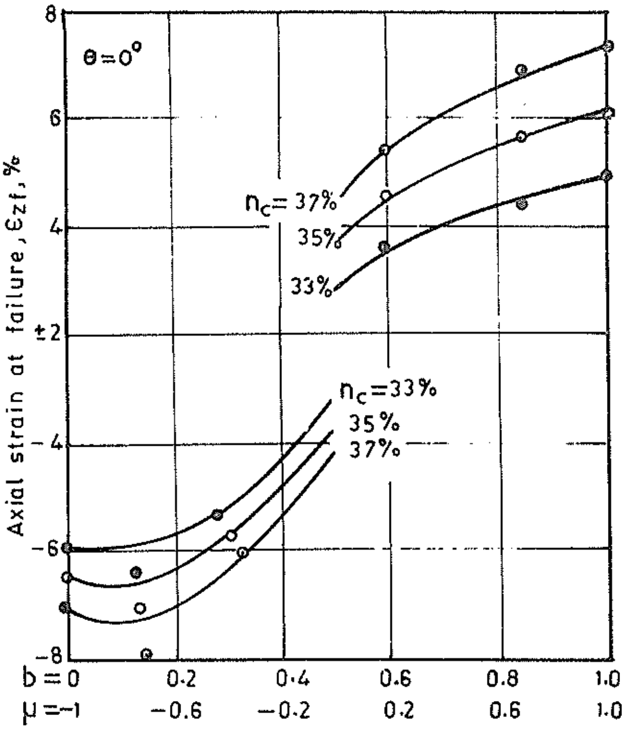

Figure 6 Axial Strain $\epsilon_{zf}$

4.5 Volumetric Strain $\epsilon_{vf}$

The material dilates at all porosities (Fig. 7). The dilation is maximum under axisymmetric conditions. With increasing deviation from axisymmetric conditions the volumetric strain at failure decreases. As has been mentioned earlier, the particles have maximum freedom of movement under axisymmetric conditions and hence produce higher dilation. As the deviation from this state increases, the freedom of the particles is restricted resulting in lesser dilation.

5 CONCLUSIONS

The angle of shearing resistance increases with increase in the value of b upto plane strain condition and then decreases continuously upto axisymmetric extension. However, the shape of the curve is not unique for all materials and under all type of test conditions. The structural anisotropy of the material effects the shape of the curve. The material exhibits maximum strength anisotropy under axisymmetric conditions and as the deviation from the axisymmetric conditions increases, the anisotropy decreases, ultimately vanishing at plane strain condition. With increase in the value of b, $\epsilon_{xf}$ decreases in compression culminating in plane strain condition when $\epsilon_x \approx 0$. In extension also $\epsilon_{xf}$ decreases with increase in b. $\epsilon_{yf}$ increases both in compression and

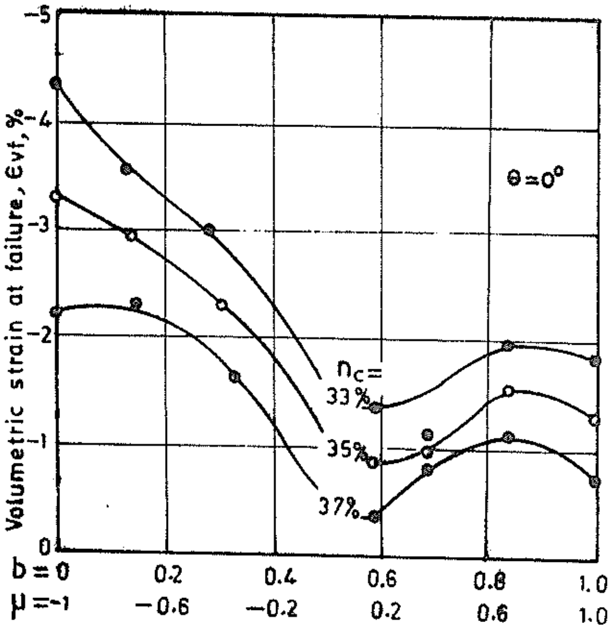

Figure 7 Volumetric Strain $\epsilon_{vf}$

extension with increase in the value of b. The axial and volumetric strains are maximum under axisymmetric conditions and with increasing deviation from axisymmetric conditions both of them decrease.

If General Compression and extension tests are carried out to reach plane strain conditions and beyond, the results of compression and extension in the overlapping region of b could be compared.

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